

Confusions and questions about the information paradox

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The quantum theory of black holes has proved to be an extremely confusing subject, especially for students trying to learn what has been achieved in the field. One of the principal difficulties arises from the fact that to understand Hawking's paradox one must know both general relativity and field theory, and to understand the resolution of the information paradox one must also have a grasp of string theory. In lecturing about black holes I have found a large body of confusions that are common to many students. In these notes I have tried to describe these confusions (and my answers to them) in a discussion format. I hope this method of presentation will be useful in bringing out the relevant issues.

1 AdS/CFT and the information paradox

A common statement that I have heard is the following: *We can make a black hole in AdS. Since AdS is dual to a CFT, and the CFT is unitary, there cannot be any information loss, and so there is no information paradox to solve in string theory.*

This is a completely circular argument. AdS/CFT duality is arguably one of the most interesting insights to emerge from string theory. It is also a very useful tool in understanding black hole behavior. But we cannot simply invoke this duality to bypass the information paradox. Since this is a very common confusion among students of string theory, we present it as the following discussion:

Student: I don't see why I should worry about Hawking's paradox. Now that we know that gravity is dual to a CFT, and the CFT is unitary, there cannot be any information loss, and so there is no problem.

Hawking believer: That is an entirely circular argument, as I can easily show. Suppose I say: Quantum mechanics is unitary, so there can be no information loss. Would I have resolved Hawking's paradox?

Student: No, that would be silly. Hawking agrees that quantum mechanics is valid in all laboratory situations. All he argues is that once we make a black hole, then quantum

mechanics is violated. So we cannot use our tests of quantum mechanics in the everyday world to argue that there will be no problem when black holes form.

Hawking believer: Good, that is correct. So now let me ask the same question about AdS/CFT. You have computed the spectrum, 2-point functions, 3 point functions etc. and found agreement between the CFT and gravity descriptions. I understand that you have numerous such computations. But these processes do not involve black hole formation, and so do not address Hawking's argument. Is that correct?

Student: Yes, that is correct. But we also have a black hole solution, called AdS-Schwarzschild, which is similar to the standard Schwarzschild metric in its essential respects.

Hawking believer: Excellent. So I will now apply the Hawking theorem, proved in arxiv/0909.1038, and *prove* that normal assumptions about locality gives mixed states/remnants. Since your black hole has an 'information free horizon' just like the Schwarzschild hole, my arguments go through in exactly the same way. Thus you have three choices: (a) You can tell me why local Hamiltonian evolution breaks down under the niceness conditions N listed in that proof (b) you can agree to mixed states arising from pure states, which violates quantum theory; in that case you lose AdS/CFT and string theory as well, since these are built on a foundation of usual quantum theory (c) You can agree to have remnants in your theory, and explain why they do not cause the problems that people feared. Now which will it be?

Student: I don't know ... I see that you have forced me into a corner by using the Hawking theorem, and I will have to work as hard to solve it in my AdS case as I would have had to in the usual asymptotically flat case. So let me try to evade the problem by trying a different argument. I will use the CFT to *define* my gravity theory. Then I will get a gravity theory that has the expected weak field behavior, and I will never violate quantum mechanics, and I can never get information loss.

Hawking believer: Excellent. With this definition of your gravity theory, you will by construction never have the 'mixed state' possibility in Hawking's theorem. So now tell me: (a) Will you claim that traditional black holes do not form in this gravity theory (b) The black hole horizon forms, but the niceness conditions N do not give locality; in this case you should be sure to tell me how this happens and what niceness conditions you will add to recover conditions for the solar system limit (c) Do neither of the above but say that the theory has long lived remnants.

Student: Well ... I always assumed that I could have a normal black hole horizon, usual notions of niceness conditions, and still get all the information out in Hawking radiation so there are no remnants. But I see now that the Hawking theorem forbids exactly this

possibility. I don't know how I can say anything about the options you list without studying the black hole formation/evaporation process in detail in either the CFT or the gravity theory.

Hawking believer: Exactly. You are welcome to do your analysis in either the CFT or the gravity theory, but at the end you must show me what happens when a black hole forms and evaporates in the gravity description.

Student: I see now that to solve Hawking's paradox I will have to understand the interior structure of the black hole. I cannot get by with any abstract arguments like 'AdS/CFT removes the paradox.'

Hawking believer: Exactly; in fact abstract arguments in general cannot distinguish between whether locality broke down and information came out in the Hawking radiation or if information leaked out from a long lived remnant. Solving the information paradox implies that you tell us which happens, and if you want the information to come out in the radiation, to show explicitly the process by which 'solar system physics' broke down while the niceness conditions N were still valid.

2 Hawking's puzzle

Many people do not understand the fuzzball proposal, mainly because of an unfamiliarity with Hawking's original proof of information loss. In this section we recall (in dialogue form) what the information loss problem really is.

2.1 The puzzle

New discussion

Student: What is the fuzzball proposal and how does it solve the information paradox?

Fuzzball person: To understand the proposal you first have to understand the information paradox itself very clearly. Have you worked through Hawking's derivation of this paradox?

Student: No, I have not. What are the main steps?

Fuzzball person: There are three things that you need to understand. The first is the fact that the geometry of a collapsing shell can be foliated by a set of 'good slices' which satisfy all the same smoothness conditions that arise for physics in a lab here on earth. Thus if someone claims that evolution in the traditional black hole geometry is different

from the evolution in a lab, then he will have to find a concrete reason to distinguish what is happening in the black hole from what is happening in the lab.

Student: That's fine with me ... I am happy to agree that in the traditional picture of gravitational collapse there is nothing happening at the horizon; its just like any other region of empty space.

Fuzzball person: Its not that simple. What you have to see is that there is a slicing that is good everywhere, and still catches (at low energy as measured on the slice) all three things: the infalling shell, the infalling members of the Hawking pairs, and the outgoing Hawking pairs carrying say 90% of the shell mass. You can see the details in [3].

Student: Okay, I will have to work through the construction of those slices. What is the second thing?

Fuzzball person: The second point is that this Hawking evaporation process leads to the formation of state with entanglement between the region inside $r = 2M$ and outside $r = 2M$. It is crucial that you be able to write down explicitly this entangled state, and see exactly what is entangled with what. This computation can be found in Hawking's original paper, or in the review [3]. With this state, one finds that when the hole evaporates, one will either have remnants or loss of unitarity.

Student: Okay, I see that I will have to learn how to write down that state. But I always thought that Hawking did a leading order computation, and careful corrections to his computation might encode the information in the outgoing radiation.

Fuzzball person: This is the third important thing to understand about Hawking's argument. Such small corrections *cannot* bring the information out in the radiation; if it could, there would have been no paradox to worry people for 30 years.

Student: I dont understand ... maybe all the small corrections that people tried to find so far did not work, but there may be others, and once we find those, we would find the information comes out. Is that incorrect?

