The fuzzball paradigm for black holes: FAQ

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1 A brief summary

The information paradox is a sharp *contradiction*: we want quantum theory to conserve information, yet Hawking *proves* by an explicit computation that information will not come out in Hawking radiation. This proof uses only the fact that there is a horizon where the state of that of a *vacuum*; thus the radiation arises from the natural evolution of vacuum modes of quantum fields on gently curved space. Since there can be no ambiguity in this evolution, Hawking says that we must give up quantum mechanics.

To resolve the paradox, we have to show where the argument of Hawking can go wrong. From the earliest days of the paradox, people have tried to see if quantum gravity effects could modify the evolution of vacuum modes near the horizon, so that the naive semiclassical evolution used by Hawking would be invalidated. The problem is that we need an *order unity* correction to the evolution of these modes, since they have to go from a fully entangled state to a non-entangled state. On the other hand, all quantum gravity effects are expected to be of order (l_p/R) to some power, where l_p is planck length and R is the curvature radius. Thus despite a lot of effort in this direction, a resolution could not be found. These attempts included trying to construct 'hair' around the horizon (made from scalar, vector, tensor fields), but such attempts did not work ('no hair theorem') because any time independent solution to a field around the hole picked up a divergent energy density at the horizon.

The fuzzball proposal resolves the paradox by an explicit construction. One constructs explicit examples of black hole microstates and finds that there is no 'dataless horizon region'; i.e., there is no region where one gets a horizon with a vacuum state in its vicinity. One can identify a rough 'boundary' outside which the metric approaches the exterior black hole metric, but the fields in the neighbourhood of this boundary do carry the information of which microstate we considered. Thus we do not get pair production from a 'dataless vacuum', and we evade Hawking's paradox.

How do such microstate constructions arise? The simplest cases are for the case of extremal holes, though now some nonextremal cases have been made as well. The traditional geometry of the extremal hole has an infinite throat ending in a horizon, and there is a further region inside this horizon with a singularity inside. What one *wants* is to have some data about the microstate in the throat; this data can be deep down, but should not be past the horizon, since then we again get into the same information problems if we excite the hole and let it radiate.

Traditional attempts to resolve the problem would try to construct 'hair' by solving the waveequation in the throat, but one again finds that there are no solutions that do not have a divergence at the horizon. So to restate the problem that we have to solve: we *want* the throat to 'end' or carry some 'data', but any time we try to make such a modification to the traditional extreme Reissner Nordstrom metric we find that we cannot add 'hair'.

The fuzzball construction *does* succeed in constructing solutions where the data is found deep down in the throat; thus there is no traditional horizon. Moreover, these solutions are are not just arbitrary constructions, but are solutions that can be shown to correspond to specific bound states of the charges that make the hole. In simple cases these constructions exhaust all bound states of these charges, and in more complicated cases a fraction of the states have been constructed (those having an additional symmetry for instance) and these states all have data in the throat, no horizon.

So why does the fuzzball construction work when earlier attempts failed? Work with string theory leads to an understanding of how the microstates behave. There are many microstates of the hole. Let us start with some particularly simple looking ones. These have the following structure. In string theory, there are *compact directions* in addition to the noncompact spacetime directions that we usually think about. Let there be a compact circle S^1. This circle is a trivial product with the rest of the metric near spatial infinity, but as we go far down the throat, something interesting happens. This circle fibers nontrivially over the noncompact directions, making a KK monopole at one point, and an anti-KK monopole somewhere else. The charges of the hole distribute in such a way that the entire geometry is stable and time-independent. There are no horizons or closed timelike curves. The throat has thus ended, without any horizon or region interior to the horizon. Note that the KK monopole is a nonperturbative construction, in the sense that if we had looked at perturbative solutions to the gauge field A_μ arising from the compact S^1, then we would not have found this solution.

This microstate was 'simple' because only 2 monopoles were involved, but there are states with more and more such monopoles, and as we move to the generic microstate, the locations of these monoples are so close to each other that we should not ignore the large numbers of quantum effects that can arise: fluctuations of their centers, quantum fluctauations made of branes extending from one monopole to another, etc. People often ask if the generic state with such quantum fluctuations can be properly written down. While there are some studies of such quantum corrections, the issue is that *the exact nature of this 'quantum fuzzball' is not the issue at all.* Our goal was to solve the information paradox. Thus what we had to do was to find a flaw in Hawking's reasoning. Once we show that there are states of the hole which do not have a 'dataless horizon', the *boot is on the other leg.* If someone now wants to still argue that there is a paradox, he has to show that somehow the other states of the holes *develop a dataless horizon after these quantum effects are taken into account.* Leading order quantum corrections have been checked, and do not give any obvious reason that would lead to the fuzzball changing back to an infinite throat with horizon. So the task of a 'fuzzball nonbeliever' at this stage is to show that the structure of microstates constructed so far does not continue in a natural way to all microstates after all possible orders of corrections are taken into account. That is, he has to show that the generic fuzzball state is not just a 'messy quantum fuzzball'; he has to show that these quantum corrections completely change the structure of the state to one where the infinite throat with 'dataless horizon' *reappears* after the quantum corrections are considered.

To summarize, once we have found microstates in which the throat 'ends', we have resolved the *paradox*. The paradox was a contradiction, so all we needed was to see what is the way out. If we wish to argue that fuzzballs do *not* solve the paradox, then we have to take on the burden of showing why other states of the hole should not behave in the manner suggested by the states that *have* been understood. It is not a requirement on the fuzzball folks to get an explicit description of the generic microstate, which is going to be a very quantum wavefunctional of all the string fields. The fuzzball folks simple have to show the path by which simple states change to more generic ones, and then it is upto the 'fuzzball nonbeliever' to prove that the logical limit of these states will somehow generate a 'dataless horizon'.

Two final comments: The information problem is often confused with the 'infall problem'. The infall problem asks 'What does an infalling observer feel'? The motion of heavy ($E \gg kT$) infalling object over the crossing timescale has little to do with the question of whether we can change the entangled state of the low energy *outgoing* Hawking radiation quanta. If we have data at the horizon, then Hawking's computation for the entangled nature of outgoing modes is invalidated. What effect this data has on the infalling observer is *irrelevant to the information question*. It is however an interesting question in itself, and so I term in the 'infall question' in the notes below.

Equally often, one finds 'circular arguments' purporting to say something about the paradox. One which we discuss below in more detail is the statement that because we have AdS/CFT duality, there is a unitary dual theory to the black hole, and so there is no paradox. This is equivalent to saying that there is no paradox because quantum theory is unitary, a clearly circular argument. In actual fact, unless we can find a flaw in Hawking's gravity computation, we lose quantum theory, so that as a by product we lose string theory and AdS/CFT duality as well. To rescue the entire structure of quantum theory, we *must* go back to the gravity picture and show why gravity microstates need not behave the way they were expected to by Hawking, and this is what the fuzzball proposal does by an explicit construction.

2 What is the black hole information paradox?

Consider a ball of matter. This ball can collapse under its own gravity, and make a black hole. In classical general relativity, the matter falls to an infinite density point at the center, called the central singularity. Around this point we can draw a spherical surface such that any object inside this surface will necessarily be sucked into the singularity. This surface is the horizon.

If we consider quantum mechanics in this black hole background, then we find the following effect. Vacuum fluctuations cause particle-antiparticle pairs to be continuously created and annihilated at all locations. Near the black hole horizon, one member of this pair can fall into the hole, lowering its mass, while the other member escapes to infinity as 'Hawking radiation'. The quanta constituting the radiation are in an entangled state with the quanta which fell into the hole. When the black hole evaporates away, the Hawking radiation quanta are left in a state that is 'entangled with nothing'. This is a 'mixed state' in quantum mechanics, so the initial pure state of the infalling matter has evolved to a mixed state, which is a violation of the usual Unitary evolution of quantum mechanics [1].

Thus it would appear that general relativity and quantum mechanics are incompatible: general relativity tells us that black holes will form, and quantum mechanics in the geometry of these holes will give non-unitary evolution.

