Name: _____

Physics 7501 Quantum Mechanics Fall 2018

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Quiz 4

Given: Wed, Oct 31, 2018 *Time: 20 minutes*

Problem 1: Given that $[\hat{x}, \hat{p}] = i\hbar$, compute

$$[\hat{x}^2\hat{p}, \hat{p}] \tag{1}$$

Solution: We have

$$[AB, C] = A[B, C] + [A, C]B \tag{2}$$

Thus

$$[\hat{x}^2\hat{p}, \hat{p}] = \hat{x}^2[\hat{p}, \hat{p}] + [\hat{x}^2, \hat{p}]\hat{p} = [\hat{x}^2, \hat{p}]\hat{p} \tag{3}$$

But

$$[\hat{x}^2, \hat{p}] = \hat{x}[\hat{x}, \hat{p}] + [\hat{x}, \hat{p}]\hat{x} = 2i\hbar\hat{x} \tag{4}$$

Thus we get

$$[\hat{x}^2\hat{p}, \hat{p}] = 2i\hbar\hat{x}\hat{p} \tag{5}$$

Problem 2: Consider the Hamiltonian

$$\hat{H} = \frac{\hat{p}^2}{2m} + \alpha\, t^2\hat{x} \tag{6}$$

where α is a constant.

(i) Find $\frac{d}{dt}\langle\hat{x}\rangle$ in terms of $\langle\hat{x}\rangle, \langle\hat{p}\rangle$.

(ii) Find $\frac{d}{dt}\langle\hat{p}\rangle$ in terms of $\langle\hat{x}\rangle, \langle\hat{p}\rangle$.

(iii) Find $\frac{d}{dt}\langle\hat{H}\rangle$ in terms of $\langle\hat{x}\rangle, \langle\hat{p}\rangle$.

Solution: (i) We have

$$\frac{d}{dt}\langle\hat{O}\rangle = -\frac{i}{\hbar}\langle[\hat{O}, \hat{H}]\rangle + \langle\frac{\partial\hat{O}}{\partial t}\rangle \tag{7}$$

Thus

$$\frac{d}{dt}\langle\hat{x}\rangle = -\frac{i}{\hbar}\langle[\hat{x}, \hat{H}]\rangle = -\frac{i}{\hbar}(i\hbar)\frac{1}{2m}2\langle\hat{p}\rangle = \frac{\langle\hat{p}\rangle}{m} \tag{8}$$

(ii)

$$\frac{d}{dt}\langle\hat{p}\rangle = -\frac{i}{\hbar}\langle[\hat{p},\hat{H}]\rangle = -\frac{i}{\hbar}(-i\hbar)\alpha t^2 = -\alpha t^2 \tag{9}$$

(iii)

$$\frac{d}{dt}\langle\hat{H}\rangle = +\langle\frac{\partial\hat{H}}{\partial t}\rangle = \langle 2\alpha t\hat{x}\rangle = 2\alpha t\langle\hat{x}\rangle \tag{10}$$