Fuzzball person: Yes, that is incorrect. One makes the Hawking argument rigorous as follows. First one has to make precise the definition of the horizon in the traditional black hole geometry. You have said that this is a place that looks to leading order as if it was a region of gently curved spacetime. To make this rigorous, let ψ_i be states of low energy quanta; i.e. quanta with wavelength $l_p \ll \lambda \lesssim 2M$. Let the evolution of these quanta over time intervals of order $\lesssim 2M$ be given by the matrix elements $\langle \psi_i | H | \psi_j \rangle$. Let $\langle \psi_i | H_0 | \psi_j \rangle$ be the corresponding matrix elements when the computation is done in the semiclassical approximation of quantum fields on curved space, or any other scheme that one may adopt

for quantum physics in gently curved spacetime. Then we say that we have a traditional horizon if

$$\langle \psi_i | H | \psi_j \rangle = \langle \psi_i | H_0 | \psi_j \rangle + O(\epsilon), \quad \epsilon \ll 1 \quad (1)$$

Student: I understand this ... you are just making precise the fact that evolution in the traditional black hole geometry around the horizon region is the same as the evolution in gently curved space, upto small corrections which may arise from the fact that we are in the black hole geometry.

Fuzzball person: Exactly. Eq. (1) makes precise the property of the horizon region in the traditional picture of the black hole. We are talking of low energy modes, like those in the lab, evolving for lab timescales, in a region with the same curvature as that on earth. If you dont write (1), then you have not quantified the fact that the neighbourhood of the horizon is ‘empty space’. Now you come across the important final point: If (1) holds in the neighbourhood of the horizon, then you will *necessarily* have either remnants or information loss.

$$Eq.(1) \rightarrow \text{remnants/information loss} \quad (2)$$

2.2 Analyzing (2)

Discussion continued

Student: I see, that is indeed a serious problem. But how could you prove (2) without knowing the details of all the possible small corrections ... is it not possible that some small correction can bring the information out?

Fuzzball person: No, it is not possible. The relation (2) was not proved in Hawking’s original paper in quite this form, but a proof is given in [4]. The members of the Hawking pair are produced in an entangled state, which adds say $\ln 2$ to the entanglement entropy with the production of each new pair. If (1) is true, then one finds that this increase in entanglement is $> \ln 2 - 2\epsilon$. Thus the entanglement cannot go down ever, and thus the information cannot emerge in the Hawking radiation.

Student: I guess I always believed that the $O(\epsilon)$ corrections in (1) can cumulate to a significant value since there are so many quanta being emitted, and so ultimately the information will be able to get delicately encoded in the outgoing photons ... what do I have to do to see that this cannot happen?

Fuzzball person: The essential ingredient in the physics is the fact that each new pair is produced in the same state Ψ upto corrections of order ϵ . The essential mathematical step is the use of a pair of relations from quantum information theory called ‘strong subadditivity relations’. (The text of Nielsen and Chuang notes that there is no elementary proof known

for these relations, so while they are very well known and widely used relations, one may not be invoking their power in intuitive thinking; this may be one reason for the common belief that information can be delicately encoded in the $O(\epsilon)$ corrections in (4).)

Student: Okay, I will work through that proof. But given this proof, have you not established that if I ever make a black hole with horizon (as defined rigorously in (1) then I will necessarily violate unitarity (or at best get remnants)?

Fuzzball person: Exactly, that is the the statement (2) which we wrote above.

2.3 Relation to the work of Page

Student: Some years ago Don Page had analyzed the nature of radiation from a burning piece of coal, and argued that at first the entanglement between the radiated photons and the coal will go up, and then (after the halfway point) it will go down, so at the end when there is no coal left, the photons in the radiation are entangled only with themselves. As a by-product of this computation one can deduce that it will be impossible to detect the information in the radiation by looking at a small sample of the photons; the information is delicately encoded in correlations between the complete set of *all* photons. What is the relation between these statements and what you are saying about information loss in the evaporation of black holes?

Fuzzball person: Page quantified a fact which was intuitively obvious about radiation from hot bodies: at first the emitted photons will obviously be entangled with the emitting body, but this entanglement will have to start decreasing when there are insufficient degrees of freedom left in the burning object: the photons emitted in this later phase correlate with the state of the coal left after the initial emissions, and so the early photons manage to correlate with the later ones, as they must if the coal burns away completely in a unitary quantum process. Hawking of course knew this obvious fact, and there would have been no information paradox if the black hole radiated like any other burning object. The proof in [4] establishes Hawking's argument rigorously, by proving that for Hawking radiation the entanglement entropy does *not* go down after the halfway point; in fact it increases by $> \ln 2 - 2\epsilon$ in each emission until the hole becomes planck size.

Student: So I cannot use Page's arguments to help me in any way with the information paradox?

Fuzzball person: You cannot. Page is describing a normal burning body, and the difference between this case and the case of a black hole is explained clearly in [4]. Page's original paper noted that a black hole was essentially different from a piece of coal, and some new effect (like wormholes) would be needed to get information out of the black hole.

But his paper has been used sometimes to make the following incorrect argument: (a) *Correlations between photons are delicately encoded for normal burning bodies, where information is of course preserved* (b) *Hawking’s computation must always have some delicate corrections which he missed* (c) *Thus these delicate corrections can encode the information in the radiation, and so there will be no information problem.* This argument is incorrect, since as we saw above small corrections to the Hawking process do not make black hole evaporation unitary.

3 What does the fuzzball program do?

New Discussion

Student: Now that I understand the strength of the information paradox, let me return to the fuzzball proposal. What does the proposal say?

Fuzzball person: The traditional belief was that the black hole has a horizon which is empty space; a concept which we have made rigorous in (1). The fuzzball proposal says that that this is false: *there are order unity corrections to the evolution of low energy modes at the horizon.* Technically,

$$\langle \psi_i | H | \psi_j \rangle \neq \langle \psi_i | H_0 | \psi_j \rangle + O(\epsilon), \quad \epsilon \ll 1 \quad (3)$$

Student: Okay, I see that this is a very concrete statement, but you have proved to me that if I don’t agree to this then I will have remnants/information loss (eq. (2)). Thus I seem to have no choice: if I want to preserve quantum mechanics, I have to agree to the fuzzball proposal, so why is the proposal not the same as just saying that we want quantum mechanics preserved in black holes?

Fuzzball person: Because the power of the proposal is not just in its statement: the fuzzball program performs an explicit *construction* of black hole microstates which are seen to satisfy (1). In fact we now have examples of nonextremal microstates where the radiation is explicitly seen to carry out the information of the microstate [37, 38, 39].

3.0.1 Fuzzballs and the ‘no hair theorems’

Discussion continued

Student: But why did people not write down such microstate solutions before? As you say, if they had a horizon they would have information loss, so the obvious thing to do would be to look for microstates without horizons.

Fuzzball person: People did indeed try to find black hole solutions where the horizon region carried information about the microstate, and was thus different from the vacuum. This program was called searching for ‘black hole hair’. They performed this search by looking at the linearized perturbations around the black hole background, and asking for finite energy excitations regular everywhere. It turned out that requiring the perturbation to die off at infinity always made the energy density diverge at the horizon, so no ‘hair’ could be found for scalar, spinor, vector or graviton fields.

Student: That looks pretty conclusive ... so black holes must have a horizon satisfying (1) and we will have information loss?

Fuzzball person: No, because the fuzzball program finally found this hair. The earlier searches were just not looking in the right place: the construction turns out to be nonperturbative in the concerned fields, and so could not be found from looking at the linearized solutions.

3.0.2 The nature of the ‘hair’ found in the fuzzball program

Discussion continued

Student: That is interesting ... can you describe the structure of this hair in some more detail?