When a normal body like a piece of coal burns away to radiation, the entire evolution is unitary. The radiation is in a pure state, and carries the information of the initial matter. This can happen because the radiation is generated from the atoms in the coal, and thus carries information about the exact state of these atoms. In the black hole the radiation is created from vacuum fluctuations at the horizon, while the initial matter fell into the central singularity. Since the horizon and singularity are separated by a large distance, it would appear that the matter at the singularity cannot imprint its information on the outgoing radiation. The radiation quanta just stay in an entangled state with their infalling partners, and there is no effect of the initial matter on this entangled state. Since the final radiation does not carry the information of the initial state, we term the problem the black hole information paradox.

2.1 Can small quantum gravity effects encode information in the outgoing radiation?

No, as we will explain below, there is a sense in which the needed corrections cannot be small.

The derivation of Hawking radiation uses 'quantum fields on curved space', which means that matter is quantized but gravity is not. Thus one might therefore think that when quantum gravity is taken into account, there might be small corrections to the emitted radiation which would restore Unitarity.

If such were really the case, there would have been no paradox to worry about: when we learnt enough about quantum gravity to be able to compute these subtle corrections, the information would be seen to emerge in the radiation. But in fact one can convert Hawking's argument to the following 'theorem'

If we are given that

(a) Quantum gravity effects are confined to within a bounded length like planck length l_p or string length l_s

(b) The vacuum is unique

Then there *will* be information loss.

The details of the argument in this form can be found in [7].

An essential point is that these quantum gravity effects need to change the state of the created particle pairs by *order unity*. This large change is needed to change the entangled state of the particle pairs to a pure state carrying the information of the matter. Note that this does not mean that there have to be large corrections to the motion of heavy objects ($m \gg T$) over short periods of time ($T \sim GM$). The order unity changes are needed for quanta with $E \sim T$ and affect the evolution of the state over the Hawking evaporation time scale $t_{evap} \gg GM$. (We will come back to this in more detail later.)

We now see why the information paradox is so robust. Even if we do not know anything about our theory of quantum gravity, we would traditionally accept the conditions (a) and (b) listed above as being natural. In that case we would be stuck with the paradox. We will see later that in string theory (a) is not true, so that it is possible for information to be encoded in the outgoing radiation.

If I have to give up condition (a) then it looks like a fundamental change in how we think about quantum gravity. Should I have to make such a big change in my thinking just to solve the paradox?

It is the nature of any paradox that when we use our conventional ideas about things, we run into a contradiction. So we might have expected that when we resolve the paradox, we will learn something new about the essential way that gravity operates. In fact one of the main reasons for which we should be interested in the paradox is that we are likely to learn something new when we solve it.

2.2 What will it take to resolve the information paradox?

We must show that the evolution of vacuum modes at the horizon gets altered by order unity effects. We must do one or more of the following things:

(a) Show that there is a new length scale in the theory can give new effects across horizon distances.

(b) Construct examples of the actual states of black hole, and see if the structure of these states gives a way for the information to come out.

Finally, one should look at Gedanken experiments with black holes to see that the suggested structure of states can be consistent with dynamical processes that we can imagine with black holes.

It is important in all this that we have to directly address the problem in the *gravity* picture, and in *Lorentzian* signature. This is because the contradiction shown by Hawking is observed by an explicit dynamical process in the actual metric of the black hole. To resolve the paradox, we have to show where his computation breaks down. Since the paradox claims that quantum mechanics is destroyed, we cannot for example use a dual CFT description to argue that information will come out: if quantum mechanics is incorrect in the presence of black holes, then duality maps to a CFT would also be incorrect for states with black holes.

Similarly, Euclidean continuation is a trick we can use with quantum mechanics, but for this we have to be first given that quantum theory is true. If the issue is that quantum mechanics might be incorrect, then we should be careful to check what the Euclidean continuation means.

3 What is the fuzzball proposal?

Consider a black hole created by a collapsing shell. In the traditional picture of the black hole the region around the horizon is in a vacuum state. We picture this in fig.1(a), and term this vacuum state $|0>$. The shell of matter has swept past the horizon radius, and the state $|0>$ describes gently curved space in the circled region. Matter fields in this region started in the natural vacuum state, and evolve semiclassically in the low curvature horizon region to create the entangled pairs.

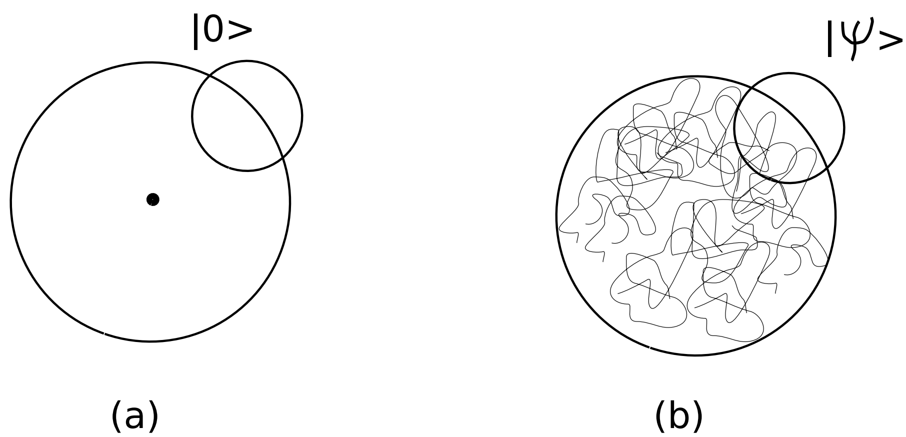

Figure 1: (a) The traditional black hole. The region around the horizon is in the vacuum state $|0\rangle$. (b) The fuzzball paradigm. The region around the horizon is *not* in the vacuum state, and $\langle 0|\psi\rangle \approx 0$ for a generic state.

The collapsing shell was an initial state of low entropy. The fuzzball proposal says that after the black hole has stabilized to an equilibrium configuration, the state in the circled region is not the vacuum state $|0\rangle$. Instead, it is a state $|\psi\rangle$ with

$$\langle 0|\psi\rangle \approx 0 \tag{1}$$

This situation is depicted in fig.1(b). The traditional hole has been replaced by a horizon sized quantum 'fuzzball'. The state $|\psi\rangle$ depends on the matter which fell in to make the hole. Radiation from the 'fuzzball' thus does not arise by pair creation in a region devoid of information. So this radiation can carry information just like the radiation from a piece of burning coal, and there is no information loss.

How do we argue that the traditional picture of fig.1(a) should be replaced by fig.1(b)? We can do two things:

(a) Show that in our theory of quantum gravity (string theory) there is a length scale that is order horizon radius rather than a fixed length like l_p or l_s.

or

(b) Construct examples of the interior state of the black hole to see if they are like fig.1(a) or like fig.1(b)

We will in fact do both of these. The first step (a) will serve as a motivation for looking for the 'fuzzball solutions' (b). Once we find the fuzzball solutions of (b) then we have to design our understanding of black holes around them.

Finally, we can think about 'Gedanken experiments' involving black holes. Such thought experiments have been important in developing the properties of black holes. Typically such Gedanken experiments are dynamical, time dependent questions, while the fuzzball solutions give the time independent states of the hole. We will consider some Gedanken experiments later on, and conjecture how the fuzzballs might behave under these dynamical conditions.

3.1 What does the proposal say about extremal holes?

Extremal black holes give us a good laboratory to start exploring the information paradox. In fig.2(a) we depict the traditional picture of such an extremal hole. We have flat space at infinity, then a 'neck', then an infinite throat. The throat ends in a horizon, and inside this horizon is a singularity. In fig.2(b) we depict the picture of the extremal hole as given by the fuzzball proposal. We have flat space at infinity and the 'neck'. The throat is long but not infinite, and ends in a quantum 'fuzzy' region which we call the 'cap'. The details of the cap depend on which of the $e^{S_{bek}}$ states of the hole we choose.

4 The 2-charge extremal hole

To address the information paradox, we will start with the simplest black hole: the 2-charge extremal hole. This will give us an idea of the fuzzball paradigm, and offer us many technical tools that we can later use for studying 3-chareg and 4-charge holes, and also near-extremal holes.

