Physics 7501 Quantum Mechanics Fall 2018

Quiz 1

Given: Friday, Aug 31, 2018 *Time: 20 minutes*

Consider an electron of mass m moving non-relativistically in the potential $V(r) = \alpha\, r$, where α is a positive constant.

(a) Solve Newton's laws for circular motion around $r = 0$, to get the energy E as a function of the radius r. (8 points)

(b) Apply Bohr's condition $L = n\,\hbar$ to get the allowed energy levels for the quantum problem. (5 points)

(c) Consider the transitions $n \to n-1$ for n large. Check whether these satisfy the correspondence principle or not. (7 points)

Solution: (a) We have

$$\frac{mv^2}{r} = \frac{\partial V}{\partial r} = \alpha \tag{1}$$

$$v = \sqrt{\frac{\alpha r}{m}} \tag{2}$$

$$E = \frac{1}{2}mv^2 + V = \frac{1}{2}\alpha r + \alpha r = \frac{3}{2}\alpha r \tag{3}$$

(b) We have

$$L = mvr = m\sqrt{\frac{\alpha r}{m}}r = \sqrt{m\alpha r^3} = n\hbar \tag{4}$$

$$r = [\frac{n^2\hbar^2}{m\alpha}]^{\frac{1}{3}} \tag{5}$$

$$E = \frac{3}{2}\alpha r = \frac{3}{2}\alpha[\frac{n^2\hbar^2}{m\alpha}]^{\frac{1}{3}} = \frac{3}{2}\frac{\alpha^{\frac{2}{3}}\hbar^{\frac{2}{3}}}{m^{\frac{1}{3}}}n^{\frac{2}{3}} \tag{6}$$

(c) We have

$$\Delta E \approx \frac{3}{2}\frac{\alpha^{\frac{2}{3}}\hbar^{\frac{2}{3}}}{m^{\frac{1}{3}}}\frac{d}{dn}n^{\frac{2}{3}} = \frac{3}{2}\frac{\alpha^{\frac{2}{3}}\hbar^{\frac{2}{3}}}{m^{\frac{1}{3}}}\frac{2}{3}n^{-\frac{1}{3}} = \frac{\alpha^{\frac{2}{3}}\hbar^{\frac{2}{3}}}{m^{\frac{1}{3}}}n^{-\frac{1}{3}} \tag{7}$$

Writing

$$\Delta E = h\nu_{emission} = 2\pi\hbar\nu_{emission} \tag{8}$$

we get for the frequency of emission

$$\nu_{emission} = \frac{1}{2\pi\hbar}\frac{\alpha^{\frac{2}{3}}\hbar^{\frac{2}{3}}}{m^{\frac{1}{3}}}n^{-\frac{1}{3}} \tag{9}$$

The time period of rotation is

$$T = \frac{2\pi r}{v} = 2\pi r\sqrt{\frac{m}{\alpha r}} = 2\pi\sqrt{\frac{m}{\alpha}}r^{\frac{1}{2}} = 2\pi\sqrt{\frac{m}{\alpha}}[\frac{n^2\hbar^2}{m\alpha}]^{\frac{1}{6}} \tag{10}$$

Thus the frequency of rotation is

$$\nu = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{\alpha}{m}}[\frac{n^2\hbar^2}{m\alpha}]^{-\frac{1}{6}} \tag{11}$$

We see that

$$\nu_{emission} = \nu \tag{12}$$

so the correspondence principle is satisfied.