Fuzzball person: Here is the nature of the simplest microstates. There are compact circles in the geometry, and the circles fiber nontrivially over the noncompact direction to make a KK monopole structure. We actually get this KK monopole tensored with a S^1 in the noncompact directions, so that the overall KK charge is zero; thus we call it a ‘dipole charge’. Different shapes of this S^1 describing the dipole charge correspond to different microstates. The compact circles give U(1) gauge fields A_μ by the normal process of dimensional reduction. People knew that of course, and so they did not worry much about including compact directions in their study of black holes; they assumed that if they have looked for gauge field hair and found none, then compact circles cannot help. But the KK monopole is a *magnetic charge* for this U(1) gauge field, and is an essentially nonperturbative construction in the metric. In fact if we dimensionally reduce to just the noncompact dimensions then the metric would be singular; it is smooth only in the full 10-d spacetime. Thus this ‘hair’ found in the fuzzball constructions was missed in the earlier search for hair, where it was implicitly assumed that if no perturbative deformations could be found then probably no deformations existed at all.

Student: Okay, I understand what the hair look like. But given how much work had been done with gravity solutions, why could people not write such nonperturbative solutions before?

Fuzzball person: Yes, one could write such solutions without much difficulty. But if one writes the simplest microstates, they do not look much like the black hole, and so people had no reason to identify them with black hole microstates. If one writes more complicated states, then their exterior geometry looks more like that of the spherically symmetric black hole, but then these are complicated geometries, and people had no obvious reason to write them. This is where string theory came to the rescue. We can list all the states of the black hole, as bound states of branes. We can then start with the simplest such state, and make its gravity description: this will give the simplest fuzzball, which may be quite different from the generic state. We can then move on to more generic states, with the assurance that what we are looking at are microstates of the hole and not just some arbitrary solutions of string theory.

3.0.3 Moving from simple microstates to more complex ones

Discussion continued

Student: Could you make it clearer what you mean by ‘simple microstates’ and ‘more generic microstates’?

Fuzzball person: Yes, the situation is exactly analogous to the case of photons in a box. The simplest state is one where we have all photons in the lowest energy mode, say $\vec{k} = \frac{2\pi}{L}\hat{x}$. Will this state be spherically symmetric?

Student: No of course not; it is one of the microstates of the radiation in the cavity, but it is very special. For example, there will be pressure in the \hat{x} direction, but not in \hat{y}, \hat{z} .

Fuzzball person: Exactly. The 2-charge black hole arises for the D1D5 system, which can be described by a set of ‘component strings’. The simplest state of this system has all the component strings with in the same mode: the mode with smallest winding number. So it is analogous to simple mode of radiation above. The geometry for this D1D5 state has the KK monopole tube in the shape of an exact circle.

Student: I see that I can now move to more generic states of radiation in the cavity by distributing the energy among different harmonics. This will lead towards a generic configuration of black body radiation, which will be spherically symmetric in a ‘coarse grained sense’. Will this happen also from the D1D5 microstates.

Fuzzball person: Yes, that is exactly what will happen. The component strings get a distribution over different possible winding numbers, and as we go to more complicated distributions, the gravity solution has the S^1 in a more and more convoluted shape. As these convolutions increase, the stringy corrections (which are always there for *any* solution) become more and more important, till in the limit of the generic state we just have a ‘quantum fuzzball’.

3.0.4 What needs to be shown to resolve the information paradox?

Discussion continued

Student: That is quite clear; I see why you call the generic state a quantum fuzzball, and I see that it will have all kinds of stringy effects in it. But if you don't yet have a good mathematical description of these stringy effects, what can we say about the information paradox?

Fuzzball person: Let me ask the following. The people looking for 'hair' by solving linearized equations were looking for modes with spherical harmonic orders $l = 0, 1, 2, 3, \dots$. Suppose they had found such hair. Would you complain that they have not solved the paradox because they did not work at harmonics high enough that they would have a wavelength of order Planck scale?

Student: No, that would be silly. If they had shown that there are degrees of freedom at the horizon that can carry information about the hole, then they would have solved Hawking's problem. It would still be true that the generic harmonic needed to carry the information about the hole would have oscillations at the Planck scale, and for such modes quantum gravity corrections would be important, but I certainly would not demand a computation of such high frequency modes: if I find that gravity perturbations exist for $l = 0, 1, 2, \dots$ then I would assume that the set of all 'hair' will be adequate to get the information out. After all, we don't think there is an information loss problem for radiation from a planet, even though we never write down the full detailed quantum state at the surface of the planet.

Fuzzball person: Exactly; I am glad you understand this point. The same situation holds for fuzzballs. Once we have constructed simple fuzzball states, and seen that these do not possess a horizon with vacuum in its vicinity, the 'boot is on the other leg'; if somebody still wants to argue that there is an information problem then *he* has to show that there are other states of the hole that *do* have a horizon with vacuum in its vicinity. It is not the task of the fuzzball person to keep finding higher and higher order corrections to more and more general states in order to resolve the information problem. It should be understood that we are resolving a *paradox*; and the paradox arises only because earlier efforts could find *no* way of getting data about the microstate at the horizon. All microstates that have ever been constructed have turned out to be fuzzballs. So if someone wants to argue against the fuzzball resolution of the paradox then it is up to *him* to find a counterexample by constructing a valid solution in string theory which arises from a bound state of branes and has a horizon satisfying (1).

Student: I understand this point: the Hawking puzzle arose only because people could find *no* way of having data about the hole at the horizon; once you have shown that states

in actual string theory do have such data, then there is no paradox: the black hole is just like any other planetary body. Can I still ask though if the surface of the black hole is ‘hard’ or ‘soft’ for an object falling on it?

Fuzzball person: You can ask the question, as long as you understand that it has nothing to do with resolving the information problem. Talking of planets, some have a hard surface like earth, and you will die if you fall on it. Some are gaseous, and you will just fall through towards the center. In neither case do we think there is an information problem. Resolving the information problem for black holes requires us to show that low energy modes ($E \sim kT$) can be affected by order unity compared to the evolution expected in vacuum. What you are now asking about is the *infall* problem: what is the dynamics of heavy objects ($E \gg kT$) over the crossing time. There have been several attempts to extract this behavior from coarse graining over fuzzballs, and initial indications are that the object falls through the fuzzball surface much as it would fall in the traditional black hole. But note that this question has nothing to do with the real question of importance: can the outgoing Hawking modes be affected to order unity?

Student: Okay, I agree that once fuzzball solutions have been found for a subclass of states, the paradox is gone; if somebody still wants to claim that there is an information problem then he has to take the trouble to show that there are other states of the hole which do *not* transfer information to low energy outgoing Hawking modes.

4 Classical solutions vs quantum fuzzballs

Many people are confused about what a classical solution is, and what its relation may be to black hole physics. Part of this confusion stems from the fact that more than one definition of the word has sometimes been used, while part stems from a general confusion among students about how classical limits arise in quantum field theory. Thus we will try to separate these definitions, while at the same time recalling the basic facts about classical and quantum solutions in quantum field theory.

4.0.5 The classical limit in quantum theory

Student: In fuzzball papers I usually see a metric solution written like $ds^2 = \dots$. So I think of fuzzballs as classical. On the other hand I am told that fuzzballs are quantum fuzzy objects in general (hence their name). How do I understand the relation between these two things?