4.1 What is the 2-charge extremal hole?

This is the simplest model of the black hole that we can study. Let us take 10-dimensional string theory and compactify it as

$$M_{9,1} \to M_{4,1} \times \mathcal{M} \times S^1 \tag{2}$$

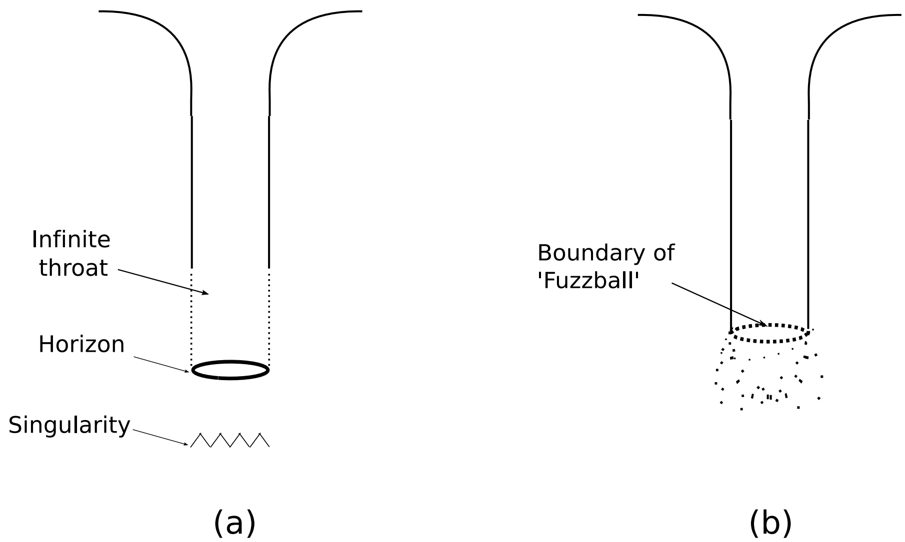

Figure 2: (a) The traditional extremal black hole. (b) The extremal hole in the fuzzball paradigm.

or

$$M_{9,1} \to M_{3,1} \times \mathcal{M} \times S^1 \times \tilde{S}^1 \tag{3}$$

The manifold $\mathcal{M}$ is T^4 or K3. The first case gives black holes 4+1 noncompact dimensions and the second in 3+1 noncompact dimensions.

We wish to wrap charges around the compact dimensions to make an extremal 2-charge black hole. The charges can be described in many dual descriptions. One choice is to wrap a string n_1 times around the S^1, and put n_p units of momentum along this S^1. We will call this the NS1-P system. Alternatively, we can wrap n_1 D1 branes on S^1 and n_5 D5 branes on $\mathcal{M} \times S^1$. This gives us the D1-D5 system. A third choice is to take n_0 D0 branes and n_4 D4 branes wrapped on $\mathcal{M}$. These systems are connected by dualities. For $\mathcal{M} = T^4$ the D1-D5 system maps to NS1-P. (For the D1-D5 system with $\mathcal{M} = K3$ we can dualize to a frame where the K3 is replaced by T^4 and the charges are given by a heterotic string carrying momentum P.)

4.2 How do we understand the entropy of the 2-charge system?

Sen [3] found the entropy of the 2-charge system in the duality frame where we have a string carrying momentum. This momentum can be partitioned among different harmonics in different ways. The count of the number of different ways gives the total number of microstates of the system for the given charges n_1, n_p. The log of this number is then the microscopic entropy S_{micro}. An equivalent computation in the D0-D4 duality frame was carried out by Vafa in [4].

We must now see if this agrees with the Bekenstein entropy of the black hole made with the same charges. Here we face the problem that the geometry of the hole receives corrections from higher order terms in the gravity Lagrangian (for example the R^2 terms that correct the Einstein action R). An *estimate* of the area entropy carried out in [3] gave $S_{bek} \sim S_{micro}$.

As we will see below, adding a third charge to the system gives a geometry that does not receive such higher order corrections, and the corresponding computations were carried out in the celebrated paper of Strominger and Vafa [43]. But in later years the higher derivative corrections became better understood. From the work of Wald we know how to compute the generalization of the Bekenstein entropy when the action has higher order terms. Dabholkar [77] computed the classical geometry of the 2-charge D0-D4 system, with the compactification $M_{3,1} \times K3 \times S^1 \times \tilde{S}^1$. He found that the Bekenstein-Wald entropy agreed exactly with the microscopic count of states

$$S_{bek} = 4\pi\sqrt{n_0 n_4} = S_{micro} \tag{4}$$

Thus in retrospect we can think of the 2-charge extremal hole as the simplest system that we can start with. After that we can look at 3-charge and 4-charge holes, near extremal holes, etc.

4.3 What is the nature of the states of the 2-charge system?

Let us work with the compactification $M_{4,1} \times T^4 \times S^1$. We can take the charges to be n_1 units of winding along S^1 and n_p units of momentum along S^1. The different states of this system are given by different ways of exciting the left movers on the string to carry the total momentum n_p; the right movers stay in the ground state.

The NS1 is 'multiply wound' n_1 times around the S^1. Consider the n_1 fold cover of the S^1, where this string looks 'singly wound'. Let the S^1 have length L. Then the 'opened up' string has total length $L_T = n_1 L$. The momentum on the string can be written as

$$P = \frac{2\pi n_p}{L} = \frac{2\pi n_1 n_p}{L_T} \tag{5}$$

The vibrations of the string come in units of $\frac{2\pi}{L_T}$, with the proviso that the total momentum be a multiple of $\frac{2\pi}{L}$ (the latter condition just comes from the natural quantization on the S^1 which had length L [51]).

The simplest state that we can think of is where we have $n_1 n_p$ units of the lowest harmonic on the string. We also have to choose the polarization of these vibrations. Let the four noncompact space directions be x_1, x_2, x_3, x_4. Let the vibrations be in the $x_1 - x_2$ plane. In the classical limit of large n_1, n_p, we can describe the vibrations by a classical profile of vibration for the string. We can choose the phases of the x_1, x_2 oscillations such that the string describes a helical profile. We will get one turn of this helix in the covering space where the string is opened up to its full length L_T (fig.3(a)). In the actual space (where the

S^1 is compactified with length L) the string will look like a 'slinky', winding around the S^1 as it wanders around in a large circle in the x_1x_2 plane.

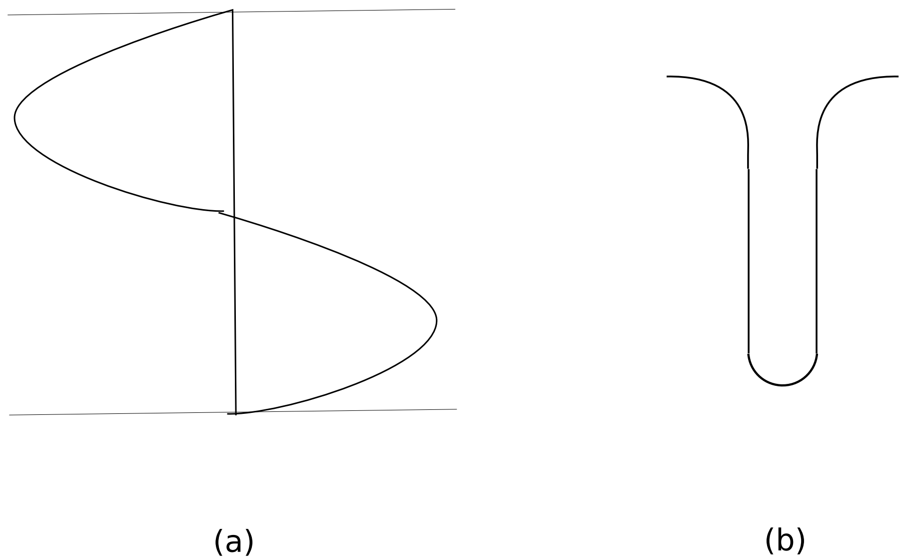

Figure 3: (a) The string vibrating in its simplest profile: a uniform helix with one turn in the covering space. (b) The geometry this creates.

This is one state of our 2-charge system. To find its metric, we go through the following steps:

(a) The metric of a straight string is known.

