1. Overview

In Newton's description of the laws of physics, te motion of bodies is governed by the forces acting upon them. These forces could be of various kinds: electromagnetic, frictional, gravitational etc. But one fact distinguishes gravity from all other forces: the fact that the *inertial mass* of any body equals the *gravitational mass* of the body. The inertial mass m_I determines the acceleration of the body under a given applied force, in Newton's description of the laws of motion:

$$\vec{F} = m_I \vec{a} \tag{1.1}$$

The gravitational mass m_G on the other hand determines what *force* the body feels due to the gravitational pull of other bodies:

$$\vec{F} = -m_G \vec{\nabla}\phi \tag{1.2}$$

$$\triangle\phi = -4\pi G\rho \tag{1.3}$$

where ρ is the (gravitational) mass density arising from of all matter other than the particle under study. Experiment shows that the ratio m_I/m_G is a constant independent of the material composing the accelerating body, to within the error of any measurement performed to this date. Assuming this ratio to be an exact constant, we can choose G, the Newton constant of gravitation, such that we set

$$\frac{m_I}{m_G} \equiv 1 \tag{1.4}$$

and write $m_I = m_G = m$. Then the motion of a particle subject to only the force of gravity is described by

$$\vec{a} = -\vec{\nabla}\phi \tag{1.5}$$

We observe that all particles, regardless of their mass, follow the same trajectory if they start at the same position with the same velocity. This circumstance suggests that we attempt a geometric description of the force of gravity along the following lines.

Consider making a graph describing the motion of particles in one dimension, plotting the position x as a functiuon of the time t. If no force acts on the body, the plot will be a straight line, with a slope equalling the velocity of the particle. In particular if two particles start off from $x = 0$ with opposite velocities, as shown in Fig. 1, then their trajectories will not meet again. On the other hand, if there was an attractive force between the particles, then their trajectories on this plot will curve (Fig. 2), and if the attraction is strong enough, the particles will meet again.

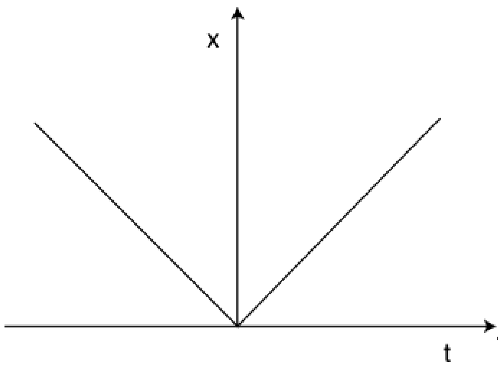

Fig. 1

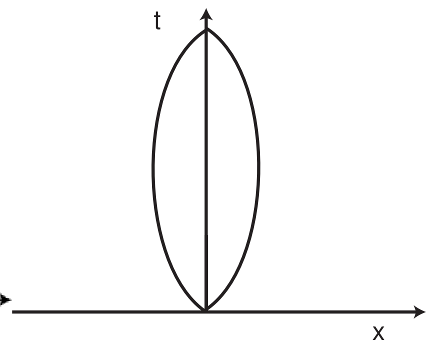

Fig. 2

From this plot it appears that the situation with force is quite different from the situation with no forces: the former gives straight line moton, while the latter does not. But consider instead of the flat t, x plane the 2-dimensional surface of a sphere, shown in Fig. 3. We can define 'straight' lines on the surface of the sphere: they are great circles, like the lines of constant longitude on the surface of the earth. These lines are straight in the sense that along each infinitesimal segment of such a line we move 'straight' according to what appears straight line motion on an infinitesimal patch of the surface around the segment. Such a notion of straight lines on curved space can be made rigorous. The lines are called *geodesics*. But now observe that two such 'straight' lines on the sphere, starting from the same point, meet again after traversing a certain distance, so that their behavior resembles Fig. 2 rather than Fig. 1.

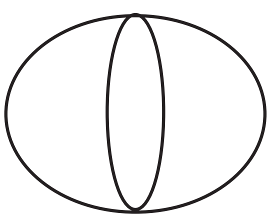

Fig. 3

We wish to describe the effect of gravity in the following way. Gravity is not to be a force in the usual sense, in that the motion of particles would be along 'straight' lines in spacetime. But the spacetime would be not a flat t, x plane, but a curved space, with the curvature reproducing the properties of the particle trajectories arising from the effect of gravitation. Note that such a description, if at all possible, requires the equality of inertial and gravitational mass: if different particles followed different paths in spacetime then we would need a differently curved spacetime to describe the motion of each kind of particle, and it would not be helpful to encapsulate the effect of gravity in this geometric manner.

Einstein's theory of gravity incorporates this geometric description of the graviational force. The geometric approach requires that space and time be treated on a similar footing, which appears very natural once we accept the lessons of special relativity, which Einstein had been instrumental in developing some years earlier. Newton's laws do not obey the postulates of special relativity - space and time are quite different entities. After the development of general relativity Cartan attempted to put Newton's law of gravitation on a geometric footing in the same spirit, but it was found that a similar unified geometric structure does not work in this case.[1]

Saying that particles move along geodescics in the curved spacetime gives the equivalent of (1.5), but we still need an equivalent of (1.3), which says how much gravitational field actualy arises from a given distribution of matter. In the geometric description this is tantamount to asking how much curvature is produced by a given amount of matter. This relationship is contained in *Einstein's equations* , which relate the curvature of spacetime to the energy and momentum of the matter: due to the relativistic nature of the theory it is this energy and momentum rather than the mass which actually determines the curvature. These properties of the matter are contained in the *energy-momentum tensor* which is alternatively called the *stress energy tensor*.

[1] For a review of Cartans attempt and the contrast between the geometric approach applied to general relativity and Newton's laws see MTW chap. 12.