Fuzzball person: You first need to understand the classical limit of quantum states in ordinary quantum theory. So first tell me: can you write the classical solution describing the ground state of a Harmonic oscillator?

Student: Of course I cannot! the ground state is a quantum wavefunction $\psi(x) \sim e^{-\mu x^2}$, with a certain width $\mu^{-\frac{1}{2}}$. That is the best that I can say.

Fuzzball person: Correct. But this harmonic oscillator describes a pendulum, and there must be a classical solution to describe the pendulum motion $x = A \cos(\omega t)$. What is the relation of this to the wavefunction you wrote?

Student: Well, I look at high energy states $|n\rangle$ of the oscillator. To describe most of the essential physical properties of these states, I can use linear combinations called coherent states. These coherent states are made as $e^{\mu(t)a^\dagger}|0\rangle$. Then one finds that $\mu(t) = Ae^{i\omega t}$, and $\langle x \rangle = \text{Re}[\mu] = A \cos \omega t$, so then we see the connection to the classical solution.

Fuzzball person: Good. So suppose I ask you something simple and approximate about the ground state, like what is its approximate spread. Would you rush to your full quantum wavefunction, or can you use the classical solution in any way?

Student: For the qualitative answer I can use the classical solution: From the classical solution I will find the mean x^2 as a function of the energy E , and then extrapolate to the case where the energy is $\frac{1}{2}\hbar\omega$. Even though the ground state is not classical at all, I will get a reasonable qualitative estimate for $\langle x^2 \rangle$ without finding the exact quantum wavefunction.

Fuzzball person: Good, you will be doing something similar for fuzzballs soon. But first I want to check that you understand how to do the same thing with field theory. Can you write a classical description of one photon?

Student: No, of course not. That is a very quantum state, and I do not know how to describe it except to write a ket $|a_\mu(\vec{k})\rangle$, giving its polarization and wavenumber.

Fuzzball person: Let's explore this further. What is the relation of the photon to a classical wave $\vec{E} = \cos(kz - \omega t)$?

Student: Each mode of the electromagnetic field is a harmonic oscillator. I will take a lot of photons in the same mode. Then I will make a coherent state for this mode just like for the harmonic oscillator above, and this state will have well defined $\langle \vec{E}(z, t) \rangle$, just like the large amplitude states of the harmonic oscillator has well defined $x(t)$.

Fuzzball person: Good. Now if somebody tells you that $\vec{E} = A \cos(kz - \omega t)$ for the laser beam, then will you say that this is a classical solution and there are no quantum effects?

Student: No, of course not. This classical \vec{E} just gives the location of the peak of the wavefunctional, and I will add a Gaussian profile around this peak to get the leading order

quantum wavefunction. Writing in terms of the vector potential, I will write the relation between the classical function you give and its actual wavefunctional as

$$A = A_0 \cos(kz - \omega t) \leftrightarrow \Psi[A] = e^{-\alpha[A(k) - A_0 \cos(\omega t)]^2} \prod_{k' \neq k} e^{-\alpha[A(k')]^2} \quad (4)$$

Fuzzball person: Its good you understand this relation between what one writes classically and what it means for the actual state of the system. So now tell me, what did you mean by asking if fuzzballs are classical metrics or quantum wavefunctionals?

Student: I now see that my question was silly. I should apply the map (4) to the metric written in any fuzzball paper, and thereby understand what I am being told about the state. So I should look at the full quantum supergravity fields and find their wavefunctionals.

4.0.6 The impossibility of separating supergravity from stringy degrees of freedom

Discussion continued

Fuzzball person: Good, but let me ask further. Suppose the metric given to you has a pair of KK monopoles close to each other. We know that there is a configuration of the M2 brane which can wrap the S^2 extending between the centers of these two KK monopoles. Won't the full wavefunctional include virtual pairs of these M2 branes?

Student: Yes, I see no way to avoid that. As the KK monopoles come closer together, the effects of these brane fluctuations and that of all other possible string modes will make one giant wavefunctional, and I see no way to look at any simple projection that will encode only supergravity fields.

Fuzzball person: Good. So the only real statement about fuzzball states is that they are wavefunctionals in the full string theory, not wavefunctions in supergravity or classical solutions of the supergravity equations. But are there some solutions that are better approximated by the classical solution than others?

Student: Of course, even in the electromagnetic case this happens. Suppose I have radiation in a box. The simplest state is a laser-like one where all the energy is in one mode. This has well defined \vec{E}, \vec{B} , even though there are quantum fluctuations $\sim \frac{1}{\sqrt{N}}$ where N are the number of photons in the mode. The next simpler states have the energy split over two fourier modes, so that the fluctuations are slightly higher. Proceeding this way, finally I come to the generic state which has just ~ 1 quantum per mode, and gives black body radiation.

4.0.7 How we use the known fuzzball states

Discussion continued

Fuzzball person: That is correct. So let me ask the same question which I asked for the harmonic oscillator. If you write down classical solutions for the electromagnetic field, can you use these to extract qualitative properties of the black body radiation state like the pressure on the walls, or will you need to solve the full quantum field theory problem to do that?

Student: I can extract that estimate from the classical solution ... I have to just put the energy in one mod with wavenumber equal to the mean wavenumber, and read off the pressure from the classical electromagnetic field ...that should be a correct estimate.

Fuzzball person: Good. Sometimes the fuzzball solutions are written in terms of wavefunctions, like in the case of the first 3-charge solution constructed. So now do you can understand why fuzzball solutions are often written as classical solutions, and not as wavefunctionals, even though the actual states are generically very quantum, very stringy wavefunctionals?

Student: Yes, I see that. But wont it still matter what information you need from the state? If the information is some simple qualitative property of the state then I can learn about it from the classical solutions, otherwise I will have to proceed further with the wavefunctional analysis.

Fuzzball person: Absolutely. But our goal is to solve the information paradox, and for this we have seen that all we need is one single qualitative fact: Is there any structure in the solution that can possibly avoid the traditionally assumed result (1)?

Student: Yes, I understand, and this I can do from the way I am given fuzzball solutions. The simplest fuzzballs are like a laser beam with all quanta in a single mode; and I can see explicitly that (1) is violated. Similarly for all the other fuzzballs, and I do not need to write the full wavefunctional to see that the state departs from the vacuum around the horizon, and thus evolution departs from the traditional one at the horizon.

Fuzzball person: Good. To summarize, fuzzball constructions have been given both as quantum wavefunctions (e.g. the one in [16]) and as classical solutions (e.g.[41, 17]). In both cases the underlying fact is necessarily the same: the full fuzzball solution is a full string theoretic wavefunctional, and the given approximation has been written because it suffices for the purpose of extracting the relevant physical quantity – in this case the size of the region where the spacetime differ from the vacuum. But there have been several people who have made the following incorrect argument: *(a) Fuzzballs are given to me as*

classical solutions (b) I expect stringy physics in black holes (c) So fuzzballs do not describe black holes. Such a confusion stems partly from not understanding the classical-quantum relation in field theory, and partly from not understanding the information problem so that they do not know what they are looking for in the solution.