(b) The metric of a string carrying a travelling wave in one direction can be found by applying a Vachaspati transform; for the fundamental string this was done in [69].

(c) We can superpose harmonic functions to find the metric of several strands carrying momentum in the same direction.

(d) For the classical limit of large n_1, n_p we replace the closely spaced strands by a continuous 'strip'.

This gives the metric for this configuration of the string (fig.3(a)). We can perform S, T dualities to find the metric in the D1-D5 duality frame. This will therefore give us the metric of a particular state of the D1-D5 system. In other duality frames the 2-charge system gives 'supertubes' [38]

The state we have chosen has maximal angular momentum for the given charges, so we would have created a D1-D5 state with maximal angular momentum. The metric for this state was found earlier in [72, 73] by taking limits of the general axisymmetric metrics found by Cvetic and Youm [74]. The geometry is flat space at infinity, then there is a 'neck', and inside this neck we find a piece of global $AdS_3 \times S^3 \times T^4$. In particular, there is no horizon or singularity.

It might seem that we got this smooth metric because the state we took had a large angular momentum (black holes typically have an upper bound on their angular momentum). So we now look at another microstate of the system which has *no* angular momentum. Consider again the string opened up to its full length in the covering space. Let the first half of the string rotate clockwise in the $x_1 - x_2$ plane, and the second half rotate anticlockwise (fig.4(a)). Now the total angular momentum of the system is zero. The geometry created by this string can be again computed, and does not have a horizon or a singularity. In fact the throat is again 'capped', with the cap being a little more complicated in shape compared to the case where all the rotation was in the same sense (fig.4(b)).

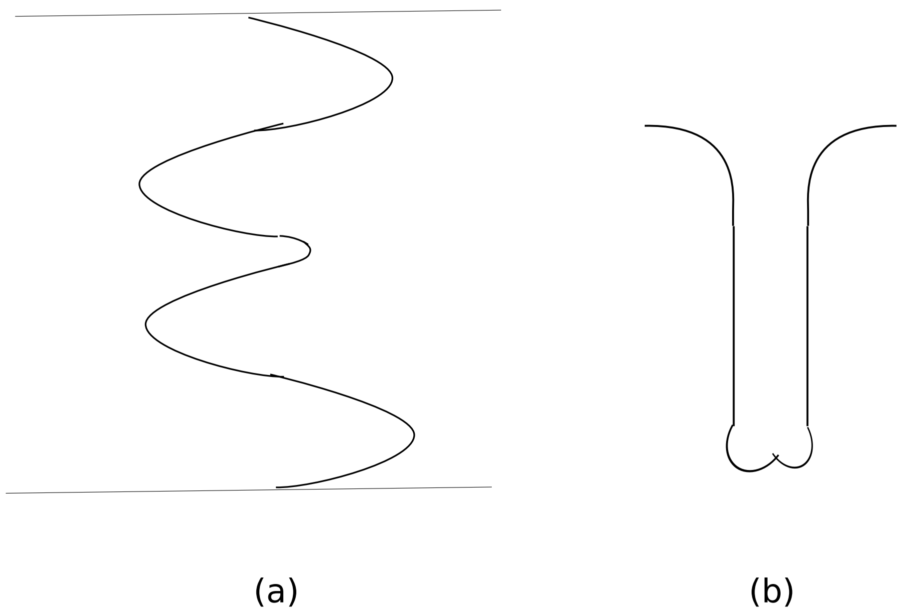

Figure 4: (a) The string vibrating in a mode which has no net angular momentum. (b) The geometry becomes more complicated, but does not develop an infinite throat or horizon.

Should I be surprised that there is a singularity in the NS1-P frame but not in the D1-D5 frame?

No, this is something that happens in different duality frames in string theory.

In the NS1-P frame, the metric and gauge field are singular at the locus of points through which the oscillating string passes. In the above examples, the string profile describes a curve in four noncompact spatial directions; this curve is topologically a circle $\hat{S}^1$. After dualizing to the D1-D5 frame, each point on this circle becomes the center of a KK monopole, so that we get the structure of a KK monopole times $\hat{S}^1$ [75]. The fiber of the KK monopole is the S^1 along which both the D1 and D5 branes are wrapped. There is of course no net KK monopole charge in the system – we can think of the KK monopoles at opposite sides of the

S^1 as forming monopole-antimonopole pairs. Thus the KK charge is a 'dipole charge' for the system. Different classical states of the D1-D5 system correspond to different shapes of the circle $\hat{S}^1$ in the non-compact directions.

In string theory we can convert any fundamental quantum to any other by dualities. Consider a string wrapped on a circle. At the location of the string, we would expect a curvature singularity. But we can dualize the string to a KK monopole, which is a regular solution in M theory. Thus the singularity at the string was an 'allowed singularity' of the theory.

Returning to our 2-charge problem, the dipole charge in the NS1-P frame is NS1, while in the D1-D5 frame it is a KK monopole. Thus the metrics look singular in the NS1 frame, and not in the D1-D5 frame.

4.4 What determines the size of the fuzzball?

We have seen that we can construct 2-charge states in the NS1-P duality frame, and then use S, T dualities to dualize the result to any other frame like the D1-D5 frame. The T-dualities are all in the compact directions, so if found a nontrivial spread for the state in the noncompact directions in one duality frame, then we will find this spread in all frames. Now we would like to ask if we can observe anything about the size of the typical fuzzball.

Thus look at the state in the NS1-P frame. The string is described by its oscillation profile. These oscillations must carry a given total momentum P, since that is fixed by the charge of the system. We find the following:

(a) The transverse *amplitude* of vibration depends on the mean wavenumber of the excitations on the string.

(b) The *quantum fluctuations* of the string depend on the occupation number of the typical mode.

Let us see in more detail what these statements mean. If we put the energy of vibration into higher wavenumbers, then we need a smaller amplitude of vibration for the same total energy. This makes the string state have a smaller transverse size, and the fuzzball size is smaller. Thus putting all the energy in the lowest harmonic (fig.3(a)) gives the largest transverse size for the fuzzball.

Note however that we have been treating the vibration profile of the string as a classically given function. When can we do that? The string vibrations are described by a set of harmonic oscillators, one for each wavenumber. Thus the energy eigenstates are given by specifying the occupation number n_k of the oscillator with wavenumber k. If we have $n_k \gg 1$ for the modes that we excite, then we can replace the energy eigenstate by a coherent state, while getting the same essential physical behavior. These coherent states for the various oscillators generate the oscillation profile of the string.

But as we move to more generic states, we find that the occupation number of the typical mode becomes $n_k \sim 1$. Then the states will not be well described by a classical profile, and will be very 'quantum fuzzy' (hence the name fuzzball). We would like to know something about the size of this generic state. How should we do this?

This is where the above observations (a), (b) become useful. Suppose we start with 1000 quanta in one harmonic k. The classical profile is a good approximation, and we can write it down and note its rough size. Now imagine that we put 500 quanta in harmonic $k+1$ and 500 in harmonic k-1. Then the mean size will not change, but the quantum fluctuations would be slightly higher, since the occupation number of each mode is a little lower. Continuing in this way, we can come to a situation where we have occupation number of order unity for each mode from $k/2$ to $2k$. Now the state will be very 'quantum fuzzy'. Since the mean size depended only on the mean occupation number, we will estimate the size of this state by finding the size of the first state we started with, which had 1000 quanta in harmonic k and so could be understood classically.

Carrying out this process, let us come to the relevant question: what is the size of the typical state of the 2-charge system, estimated in this way? Interestingly, one finds that if we draw a sphere around the typical fuzzball and compute the area A of this sphere, then we get [76]

$$\frac{A}{G_5} \sim \sqrt{n_1 n_5} \sim S_{micro} \tag{6}$$

so that the surface area of the fuzzball reproduces a Bekenstein entropy type relation with the entropy of the fuzzball. (A similar relation is obtained for black rings when we take states with large rotation.)

Is this computation just like computing the size of a typical string state in flat space?

