

Physics 5300, Theoretical Mechanics Spring 2015

Quiz 8

Given: Friday March 6

Problem 1 (a) Find the Poisson bracket of $f = p^2q + q^2$ and $g = pq$. (3 points)

(b) Suppose we wish to go from variables q, p to variables $Q(q, p), P(q, p)$. We set

$$Q = \frac{p}{1 - 3q} \quad (1)$$

Find a possible choice of P . (7 points)

Solution: (a) We have

$$\{f, g\} = \frac{\partial f}{\partial q} \frac{\partial g}{\partial p} - \frac{\partial f}{\partial p} \frac{\partial g}{\partial q} = (p^2 + 2q)(q) - (2pq)(p) = p^2q + 2q^2 - 2p^2q = 2q^2 - p^2q \quad (2)$$

(b) We use a generating function $F(q, Q)$. Then we have

$$p = \frac{\partial F}{\partial q} \quad (3)$$

$$P = -\frac{\partial F}{\partial Q} \quad (4)$$

We have

$$p = Q(1 - 3q) \quad (5)$$

The first relation gives

$$\frac{\partial F}{\partial q} = Q(1 - 3q) \quad (6)$$

This integrates to

$$F = Q(q - \frac{3q^2}{2}) + g(Q) \quad (7)$$

We set $g(Q) = 0$. This gives

$$F(q, Q) = Q(q - \frac{3q^2}{2}) \quad (8)$$

This then gives

$$P = -\frac{\partial F}{\partial Q} = -[q - \frac{3q^2}{2}] = \frac{3q^2}{2} - q \quad (9)$$

Alternatively, we can guess the form of P , and check:

$$\{Q, P\} = 0 - (\frac{1}{1 - 3q})(3q - 1) = 1 \quad (10)$$