4.0.8 What needs to be shown to resolve the information paradox

New discussion

Student: What do I need to demonstrate to convince somebody that there is no information paradox?

Fuzzball person: That is straightforward, once you have the precise formulation of the paradox that we set up above. Eq. (1) quantifies the fact that the neighbourhood of the horizon is a vacuum, and then we have the result (2) that we will have information loss. Thus to solve the paradox we have show that it is possible to avoid having a horizon satisfying (1).

Student: There seems to be no choice: if we are to avoid information loss, then we cannot have the traditional black hole in which the horizon satisfies (1), so this means that we must have fuzzballs instead of the traditional black hole. Why did anyone ever think that black holes were *not* fuzzballs?

Fuzzball person: After Hawking found his paradox, people immediately tried to construct black hole ‘hair’ as nonspherical perturbations to the black hole metric. It was hoped that the information of the hole would be encoded in these distortions of the metric and then it would be possible to violate (1). Had they found such hair, there would be no paradox; the black hole would radiate like any other body which has its information in the vicinity of the place where the radiation emerges. The problem came only because they could *not* find these hair.

Student: If they could find no such deformations (hair) then, how come the fuzzball people find them now?

Fuzzball person: They had not looked for the hair in the correct place. They had studied linear equations for scalar, vector and tensor perturbations. They did not use compact directions in any essential way: it was assumed that if there were any compact directions then their effect would be taken into account by using scalar and vector fields arising from dimensional reduction; since there were no scalar or vector hair, there was no benefit from compact directions. But we have now learnt that the hair is *nonperturbative*; the compact directions fibers nontrivially (locally) over the noncompact directions, creating ‘dipole charges’, and the positions of these dipole charges gives an example of the degree

of freedom in the hair. From a dimensionally reduced perspective such KK monopoles would be *magnetic charges* for the gauge field obtained by dimensional reduction. The KK monopole is smooth in the full spacetime, but singular after dimensional reduction, and in any case is a non perturbative deformation of the naive black hole metric. Thus fuzzballs provide the hair that people were looking for; its just that in the 1970s people had not looked in the right place for the hair.

5 What has been done to establish the fuzzball picture?

New discussion

Student: I know that you are showing me special constructions that you claim are special microstates of black holes. But this will not convince me that all states must be like fuzzballs. Can you show me enough states that I should start to think that perhaps all states of all black holes will be fuzzballs.

Fuzzball person: Indeed, there has been a large amount of progress in constructing states that have the same quantum numbers as the corresponding black hole, but which have no horizon or singularity. To accomplish this we need to have a general way of making solutions in the gravity theory. In [21] Bena and Warner developed a formalism that solves the supergravity equations for 5-d solutions in M theory. In [22, 23] this formalism was used to make a large class of solutions, generalizing the basic structure of a simple microstate found in [24]. All these solutions have the same mass and charges as the 3-charge strominger-vafa black hole, but none of the solutions have a horizon or singularity.

Student: Okay, I see that there are large families of these solutions. But I worry that they may not have the right characteristics to represent the generic states that I am interested in.

Fuzzball person: The construction of microstates has progressed from very simple ones to more generic ones. Each time we go to more generic states, the construction becomes more complex, and we have less symmetries to help us. In [25] solutions with black holes and black rings were found, and in [26] states representing the maximally rotating black hole were constructed; these states had an entropy $\sim Q^{\frac{1}{2}}$ where Q is the magnitude of any one of the three charges.

Student: Okay, this looks like a lot of interesting states, but I would think that a generic state of the hole would have a very deep throat, not just a mess with size $\sim Q^{\frac{1}{2}}$. Are there such microstates?

Fuzzball person: Indeed, in [27] it was found that when we look at states with low rotation, we can get to a set of states whose throats plunge to arbitrarily large depths. The

same can be done for black ring microstates [30]. In fact this depth would become infinity if it were not for quantum effects which cut off the throat depth [42, 28]. A general review is given in [29].)

Student: Okay, that looks like good progress. But do I need to worry about whether there are enough states here to account for the correct order of magnitude of the entropy?

Fuzzball person: The solutions made so far had too many isometries to account for too much entropy. For example if you assume that the solution must be spherically symmetric, then you get no solutions at all; you only have the traditional solution with a horizon and singularity, which as we have learnt from the 2-charge story, is not an actual state of the system at all. In [31] supertubes were added to the geometries, breaking the symmetry on one of the compact circles. It was then argued that one could get an entropy of the correct order from such states.

Student: But doesn't such a supertube look like a source added to the gravity solution? Are such solutions of the same kind as all other solutions?

Fuzzball person: As you will see in section (6.1.4), there is no fundamental difference between solutions with sources and solutions without sources. Sometimes as we increase the strength of a source, it goes over to a regular geometrical solution, with the quantum fluctuations going down as we increase the strength of the source. Sometimes a smooth solution can become a source when we perform dualities. Stringy corrections to 'capped geometries were studied in [34]. Note that *all* solutions will have quantum fluctuations, in particular stringy fluctuations. The only important thing is the question of whether a solution has a true horizon satisfying condition (1). No fuzzball solution has such a horizon, while the traditional belief was that the black hole did have such a horizon.

Student: Okay, that looks like a lot of progress has been made on extremal solutions. But can I ask what has been done for nonextremal holes?

Fuzzball person: The first nonextremal microstate was made in [37], and we now know that the rate of (information carrying) emission from this microstate [38] exactly agrees with the rate of Hawking radiation expected from this microstate [39]. The structure of this microstate was analyzed in detail in [33]. Recently nonextremal configurations have been made with no horizons or singularity, but with the mass and charges expected for a nonextremal hole [?]. The constructions of these papers suggests that a general method is emerging for the construction of nonextremal solutions, extending the methods available for constructing extremal solutions. If such constructions can be related to microstates of nonextremal holes then we will have enormous progress in our understanding of nonextremal holes.

Student: Okay, I see that a lot of progress has been made on lots of different holes. But is there any one hole where I can claim to understand all solutions?

Fuzzball person: The 2-charge case gives a simple example of a black hole about which most relevant questions can be answered. In earlier times this was not thought to be a good black hole, but now we know that when we include quantum corrections the *naive* spherically symmetric solution of the field equations develops a horizon, just like the horizon for other extremal holes. The actual states of this hole have been extensively studied, and are all ‘fuzzballs’ [41, 13].

Student: So what are my options if I want to still argue that black hole microstates are not fuzzballs?

Fuzzball person: Very limited, I am afraid. You will have to argue that the 2-charge hole is not a good example of a black hole, which looks difficult, given what we know about its structure. For 3-charge holes you will have to actually produce states of black holes which do have a horizon, and no one has succeeded in doing this so far; all states which have been made have turned out to be ‘fuzzballs’. Lastly, if you do succeed in making a state with a true horizon, then you would have established information loss, as we have seen above.

6 The Sen computation of black hole entropy

Recently Sen [40] proposed a way to compute the entropy of ‘small black hole’; i.e., 2-charge extremal holes. In this section we discuss possible confusions which can arise about fuzzballs if we interpret this computation in an incorrect way. I will first describe what I think is the computation of Sen, and then describe (in conversation format) possible confusions and their clarifications.

6.1 The Sen computation

The Bekenstein entropy is given in terms of the area of the horizon. But this is only a leading order result. How do we get the subleading corrections which will end up giving us the *exact* count of states?