No, it is important that we are working with gravity solutions with full backreaction. If we used a string in flat space, we would *not* get the above answer. Thus we are doing something quite different from the Horowitz-Polchinski correspondence principle, where we take strings in flat space and match them to black hole sized objects at a specific value of the coupling g. In the computation (6), there are several parameters *which all cancel out*. Both A and the 5-d Newton's constant G_5 depend on the parameters g, the string coupling, V, the volume of T^4, R, the radius of S^1, and α', the string tension. But all these parameters cancel out in the ratio A/G_5 so that we are left with the parameters n_1, n_5 in the combination which gives the entropy of the system.

What do we mean by saying that the gravity is strong here? in any geometry, we get functions like $1 + \frac{Q}{r}$ as coefficients of various metric terms. If gravity is weak, then we ignore the $\frac{Q}{r}$, while where gravity is strong, we can ignore the 1 compared to the $\frac{Q}{r}$. In our fuzzball geometries, we are in the latter situation, and in computing the area of the horizon the metric, dilaton etc. all have to be taken from the full gravity solution, not from values in flat space. Of course the reason why we have been able to construct the strong gravity solution for the string is that we have a special tool for computing 2-charge extremal states in string theory, by superposing the harmonic functions of many strands of the vibrating string, and then we can dualize to any other frame like D1-D5 if we wish.

4.5 What kinds of quantum corrections arise for typical states?

There are several sources of these corrections. Let us work in the D1-D5 duality frame.

(a) First of all, we have just quantum *fluctuations*, arising from the fact that the occupation number of each harmonic is small in a generic state. These do not change the size of a state, but increase $\frac{\delta g}{g}$ in the fuzzball.

(b) We have higher derivative corrections to the Einstein action. These arise from R^4 terms in the 10-d theory, where four gravitons are inserted into the one-loop diagram of an elementary string. The dominant state running along the loop is the winding mode of the string around the S^1 factor in the geometry. In the case where the compact 4-mainifiold is K3, two of the R factors can be integrated on the K3, leaving R^2 corrections to the 6-d action.

(c) Higher derivative corrections from 'string' tree level terms, but where this string is an effective string arising from taking a D3 brane, wrapping one cycle on S^1 and one on T^4. Since the S^1 becomes small in the fuzzball region for generic states, this effective string becomes very light, making the higher derivative terms significant.

The corrections (a), (b) are discussed in more detail in [**?**]. (c) is discussed in various places in the literature on higher derivative corrections.

4.6 What is the role of such quantum corrections?

Such corrections change the internal structure of the fuzzball, distorting it and making it quantum and fuzzy. However the question of interest to us is whether these corrections can change the fuzzball paradigm, which would require that they change the finite depth 'capped' throat back to an infinite throat ending in a horizon. (Recall that the horizon is a region with the vacuum $|0\rangle$ in its vicinity, and the metric continues past the horizon to an interior where it would have a singularity.) This seems to be very hard to do. In [78] it was found that the corrections of type (b) were bounded because of the interesting underlying structure of the capped geometry; this is important because if there is a divergence in the correction at some point then the qualitative nature of the geometry can change. No such divergences are found. Thus it does not seem possible to get back to an infinite throat by taking into account quantum corrections.

Thus the important point is the following. We can write down the structure of the geometry when we put 'many quanta in the same state' and make simple classical geometries. As we move towards generic geometries, the quantum corrections increase to order unity. But now 'the boot is on the other leg' in the following sense. In the absence of the simple fuzzball geometries, one would wonder how we would find any significant change at the horizon of a black hole, since this horizon looked like a region where the traditional black hole solution would remain valid. Once we have found the set of fuzzball geometries, we need a reason to argue why the generic quantum geometry should revert back to the traditional black hole. For all the states that we can reliably construct, we find that the state at the horizon is *not* the vacuum $|0\rangle$; instead it is a state $|\psi\rangle$ with

$$\langle 0|\psi\rangle \approx 0 \tag{7}$$

Now when we take the limit to the generic geometry, we would still expect the above relation to be valid, and if one wishes to argue that the generic state is *not* a fuzzball, then one has to show that for this generic state

$$\langle 0|\psi\rangle \approx 1 \tag{8}$$

In this sense the 'boot is on the other leg': one needs the quantum corrections to completely alter the leading order structure of the geometry. We have found no indication of such a qualitative change in out studies of the quantum corrections, so it looks logical to extrapolate the capped nature of states to all black hole microstates.

5 What is known about 3-charge and 4-charge holes?

The 2-charge hole requires R^2 corrections at its horizon to get the exact Bekenstein-Wald entropy. We would therefore like to study 3-charge holes, which have a larger horizon and do not require such corrections.

For 3-charge holes we do not yet have a systematic way of constructing all states in the gravity description. But for all those which have been constructed, we find that we get 'fuzzball' states: the throats are finite and capped, not infinite and ending in a horizon.

5.1 Do we find capped geometries for 3-charge states?

Yes, the simplest cases for 3-charge can be made in exactly the way the axially symmetric 2-charge case was constructed.

The simplest states are those with $U(1)\times U(1)$ axial symmetry, and these have large angular momenta (j_1, j_2). There geometries were constructed in [19, 20], using techniques similar to those used in [72, 73]. The fact that the throat was capped again helped remove the problem with travel times and energy levels, just as in the 2-charge case. Thus the spectrum of energy excitations found in the CFT were found to agree exactly with the energies of excitations in the capped geometry. If the cap was not there and we had an infinite throat instead, then the excitation spectrum would not agree at all between the CFT and the gravity theories.

5.2 What would more general 3-charge states look like?

How do we make more general 3-charge solutions? It can be shown that any BPS solution for N=1 supergravity in 6-d can be written as a 2-d fiber over a hyperkahler base [90]. The solutions of [19, 20] can be dimensionally reduced on the T^4 to give 6-d solutions, and so it was interesting to see what this base-fiber split would look like. Interestingly, the base turned out to be 'pseudo-hyperkahler': the signature of the base jumped from being $(++++)$ to $(----)$ across a hypersurface in the base. The fiber degenerated at this hypersurface too, in such a way so that the overall 6-d metric remained smooth. Thus the lesson is that while local supergravity equations tell us that the solution will have a hyperkahler base and a 2-d fiber, in the actual solutions corresponding to D1-D5-P extremal states this split cannot be performed globally; it degenerates along certain surfaces.

The $U(1) \times U(1)$ symmetric solutions mentioned above were also obtained independently by Lunin [21] who solved these base-fiber equations directly, after imposing $U(1) \times U(1)$ symmetry to simplify the solution.

5.3 What is the technique to make large classes of 3-charge solutions?

In a very interesting series of papers [87], Bena and Warner took this story to a new level. They started from the equations of 11-d M-theory, and obtained a more detailed version of this base-fiber split. Specializing the hyperkahler base to Gibbons-Hawking spaces (which have an extra U(1) symmetry), they managed to get a complete solution of the supergravity field equations. The fact that the space was pseudo-hyperkahler (rather than hyperkahler) could be easily built in with their formalism: the solutions were written in terms of harmonic functions and the sign of the sources in these harmonic functions determined the local signature of the base. With this formalism, it became possible to write down explicitly large families of BPS solutions to string theory, all having the mass and charges of the 3-charge black hole. None of the solutions had a horizon or 'black hole singularity'. The sources of the harmonic functions are held apart at fixed distances by fluxed running on spheres joining them; these constraints are given by 'bubble equations', which contain the essence of the supergravity equations in the present ansatz.

The above mentioned solutions had one $U(1)$ symmetry – the one needed to make the pseudo-hyperkahler base a Gibbons-Hawking space. Thus they were more general solutions than the ones in [19, 20, 21] which had ($U(1) \times U(1)$ symmetry. They are solutions in 4+1 noncompact dimensions. We can do a dimensional reduction along the circle corresponding to the remaining $U(1)$ symmetry, thus getting solutions in 3+1 noncompact dimensions. The way to do this compactification is to make the circle into the fiber of a Kaluza-Klein monopole. Thus the solutions acquired a fourth charge, that of the KK monopole, and we get 4-charge solutions in 3+1 noncompact dimensions. (Recall that if we want to make an extremal black hole with classical horizon size in 3+1 dimensions, then we have to use four charges.) Such solutions were constructed in [?, 87]. The bubble equation in this 3+1 dimensional setting become similar to equations studied earlier by Denef [35].

