Physics 5300, Theoretical Mechanics   Spring 2015

Quiz 4

Given: Friday Feb 6

Problem 1   A mass m is suspended from a massless string, the other end of which is wrapped several times around a horizontal cylinder of radius R and moment of inertia I, which is free to rotate about a fixed horizontal axle. Using a suitable coordinate, set up the Lagrangian and Lagrange equation of motion, and find the acceleration of the mass m. [The kinetic enery of the rotatong cylinder is $\frac{1}{2}I\omega^2$.]

Solution: We use the vertical position z of the mass m as our coordinate; we orient z upwards.

The kinetic energy olf the mass is

$$T_1 = \frac{1}{2}m\dot{z}^2 \tag{1}$$

The kinetic energy of the cylinder is

$$T_2 = \frac{1}{2}I\omega^2 \tag{2}$$

We have

$$\dot{z} = R\omega \tag{3}$$

Thus

$$T_2 = \frac{1}{2}\frac{I}{R^2}\dot{z}^2 \tag{4}$$

Thus the kinetic energy is

$$T = T_1 + T_2 = \frac{1}{2}m\dot{z}^2 + \frac{1}{2}\frac{I}{R^2}\dot{z}^2 = \frac{1}{2}(m + \frac{I}{R^2})\dot{z}^2 \tag{5}$$

The potential energy is

$$V = mgz \tag{6}$$

Thus

$$L = T - V = \frac{1}{2}(m + \frac{I}{R^2})\dot{z}^2 - mgz \tag{7}$$

The Lagrange equation for z is

$$\frac{d}{dt}(\frac{\partial L}{\partial \dot{z}}) - \frac{\partial L}{\partial z} = 0 \tag{8}$$

which gives

$$(m + \frac{I}{R^2})\ddot{z} + mg = 0 \tag{9}$$

This gives

$$\ddot{z} = -\frac{mg}{m + \frac{I}{R^2}} \tag{10}$$

$$\dot{z} = -\frac{mg}{m + \frac{I}{R^2}}t + c \tag{11}$$

$$z = -\frac{1}{2}\left(\frac{mg}{m + \frac{I}{R^2}}\right)t^2 + ct + d \tag{12}$$