We have a general expression for the entropy given By Wald. But to apply Wald’s formula, we need two things: an action $S = R + R^2 + \dots$ and a solution g_{ab} to apply the expression to. How do we get these?

Sen’s conjecture (as I understand it) is the following:

- (i) Go to Euclidean spacetime
- (ii) The $g_{ab}(x)$ are functions on spacetime. To get these, look at the world sheet theory of a string moving in this Euclidean background. Require that the beta functions of the

string world sheet theory vanish, giving conformal invariance on the world sheet. Let us call such solutions ‘vanishing beta function metrics’ (VBFM) (this terminology is not in Sen’s work; I am adding it here to avoid confusion with the term ‘classical solutions’ which may have other connotations as well).

(iii) Choose a VBFM that has the charges $n_1, n_2, n_3 \dots$ of the hole, as measured by fluxes from infinity.

(iv) Perform the Wald procedure on this VBFM to get an entropy.

(v) Adding contributions from degrees of freedom arising from zero modes in this background we get a number; this is claimed to be the exact count of bound states for the charges n_i .

6.1.1 Problems with using a Lorentzian interpretation of the Sen solution

New discussion

Student: I wish to use the Sen construction not in the Euclidean section but in the Lorentzian section. Thus I would claim that bound states of the 2-charge system in the FP (fundamental string+momentum) duality frame have horizons and are given by a traditional black hole solution. Thus I would not get fuzzballs in this frame, but black holes, and more generally for 3-charge and other systems I would also get solutions with horizons (not fuzzballs), though there might be a few fuzzball solutions. Will I encounter any difficulty with such an interpretation?

Fuzzball person: Yes, there are several difficulties with such an interpretation. First let me ask: if you have a horizon for most states of extremal holes, you will also agree that there will be horizons for most states of near-extremal holes?

Student: Yes, I would be forced to agree, since otherwise I will have to produce a reason why nonextremal states are fuzzballs while extremal ones are not.

Fuzzball person: Then the first problem you face is the following. In [4] it was proved rigorously that if a nonextremal hole possesses a horizon, then there will necessarily be information loss in the resulting Hawking radiation. The meaning of ‘horizon’ was defined very precisely: low energy modes must evolve according to the Hamiltonian of vacuum spacetime upto corrections that are order ϵ with $\epsilon \ll 1$. It was then proved that only a fraction 2ϵ of information can be retrieved in the Hawking radiation. The opposite situation holds with fuzzballs: the evolution of low energy modes ($\lambda \sim kT$) gets modified by order *unity*, and there is no information loss. (High mass objects followed over the crossing time do not have to suffer such a correction; this is the infall problem, discussed elsewhere in these notes.)

Thus by claiming that you find Lorentzian solutions with horizon (instead of fuzzballs) you have forced yourself into the category of people who believe that information will be lost. Is this what you wanted?

Student: No, I would not be happy with that, since then I would lose string theory, AdS/CFT and all the other things I have gotten used to. But how can I avoid my conclusion? I have these explicit solutions of the low energy equations that have a horizon, and I guess I could make this rigorous by saying that these are VBFM solutions (vanishing beta function backgrounds).

Fuzzball person: Not quite. You have insisted on not allowing a fundamental string source anywhere in your spacetime, saying that this would spoil the regularity of the VBFM solution. But in the solution you actually have, there is a singularity inside the horizon (this follows from the Hawking - Penrose theorems: once there is a horizon, there will be a singularity). This singularity is much worse than the fundamental string source that you were trying to avoid: the fundamental string was at least an allowed source in the full string theory, while I do not know any way of saying the same for this singularity that you now have.

Student: Well, I thought that the throat of the extremal hole was infinite, so I could limit my VBFM to the throat and not worry about the singularity which is in some sense infinitely far away.

Fuzzball person: But this is not the case. For extremal holes, the distance to the horizon is infinite as measured along a constant t surface, but it actually takes only a finite proper time for a geodesic to pass through the horizon and reach the singularity. When you start your string worldsheet beta function computation, won't your string world sheet be able to cross the horizon and reach the singularity with finite action? You are trying to make a well defined VBFM, but I do not see how your traditional black hole geometry can be one.

6.1.2 Lack of an argument for the effect of quantum corrections

Discussion continued

Student: Okay, I see that I have a problem in claiming that the traditional extremal black hole solution is a good VBFM. But Sen has argued that quantum corrections (like mixings of non-BPS states with BPS states) can lead to large changes, and a horizon can form. What do I do about that?

Fuzzball person: First, let me note that until recently many opponents of the fuzzball proposal were arguing that quantum corrections to the D1D5 2-charge extremal solutions

will lead to the formation of a horizon and so the fuzzball proposal would be invalid. Now Sen has noted that such an eventuality will not happen in the D1D5 frame. What happened to all those earlier arguments, and what new computation has convinced these people that the D1D5 frame states do not develop horizons?

Student: I do not see any such computation; just an argument that since the generic solution can be approached through a set of regular solutions, there is no obvious reason why a horizon should be generated by quantum corrections. I agree that I cannot prove this to all orders in the correction terms, but it certainly sounds reasonable, given that I started by trying to quantize regular solutions.

Fuzzball person: This was the argument made by the fuzzball people, but the opponents kept demanding a proof that corrections of higher and higher orders will not generate a horizon. So it is indeed interesting that the entire question seems to have vanished now, for these same people. But now I see a related question being raised, which I find equally puzzling. At leading order there was no horizon in the solution of [41] in the FP frame, but you claim that after α' corrections a horizon will be generated. How will you prove the claim this time?

Student: I do not have any computation this time either. The claim was based mainly on what I wanted: to get a VBFM I would like to have no sources anywhere, and so the simple solutions written down for the vibrating string must get corrected, at least for BPS states, so that all the string falls behind a horizon.

6.1.3 Problem with orthogonality

Discussion continued

Fuzzball person: So let me understand this. Suppose I start with two orthogonal states of the FP string given in [41]. After your α' corrections, they will become the same state?

Student: No, I know that cannot happen, orthogonal states must remain orthogonal. With fuzzballs, whatever be the corrections from α' effects, I just got fuzzballs, and the wavefunctions would remain orthogonal. With the Sen solutions I cannot really tell you how that will work in detail, since I cannot tell you where the two solutions will differ from each other.

6.1.4 Sources in string solutions

Discussion continued:

Fuzzball person: Okay, this would be a problem that you need to explain. But let me ask you a different question. You have said that your claim that a horizon will form is based on the fact that good solutions in string theory have no sources. If I have a straight string wrapped along a circle, without momentum, will it look like a solution with source, or without source?

Student: This time it will have a source, but this is a special case.

Fuzzball person: Take the simplest case of the FP solution, where the string rotates with maximal angular momentum. In this case, if you make a horizon, what will the area of the horizon be? Note that there is no entropy, since there is only one state with these quantum numbers.

Student: Again there will be a solution with source, but this case is also special.

Fuzzball person: Okay, let us take a case where the string is vibrating, but with less than maximal angular momentum. In [41] the leading order solution for this case was written down. Do I understand that you will now argue that α' corrections to this solution will generate a horizon?