A general philosophy that emerges from all these constructions is the importance of 'dipole charges'. The BPS solutions has some charges that we measure from infinity; let us call these the 'true charges' of the solutions. When we look at the actual microstate solutions, we find that we have flat space at infinity, then for some distance the uniform throat expected of the traditional black hole, and then a 'cap' region. In this cap we find a set of charges that are not measured as charged at infinity. These are 'dipole charges' and their net value adds up to zero. But their locations can be varied, and this gives us different solutions corresponding to the same total mass and 'true' charges. Exploring the space of such allowed solutions is therefore relevant to exploring the structure of general black hole microstates.

With this wealth of available tools, a large variety of BPS solutions were constructed for the 3-charge and 4-charge cases. One can make structures that look like microstates of holes, or rings, or a collection of holes and rings. Some choices of fluxes lead to 'deep throat' solutions, which may account for a large fraction of the microstates of the hole. Solutions depending on a continuous parameter were recently found by putting a supertube inside a deep throat. With such a construction it may be possible to get enough solutions that their number will go like $\sqrt{n_1 n_2 n_3}$ for charges n_1, n_2, n_3; in that case one would have an entropy from these solutions that would account for the black hole entropy, and we would be in a situation similar to the one that we had for the 2-charge case.

5.4 What are 'hybrid' models?

Our goal in the above mentioned constructions has been to compute gravity duals to CFT states. In all the cases where this could be done, we are then able to match properties of the gravity solution with those of the CFT state. To see the essential property of the solution – having a 'cap' rather than an infinite throat – we had to construct the gravity solution with all charges producing their full backreaction.

Such construction are of course difficult, and the three charge set has not been completely understood yet. But one may look for something that is a little easier and not so rigorous: let some of the charges back-react to produce a geometry, and consider the remaining charges as test charges (without backreaction) in this background geometry.

If we do not adopt a fuzzball type picture, then we will have an infinite throat in the geometry of the extremal hole. There were several attempts to identify the entropy of the hole as coming from D-branes wrapping the horizon. But the problem was that these D-branes would keep falling down the throat and into the hole; there is no stable point for such a brane in the traditional black hole geometry.

Note that the near horizon geometry of the 3-charge hole is the Poincare patch of AdS_3 times $S^3 \times T^4$. The Poincare patch is not a complete space; geodesics exit the space through the horizon, which is why one is encountering the problem of where the D-branes could be places. If we could replace the Poincare patch of AdS_3 by *global* AdS_3, then we would be okay, since now the test branes could sit at the center of this global AdS space.

But how do we change from the Poincare patch to global AdS? This is of course an important issue for the black hole question, since it involves asking why we do not have a horizon that we will fall through. In [54] it was pointed out that we could study this problem perturbatively in the following situation. We can start with a 2-charge D1-D5 system, in a particular state; we choose the state where the D1-D5 branes are in the state with maximal rotation. Thus they create a geometry that is flat space at infinity, then a neck, then a throat, and finally a 'cap'. The cap and throat together make up a piece of global $AdS_3 \times S^3$, so we do have a global AdS space to work with. We can now place the test D-branes in this space, and try to count states. But other problems arise; for example the flux created by these test branes continues to be nonzero at spatial infinity, and makes the solution have a divergent energy.

In [53] the authors started with a similar 'capped geometry' carrying D2 charges around three different cycles; the cap was created by a $D6 - \overline{D6}$ dipole charge pair. The test branes were a collection of D0 branes, which added the fourth charge needed to make a 4-charge solution, but this fourth charge was not allowed to back-react on the geometry. While we lose the backreaction in this 'hybrid' construction, we can count the D0 brane states more easily, and see effects like their interactions among each other due to open strings stretching between them.

Such hybrid constructions have been used several times before in the study of giant gravitons [55]. One again takes global AdS space, and puts giant gravitons in this space to add new charges. The giant gravitons are treated as test particles, so they do not back-react on the geometry. In this approximation one can carry out an estimate of the entropy that can be carried out by these giant graviton states, and relate this to the entropy in the dual gauge theory, or to the entropy of black holes carrying the same charges.

5.5 What is known about nonextremal holes?

In [26], a family of nonextremal microstates was constructed, using the same techniques which were used to make $U(1) \times U(1)$ symmetric extremal states [74, 72, 73, 19, 20]. Thus these give states with high rotation, which makes them rather special states of the CFT.

In [27] it was found that these geometries are unstable, and radiated energy at a very specific set of frequencies through an 'ergoregion instability'.

In [28] it was shown that this instability radiation was *exactly* the Hawking emission expected from these specific microstates. It is known that the CFT computation of emission from a generic microstate reproduces all the properties of low energy Hawking radiation from the corresponding black hole [?]. The microstates we have are nongeneric in that they have 'many component strings in the same state' (in fact all component strings are in the same state). We take the CFT emission vertex that was used in the computation of Hawking emission from the generic state, and apply it to the specific microstates that we now have. The emission is highly amplified because of 'Bose enhancement' caused by many component strings being in the same state. This enhancement makes the quantum Hawking radiation turn into an instability leading to coherent emission from our specific microstates.

In summary, what we have now is a simple example of a non-extremal microstate where we can explicitly see the interior structure of the state, and observe 'Hawking emission' emerge from this interior and carry information out to infinity. It is reasonable to conjecture that in more general states (which do not have axial symmetry) there can be non-axially symmetric ergoregions. These will lead to a similar emission. but with a lower emission rate, so that when we come to states with the complexity of the generic state then we will recover the low energy Haking radiation rate.

5.6 Can a generic state be approached through supergravity solutions?

This is not obvious. We have seen that 2-charge states could be approached through supergravity solutions: we can choose low harmonic orders to make the fuzzball large, and put the energy in only a few harmonics to get a coherent state. This gave supergravity solutions, and we starting here, we could approach the generic state while observing the rise of fluctuations and quantum corrections. In this sense we could approach the problem by starting with supergravity solutions, and this was possible because we had a picture of all states as arising from vibrations of a string.

For 3-charges, it is *not* obvious that we can start with supergravity solutions and take limits to get the generic solution. On the other hand, we cannot rule this possibility out either. Some recent interesting work of Bena, Warner and collaborators suggests that one may be able to approach the generic solution through supergravity solutions. This would mean that we make supergravity solutions with curvature length scales down to say 10 l_p, and then observe that quantum corrections would start to become order unity as we do down to curvature length scales $\sim l_p$. If we have a reason to expect that these limiting (quantum) solutions give all states, then we would say that 3-charge solutions can also be approached through supergravity.

5.7 Is it important that the generic state be approachable through supergravity solutions?

No, this is not important for the fuzzball paradigm. The paradigm (for extremal holes) says that the throat ends in a quantum fuzz containing information about the state. The conventional paradigm says there is no data at the end of the throat, and we pas smoothly through the horizon to the interior of the hole. It is not important that the generic fuzzy state be approachable through supergravity. In arriving at evidence for the paradigm,, we have used supergravity solutions to study the simplest cases, and in this sense the supergravity solutions are useful.

6 What is the relation between AdS/CFT and the information paradox?

Two of the major directions to emerge from nonperturbative string theory are an understanding of black holes and the gauge-gravity correspondence. These two developments thus share many of the same tools, and each has helped the development of the other. In particular the theory of black holes has used the tools of AdS/CFT in many ways. Before we look at what AdS/CFT can do for black holes, let us first note what it *cannot* do.

6.1 Can we use AdS/CFT duality to argue that there is no information paradox?

No, we cannot do that, since the argument would be circular. Let us see this in more detail.

