# Physics 5300, Theoretical Mechanics   Spring 2015

Quiz 2 Solutions

*Given: Friday Jan 23*

*Problem* 1   Write down the potential energy $U(\phi)$ of a simple pendulum (mass $m$, length $l$) in terms of the angle $\phi$ between the pendulum and the vertical. (Choose the zero of $U$ at the bottom.) Show that for small angles, $U$ has the Hooke law form $const + \frac{1}{2}kx^2$. What is $k$?

*Solution:*   The height of the pendulum above its lowest point is given by

$$h = l - l\cos\phi = l(1 - \cos\phi) \tag{1}$$

Thus we have

$$U = mgh = mgl(1 - \cos\phi) \tag{2}$$

We have

$$\cos\phi = 1 - \frac{\phi^2}{2!} + \dots \tag{3}$$

Thus we get

$$U = mg\frac{\phi^2}{2} \tag{4}$$

Writing this as $\frac{1}{2}k\phi^2$, we get

$$k = mgl \tag{5}$$

*Problem* 2   A mass on the end of a spring is oscillating with an angular frequency $\omega$. At $t = 0$, its postion is $x_0 > 0$ and I give it a kick so that it moves back towards the origin and executes simple harmonic motion with amplituide $2x_0$. Find its position as a function of time in the form

$$x(t) = A\cos(\omega t - \delta) \tag{6}$$

*Solution:*   We have

$$x(t) = A\cos(\omega t - \delta) \tag{7}$$

At $t = 0$, the postion is $x_0$. Thus

$$x_0 = A\cos(-\delta) = A\cos\delta \tag{8}$$

The amplitude is $2x_0$, thus

$$A = 2x_0 \tag{9}$$

Thus

$$\cos\delta = \frac{x_0}{2x_0} = \frac{1}{2} \tag{10}$$

Thus

$$\delta = \pm\frac{\pi}{3} \tag{11}$$

These two choices give

$$x = 2x_0\cos(\omega t + \frac{\pi}{3}) \tag{12}$$

and

$$x = 2x_0\cos(\omega t - \frac{\pi}{3}) \tag{13}$$

respectively. Now we compute the velocity at $t = 0$. In the first case we have

$$\dot{x}(t=0) = -2x_0\omega\sin(\frac{\pi}{3}) \tag{14}$$

while in the second case we have

$$\dot{x}(t=0) = -2x_0\omega\sin(-\frac{\pi}{3}) = 2x_0\omega\sin(\frac{\pi}{3}) \tag{15}$$

We are given that this initial velocity is negative (since it is towards the origin). Thus ew must take the first possibility, and we get

$$x(t) = 2x_0\cos(\omega t + \frac{\pi}{3}) \tag{16}$$