Student: Yes, though I have no concrete computation to show that this will happen. But somehow I want it to be the case that apart from the special solutions you have forced me to agree to, all other states develop a horizon. My basic reason for this is that I do not want to have any sources in my solution.

Fuzzball person: Okay, suppose you have a smooth D1D5 solution. In this geometry I place a single D1, wrapped on the S^1 , with some position in the noncompact space. Will this be a solution with source or without?

Student: With source, obviously, and I will disallow such solutions for the same reasons as above.

Fuzzball person: What if the D1 is replaced by a bound state of one D1 and one D5 (the D5 is wrapped on $T^4 \times S^1$)?

Student: This is also a solution with source, and I don't want it in my count of smooth solutions that will give hair.

Fuzzball person: Okay, what if I have 100 D1s and 100 D5s in the bound state?

Student: What difference does that make? This would be an even more singular source

..

Fuzzball person: Well, these branes actually create a gravity solution just like the D1D5 that you started with, so you will get just one geometry with *no* sources; the original D1D5 geometry with a new ‘throat’ and ‘cap’ created at the location where this new brane bound state has been placed. So where is the source now?

Student: I see that I have a problem now ... you are showing me that there is no sharp distinction between sources and smooth backgrounds. If I have many branes then it is more useful to use a smooth background description, while if I have only a few branes, then I might as well think of them as sources in the rest of the background.

Fuzzball person: Exactly. I do not see why you would want to separate allowed sources in string theory from smooth solutions. For example the fundamental string is an ‘allowed source’, but it can be dualized to a KK monopole, whereupon it looks like a smooth manifold.

6.1.5 Difficulty with separating Lorentzian hair from Euclidean black holes

New discussion:

Student: Given the problems that you have shown me, I would like to retreat from my original suggestion that Sen’s solutions should be thought of as Lorentzian ones. Can I not think that I am talking Euclidean all the time, even though I do not say that explicitly anywhere?

Fuzzball person: I see one problem with that. The geometries of [41] have terms in the metric $dt dx_i$, so if you Euclidize then you will get complex solutions. This is fine for the fuzzball people because they are dealing with the full string theory wavefunctionals instead of VBFMs. Thus the term $dt dx_i$ in the metric just changes the definition of canonical momentum, and this changed definition goes into the wavefunctional the wavefunction is then written as $\Psi[g]$. Then the *evolution* is Euclidized. This is the same way they will treat a time dependent solution in any quantum theory. But you are looking for a VBFM, so I dont see what you will do.

Student: Okay, that is a problem. I was trying to write some states as ‘hair’, using Lorentzian signature, and then write the others as black holes, which would also have to be Lorentzian. I understand that the fuzzball people have no such problem because they find that *all* states are ‘capped’ i.e. there are no horizons. But what do I do about the fact that Sen find a split between his states: some are hair and some are ‘horizon degrees of freedom’? If there is such a split, then it seems somehow reasonable to find a ‘black hole horizon description’ for the horizon degrees of freedom, and keep the other as ‘hair’.

Fuzzball person: You have to first make sure that your split is not coming because of some asymmetrical choice that you have made in describing your states. Sen takes a 5-d hole places this at the center of a KK monopole, getting a 4-d hole as seen from infinity. Then he finds that the entropy of the 4-d hole is reproduced by counting the entropy of the 5-d hole from the horizon area of the 5-d hole, and adding ‘hair’ degrees of freedom which live much further out, at $r = r_H$ where r_H is the radius of the KK monopole. Is that correct?

Student: Yes, that shows a split between the two kinds of degrees of freedom.

Fuzzball person: But note the asymmetry in what you have done. The 4-d hole has 4 charges: D1, D5 P sitting at the center of the KK, and then the KK charge itself coming from the solitary KK monopole. To get all states on the same footing, you should treat all these 4 charges in the same way. For the first three charges you have taken a large number of charges n_i , and a small mass of each charge, but for the KK monopole you have taken $n_{KK} = 1$ and a very large mass:

$$n_1 m_1 \sim n_p m_p \sim n_5 n_5 \sim m_{KK} \quad (5)$$

Had you taken $n_{KK} \sim n_1, n_5, n_p$ and $m_{KK} \sim m_1, m_5, m_p$ would you be able to find these ‘hair’ at large radius, and the other degrees of freedom at a ‘horizon’?

Student: I see that that could be a problem for me ... I should go back and do the computation in a more symmetrical way. I understand that the fuzzball people have made microstates using the same 4-5-d lift, and they find that *all* solutions are hair; where exactly the hair appears to be centered depends on the details on the microstate, but the important point is that there is never any horizon. So it is possible that when I make my charges symmetrical my ‘hair’ on the KK will descend down to where I thought my horizon degrees of freedom were, and when I look carefully at that location I will find just a set of fuzzball states and no horizon.

Fuzzball person: Yes, you should do that computation and check. There is one other fact you must note about your ‘hair’ degrees of freedom. The degrees of freedom in the CFT split into a “ $SU(N)$ ” type set and a “ $U(1)$ ” type set. The latter are very few, and arise from diffeomorphisms of $AdS \times S$. These states can *always* be considered to be localized at the ‘neck’ where the AdS space joins asymptotically flat space. (Being diffeomorphisms, there is some ambiguity in where we take the excitation to be localized). So you should be careful that you are not looking at $U(1)$ type states, and be careful that you have not introduced an asymmetry by your choice of KK size, and then come and tell me if you find a separation between ‘hair’ and horizon degrees of freedom.

6.1.6 Three charge holes

New Discussion

Student: 2-charge holes are in some sense on the borderline of being good black holes. Thus would like to extend my claims about 2-charge holes to 3-charge holes, which have all the properties of general black holes. Since I am trying to use Sen's computations in the Lorentzian section, let me extend my claims about Lorentzian states to 3-charge extremal states. Suppose I claim that the states can be split into (i) a small set described by classical solutions (which I will call 'hair') and (ii) the remainder (which I call 'horizon degrees of freedom') which will be described by Lorentzian black holes with a true horizon satisfying (1). Will I be wrong?

Fuzzball person: Yes, it is easy to show that you will have difficulties with such a claim. Start with the first 3-charge microstate that was made, in [16]. The D1D5 geometry has a 'cap' representing the 2-charge microstate, and there is a wavefunction in this cap carrying 1 unit of P charge. As we make the throat deeper, the wavefunction stays at the 'cap'; thus it gets further and further from the 'neck'. Would you call this state 'hair' or a horizon degree of freedom?

Student: This is clearly a horizon degree of freedom. Firstly it is quantum (it is a wavefunction), and secondly it stays deep in at the 'cap' instead of being localized at the neck.

Fuzzball person: Good. The 3-charge state represented by this solution is

$$[J_{-1}^+ \sigma_1^-][\sigma_1^-][\sigma_1^-] \dots [\sigma_1^-] \quad (6)$$

Thus one of the component strings of the D1D5 solution is excited, and the others are unexcited. Now let me give you two quanta placed at the cap, representing the state

$$[J_{-1}^+ \sigma_1^-][J_{-1}^+ \sigma_1^-][\sigma_1^-] \dots [\sigma_1^-] \quad (7)$$

If you like, I can even compute the small interaction between these quanta using the methods of [45]. Is this state 'hair' or a 'horizon degree of freedom'?

Student: This will also be a 'horizon degree of freedom, for the same reason.