Suppose we were to argue as follows: (Argument A): *The black hole is a state of the gravity theory. The gravity theory is dual to a gauge theory. The gauge theory is Unitary. Thus information cannot be lost in the process of black hole evaporation.*

To see where this argument breaks down consider the following argument that we could have made, right in the early days after the information paradox was encountered. (Argument B): *The process of black hole evaporation is some dynamical process. All dynamical processes are described by quantum mechanics. Quantum mechanics is Unitary, so information cannot be lost in the process of black hole evaporation.*

What is wrong with this latter argument? The point is that we cannot start by assuming that quantum mechanics is correct. It is certainly natural to start by thinking that quantum mechanics holds for black holes since it holds for so many other processes that do not involve black holes. But the power of Hawking's calculation is precisely that when we come to black holes, then we explicitly encounter a process where Unitarity is lost. Thus Hawking is not saying that quantum theory has to fail in the lab; he is saying that quantum theory will fail when we come to black holes (or have black holes involved as virtual states somewhere in the problem).

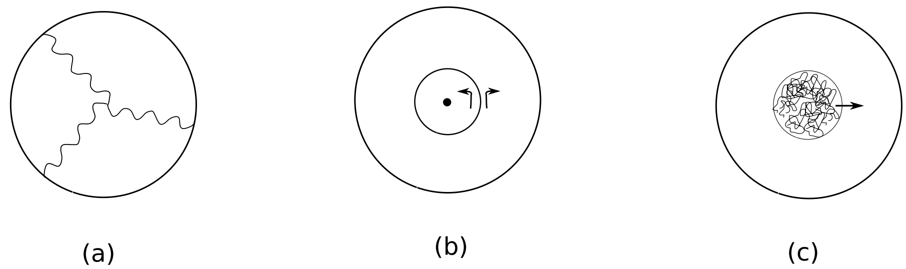

Figure 5: (a) A three point function in AdS/CFT: no black hole is involved. (b) An AdS dual where the black hole is the traditional geometry. Information would be lost, and quantum mechanics, including AdS/CFT would break down. (c) If we can show that the black hole is a fuzzball, then quantum mechanics can be saved, and we can have AdS/CFT in particular.

Let us return to the argument about AdS/CFT and information loss (argument A) . In fig.5(a) we sketch a simple process on the gravity side: a process given by a 3-point function. This process does not involve black holes, and its value agrees with the 3-point function in the dual gauge theory.

In fig.5(b) we sketch a process which does involve forming a black hole. Suppose the black hole has the traditional metric of the hole. Then we will have Hawking radiation from the horizon and lose Unitarity, by Hawking's computation. The gauge theory is Unitary by construction, so we will find that AdS/CFT has broken down.

How can this happen? Have we not checked AdS/CFT duality by hundreds of processes? But the same situation holds for argument B. Quantum mechanics has been tested not in hundreds, but in thousands of cases! Quantum mechanics has been seen to hold in laboratory physics, astronomical situations, chemistry. Yet we cannot use all these instances where quantum mechanics works, to argue that it will help us with the black hole: if we have an explicit computation that shows information loss for the black hole then we have to conclude that quantum mechanics breaks down when black holes form. Similarly, if we find that on the gravity side of AdS/CFT duality we have the traditional black hole of fig.5(b), then we have to give up quantum mechanics. The fact that we lose AdS/CFT is a small corollary, since AdS/CFT is based on having normal quantum theory. Losing AdS/CFT would be a small thing, (however sad) compared to the much more serious fact that we our losing quantum theory!

So what we need to save quantum theory (and AdS/CFT in passing) is that we find something wrong in the *gravity* description of the traditional black hole; something that will alter Hawking's computation of radiation. For example if we find that the black hole states are like the fuzzballs shown in fig.5(c) then information *can* emerge from the hole; quantum mechanics can be saved, and AdS/CFT need not fail.

To summarize: *We must understand the structure of black holes in the* gravity *picture, and see how information comes out. We cannot just say that because there is a dual CFT that is Unitary, the information must come out.*

6.2 How does AdS/CFT help with understanding fuzzballs?

Having seen what AdS/CFT cannot do, let us come to what it *can* do for us. We will see that it is an invaluable tool in understanding the structure of black holes.

(a) Understanding the CFT can help us find the gravity states of the hole. For example, in [19, 20] the goal was to look for the geometry for an extremal 3-charge state. From the CFT we can find the maximal possible angular momenta that are allowed for the given charges of the hole. We expect that the state with these maximal angular momenta will have axial symmetry, so we look at solutions of [74] with these specific values of angular momenta, and indeed find the required microstate. (This method of finding the microstate is analogous to the construction of 2-charge states in [72, 73].)

(b) The CFT can help us check the properties of the gravity solutions that we construct. For example an important issue is one of energy gaps. Any CFT state has a non-zero energy gap; this is expected since any finite energy system in a finite volume would not typically be able to have a continuous spectrum. This gap can be computed for example for the simple states where all component strings have the same winding number m. Then the energy gap in the CFT is

$$\Delta E = \frac{2}{mR} \tag{9}$$

Here R is the radius of the S^1 on which the effective string is wrapped, so that the total length of each component string is $2\pi mR$. The energy ΔE corresponds to one left moving and one right moving excitation in the lowest harmonic on any one component string.

Now consider the gravity solution for such microstates. Since all component strings are similar, the solution has axial symmetry, and the wave equation for excitations in the throat can be separated and solved. One finds *exactly* the same energy gap, now created by supergravity quanta that lie in stationary states at the bottom of the throat.

It is very important to note here that if we did *not* have the caps, but had an infinite throat instead, then we would get a contradiction with AdS/CFT duality: we can make excitations of very low energy sitting deep down the infinite throat, so that the energy gap on the gravity side would be much lower than the gap on the CFT side.

(c) We can use the CFT to take *limits* from simple gravity solutions to generic solutions.

Suppose that in the CFT we take many component strings of the same kind. Then we can make coherent states to describe the system. The supergravity solutions we write on the gravity side correspond to such coherent states. Now imagine reducing the number of component strings of each type; thus if we had 100 strings of winding w, we can replace them with 50 strings of length $w+1$ and 50 of length $w-1$. The average size of the state does not change, but the fluctuations become somewhat higher. As we keep reducing the number of strings with any given winding, from 100 to 50 to 10 to 5, the fluctuations keep growing, while the size of the fuzzball remains of the same order. What can we say when the number of strings of each type is ~ 1? On the CFT side, nothing dramatic happens, and we can use this to extrapolate on the gravity side, and say that while the fuzzball becomes very quantum, its size will not suddenly change from 'horizon size' to zero. Thus the continuity on the CFT side allows us to extrapolate to generic states from the states that we do construct.

7 Dynamical experiments with black holes: Conjectures

We have focused on understanding the $e^{S_{bek}}$ stationary states of the black hole, and have understood their structure as 'fuzzballs'. On the other hand we can perform Gedanken experiments with black holes where we imagine time dependent processes, for example the infall of a quantum into the hole. We should use our understanding of the stationary states to see if we can obtain reasonable resolutions to puzzles created by such dynamical processes. Such steps are mostly in the domain of conjectures, since only a few dynamical computations have been carried out with fuzzball states. We list below such conjectures answers to some Gedanken experiments. It should be noted that with the explicit construction of fuzzball states, it is possible to slowly start exploring dynamical questions in more detail, by starting with single quanta infalling into the simplest extremal holes.

7.1 What are the two timescales associated with black holes?

A central point to recognize about black holes is the existence of two very different timescales:

(a) The 'crossing timescale'

$$t_{cross} \sim R_{schwarzschild}/c \tag{10}$$

(b) The Hawking evaporation timescale

$$t_{evap} \sim t_{cross} \left(\frac{M}{m_{pl}}\right)^2 \gg t_{cross} \tag{11}$$

The essential point is that t_{evap} is larger than t_{cross} by a power of $(\frac{M}{m_{pl}})^2$. Thus when we perform a Gedanken experiment we should ask if we are considering physics over t_{cross} or t_{evap}.

7.2 Do two such scales also show up for extremal holes?

The extremal holes do not evaporate, but we encounter two length scales again. The traditional geometry of the extremal hole has an infinitely long throat. Let the radius of this throat be Q. The 'neck' of the geometry extends over this same length scale Q.

In the fuzzball picture the throat is long but not infinite. Thus we can ask how long the throat is in units of its radius Q. We find that the depth of the throat for a typical microstate is a power of $(\frac{M}{m_{pl}})^2$ times Q. (For details of these length scales see [**?**, **?**, **?**].)

