

Physics 5300, Theoretical Mechanics Spring 2015

Quiz 3

Given: Friday Jan 30

Problem 1 Find the equation of the path joining the point $O = (0,0)$ to the point $P = (1, -1)$ in the $x-y$ plane that makes the integral $\int_O^P (y'^2 + 3yy' + y^2 + 2y)dx$ stationary.

Solution: We wish to minimize

$$L = \int_O^P (y'^2 + 3yy' + y^2 + 2y)dx$$

The variational equation is

$$\frac{d}{dx}[(2y' + 3y)] - (3y' + 2y + 2) = 0$$

This is

$$2y'' - 2y - 2 = 0$$

or

$$y'' = y + 1$$

We write

$$z = y + 1 \tag{1}$$

getting

$$z'' = z \tag{2}$$

The solution has the form

$$z = Ae^x + Be^{-x}$$

Thus

$$y = z - 1 = Ae^x + Be^{-x} - 1 \tag{3}$$

At the origin O we have

$$0 = A + B - 1$$

which implies $A = -B + 1$. At P we have

$$-1 = Ae + Be^{-1} - 1$$

Thus

$$B = -Ae^2$$

Thus we have

$$A = Ae^2 + 1, \quad A(1 - e^2) = 1, \quad A = \frac{1}{1 - e^2}, \quad B = -Ae^2 = -\frac{e^2}{1 - e^2} \tag{4}$$