Fuzzball person: Okay, now look at the state where I excite all the component strings the same way

$$[J_{-1}^+ \sigma_1^-][J_{-1}^+ \sigma_1^-][J_{-1}^+ \sigma_1^-] \dots [J_{-1}^+ \sigma_1^-] \quad (8)$$

This state was described by a regular metric given in [17]. Is this state 'hair' or a 'horizon degree of freedom'?

Student: Now you have me confused ... By continuity I would have to argue that this is also a ‘horizon degree of freedom’, but on the other hand you have written a classical metric, so I would like to call it ‘hair’ ...

Fuzzball person: Your confusion stems from a more basic confusion about the states in field theory. At this point you should review the discussion of section (4) about the classical limit of field theory states. You will learn that (i) what is written as a classical metric is always really a wavefunctional (ii) The fluctuations in the state depend on how many quanta are placed in the same mode; less quanta per mode means more fluctuations, but these fluctuations are never zero (iii) Thus there can be no division into classical and quantum states: there is a range of allowed states, and no state of a finite system can ever be purely classical.

Student: Okay, that was my error. But Sen tells me to quantize the classical solutions I find in supergravity by ‘geometric quantization’, so in some sense I *am* taking quantum fluctuations into account. Can I not separate stringy degrees of freedom from supergravity degrees of freedom in this way?

Fuzzball person: You have already learnt in section (4) that such a separation is not possible in string theory. So can you tell me what kind of a quantization will achieve that?

Student: I guess I was trying to be approximate, but now I see that if I cannot make a proper separation between different states then I have to agree that are all ‘fuzzballs’. From the example above I see that there is no dividing line between ‘hair’ and horizon degrees of freedom, which is a good thing because if some states had been described by a horizon then I would have to accept information loss, as you have shown above.

6.1.7 Change of duality frame

New discussion

Student: Suppose I agree that all states of the 2-charge extremal system in the D1D5 duality frame are fuzzballs. But I would like to say that in the FP frame they are black holes. What is a black hole in one frame is not a black hole in another frame, and the difference stems from what quantum corrections I keep in a given duality frame. What is wrong with that?

Fuzzball person: Sen made the distinction between different duality frames because he was only looking for a formal mathematical structure: the ‘vanishing beta function metrics’ (VBFMs). These are not actual states arising from the bound states of branes, but rather formal solutions to worldsheet beta function vanishing conditions, which incorporate all

orders of α' but no corrections in g . Were you thinking of these as actual states of black holes, like the ones that the fuzzball people make?

Student: Well yes, I would like to think of these as black hole states in the given duality frame ...

Fuzzball person: Then you have serious problems. If you are talking of the actual state of n_1, n_p charges, then can I ask how much are the quantum fluctuations in your solution?

Student: Well, since I was thinking of classical solutions, I dont have any quantum fluctuations ...

Fuzzball person: Then you have a contradiction right away. You learnt above that a system with a finite number N of quanta has quantum fluctuations $\sim 1/\sqrt{N}$; in particular the system can never be completely classical. So how can a VBFM describe an actual state of your black hole?

Student: I guess I did not think much about these things ... But let me recount Sen's argument to you. We know that the α' corrections he takes into account in the FP duality frame become one loop g corrections in the D1D5 frame. If I use these g corrections, I will indeed get exactly the same physics in both frames. But Sen tells me to not count the g corrections in any frame, so now the D1D5 frame becomes different from the FP frame; this is how he ends up saying that there are no black holes in the D1D5 frame but there *is* a black hole in the FP frame.

Fuzzball person: So tell me, up there in the sky is a black hole. If I fall into it, will my future depend on which duality frame I am fond of?

Student: No, that would be silly. But Sen says that I cannot include g corrections in the D1D5 frame because they arise from string loops winding around a compact circle [15]. If I dimensionally reduce such corrections to 5-d, then they look singular. So I should not use them.

Fuzzball person: I dont understand ... if the corrections are diverging, why is that a reason to discard them? In [15] it was shown that the corrections are indeed bounded when computed correctly using the full 10-d metric. Any 5-d reduction looks singular because the expansion breaks down. Thus if for some reason you insist on working in 5-d, then you can find a way to take careful limits and reproduce the correct answer. Better still, just use the 10-d corrections derived in [15]. But your logic seems strange: you wish to work in 5-d, this makes the expansion tricky, and then you bypass this difficulty by saying that

g corrections should not be included, and you use this to argue that there is no black hole system in the D1D5 frame, while there is one in the FP frame.

Student: Yes, put that way it does look strange. But my problem is the following. I know that all microstates are fuzzballs in the D1D5 frame. But I do not want to say that there are fuzzballs in the FP frame. So I write the classical solution which you have termed the VBFM. I somehow thought that the VBFM would describe all the horizon degrees of freedom of the FP system ‘in one go’. Is that wrong?

Fuzzball person: Yes, that is wrong. If a quantum system has degeneracy N , then there must be N orthogonal states. You cannot make these into the ‘same state’ in any sense. You can have differences between the states localized in one place or another, but wherever these differences are, they must be sufficient to give N states that are orthogonal to each other.

Student: But if the differences between my states is ‘subtle’ in some sense, can I not say that all these states are the ‘same for all practical purposes’ and represent them by one VBFM?

Fuzzball person: There is only one ‘practical purpose’ that is really important in the black hole problem: understanding the evolution of low energy modes that give Hawking radiation. So tell me: are all your states supposed to behave ‘the same way’ for this evolution?

Student: I guess that would be a disaster for me ... since then I would get no information in Hawking radiation. I would prefer to have *order unity* difference in the evolution for different states; certainly not (1) since then I will get information loss. So now I don't understand why Sen is trying to separate some states as ‘horizon degrees of freedom’ and represent them by one geometry which is not a state at all, but a formal ‘VBFM’ with no quantum fluctuations.

Fuzzball person: Sen's goal is to count states, and he is doing what Gibbons and Hawking did many years ago: use the Euclidean path integral to compute the path integral and thus read off the degeneracy of states. The fuzzball people have exactly the same interpretation of Euclidean solutions [42]. Sen's method extends the traditional work with Euclidean solutions by postulating how the Euclidean path integral should be computed in full string theory. *You*, on the other hand, have serious problems since you have tried to use these VBFM solutions rotated to the Lorentzian section. In doing this your physics has been wrong in almost every conceivable way, most importantly in the fact that you are committed to having information loss.

6.1.8 Summary

New discussion

Student: I see that I cannot use Sen's approach without saying at all times that I am working with Euclidean solutions. Further, the solutions I will be working with in the Sen approach are formal solution (VBFMs) made for a specific purpose; thus they are not full solutions in string theory, but rather solutions made by ignoring g corrections. This is a correct thing to do in the Sen approach since he postulates that using such formal solutions in the Wald formula will give a correct count of states. But I have tried to think of his solutions in the Lorentzian section, whereupon I find black holes with horizon. Then you have shown me that nothing makes sense, and most importantly, I will be forced to have information loss.

Fuzzball person: Exactly. All Lorentzian solutions are fuzzballs. If we do a Euclidean path integral, then we do not look at individual states but the whole ensemble, and the saddle point expansion of this path integral starts with the smooth Euclidean black hole solution, which as you know has no horizon.

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