A classical observer who thinks of $\hbar$ as ignorably small will thus think of the throat as infinitely long.

7.3 What is the significance of having two scales in the problem?

Suppose a shell of matter collapses through its horizon radius. We know that the time independent states of the hole are horizon sized quantum fuzzballs. Does this mean that the shell should encounter some large quantum gravity effects as it falls through its horizon?

The answer is no, the fuzzball proposal does not require that. The collapse of the shell happens on the short timescale t_{cross}. But the shell is a special low entropy configuration, a very non-generic state for its total mass M. As in any statistical system, this special initial state can take time to ergodize to a generic state, which we expect to be a 'fuzzball' type state.

But this relaxation process can take much longer than t_{cross}, without creating a problem for information recovery. It is known that the Hawing radiation quanta from black hole have an entropy that is some 30% larger than the Bekenstein entropy of the hole. (This happens because the radiation 'free-stream' out of the hole, instead of being released through a reversible quasi-staic process.) Thus even if the relaxation process takes about one-third of t_{evap}, we still have enough radiated quanta to carry out the information of the hole.

7.4 Are fuzzballs classical or quantum?

The generic fuzzball is very quantum. This issue was discussed in more detail in section **??**. The classical geometries corresponded to the special situations where we put 'many quanta in the same state'. In this situation we can replace the energy eigenstates by coherent states to a good approximation, and these coherent states are well approximated by classical geometries. But putting 'many quanta in the same state' reduces the number of allowed possibilities, and thus the generic state is not expected to be well described by any classical geometry.

7.5 If a generic state is so quantum, then can it look just like the usual vacuum near the horizon?

No, it is important to understand that a very quantum fuzzball state is *not* the vacuum. Recall the discussion of section 3. The vacuum state in the region around the horizon was termed $|0\rangle$. The fuzzball state around the horizon is $|\psi\rangle$ with

$$\langle 0|\psi\rangle \approx 0 \tag{12}$$

7.6 Does this mean that an infalling observer will see something very different from the vacuum state as he falls in?

No, that is not necessary, because of the two different timecsales in the problem. The Hawking radiation quanta have an energy of the order of the temperature T of the hole and wavelength of the order of the radius of the hole. The evolution of such modes *must* suffer changes of order unity in their evolution, if information is to emerge in the radiation. On the other hand, when we consider an infalling *object*, we have in mind something with energy $E \gg T$. The evolution of this massive object over the short timescale t_{cross} need *not* be affected by the change of state from $|0\rangle$ to $|\psi\rangle$. Over the long timescale t_{evap}, the information in this massive object should get incorporated in the quantum fuzz, and be radiated away with the Hawking radiation.

A rough analogy would be the following. Suppose we toss an apple into a room. The room may either be in a vacuum state, or full of air. The motion of the apply from the door to the center of the room is hardly different in the two cases. But if the room was full of air, then the molecules of air start bouncing off the apple from the time the apple enters the room, and continue to do so later on. The apple decays, and the air molecules carry its information out to the walls, where slow effusion of air from from the room sends the information of the apple out to infinity. If on the other hand there was no air in the room, the information would never emerge. Thus having a low density 'fuzz' filling the horizon need not change the short time evolution of massive objects, yet can get the information radiated out over $t \sim t_{evap}$.

As another related example, consider introducing a sharp laser pulse of blue light into a room which is at 'room temperature' $T \sim 300^o K$. The typical quantum of black body radiation in the room has energy much lower than the energy of the blue light. The pulse of blue light travels through the room almost as if the room was in a vacuum state. The situation is different if we try to introduce a single quantum of energy $\sim T$ into the room. If its energy level was already occupied by n quanta, then the amplitude for creating another quantum in that mode is enhanced by $\sqrt{n+1}$ by Bose enhancement. If we were considering fermioic quanta, then we could not introduce another quantum in the given mode if the mode was already occupied. Thus we see that the source trying to introduce the new quantum is affected by the radiation state in the room if we are talking of a quantum with $E \sim T$, but there is virtually no effect if $E \gg T$.

Thus it is important to realize that the motion of heavy objects over the short crossing timescale is a very different issue from whether there is a slow leakage of information in the Hawing radiation.

7.7 Do fuzzballs have a horizon?

First we should note that we are now asking the question about the stationary states of the system, when the initial collapsing matter has ergodized to a generic configuration.

If we look at the special states which have 'many quanta in the same state' then we find geometries with no horizon or singularity. As explained in section **??**, when we take the limit to generic states, we get more 'quantum fuzziness', but the size of the fuzzball does not change.

Let us now be more precise by what we call a 'horizon'. The important property of the traditional horizon is that the state in a region around the horizon is the vacuum $|0\rangle$. If the state around the horizon is $|\psi\rangle$ with $\langle 0|\psi\rangle \approx 0$ then we do *not* have a horizon. The fuzzball picture tells us that after the hole has stabilized, we do not have the traditional horizon.

So in what sense can the fuzzball have a horizon? The horizon emerges as an effective concept in the fuzzball picture. If we draw a boundary around the region where the typical fuzzballs differ from each other, we find that the area of this boundary satisfies a Bekenstein type relation $A/G \sim S_{micro}$. But does this boundary behave like a 'horizon' in some way?

Consider a quantum that falls down the 'throat' of the D1-D5 system. At some point this quantum reaches the boundary of the region where generic 'caps' differ from each other. As the quantum passes into the cap region, its further evolution becomes very complicated. If we took a D1-D5 state which was still classical but where the curve $\vec{F}(v)$ was very complicated, then we can see that the quantum gets trapped in the complicated cap region for long times. Extrapolating this time for generic fuzzball states (which will in fact not be classical metrics) one find a trapping time that is a power of $(\frac{M}{m_{pl}})^2$ times the light crossing time across the fuzzball diameter [71].

The same effect can be seen in the dual CFT. The CFT has component strings of many different lengths. The infalling quantum has some amplitude to be absorbed by each of these component strings. But the time of *emergence* from different component strings is different, and so when we add the amplitudes for re-radiation from all the component strings we get 'phase cancellation' for the amplitude of the emitted wave. Thus again we find that the quantum is trapped by the state for long times.

What are we to make of this long time of trapping? For a classical observer who thinks that $\hbar$ is ignorably small, the return time is infinite. He can then model the physics as in fig.2(a), by drawing a traditional horizon at the fuzzball boundary. The quantum would not return from behind this traditional horizon because the forward light cones points towards $r = 0$. Thus motion in this simple 'effective' geometry would reproduce the lack of return of the infalling quantum. This would be the correct physics for times of order the crossing time. The important point of course is that this is *not* the correct physics for times that are some power of $(\frac{M}{m_{pl}})^2$ times the crossing time. The fuzzball does return the information of the quantum after a long time, while the effective geometry fig.2(a) does not. Thus this effective geometry is good only for physics on the short time scale t_{cross}.

This example may be part of a general 'correspondence principle'. Process over the crossing timescale may be effectively reproduced by the traditional black hole geometry, but processes on the Hawking evaporation timescale need the full fuzzball structure, and yield an evolution that depends on the choice of fuzzball state.

7.8 Do fuzzballs look the same as the traditional black hole outside the horizon?

Fuzzballs are states of a statistical system. As in any statistical system, the generic states have macroscopic parameters that are very close to some given values. There will also be non-generic states whose parameters will depart from these mean values. For example in a gas in a box, most states will give a pressure on the walls that is given by the traditional gas laws. But if we let the atoms of the gas all move in one direction, the pressure will be large in that direction and zero in others – this is an example of a non-generic state.

For 2-charge states, the state is not in general spherically symmetric. For a simple non-generic state like that pictured in fig.2(a) we have rotation in one plane, and a large size for the fuzzball. But the multipole order in a generic state is very high (a power of $n_1 n_5$, so we should say it is a power of $(\frac{M}{m_{pl}})^2$). Thus outside the fuzzball the non-sphericality effects fall off *very* fast, and we see an effectively spherical geometry.

The size of the fuzzball should also be concentrated around a given value for generic states. For 2-charge holes (which are 'small' statistical systems, in some sense) this size can depend on the dynamical process that we are using to define the size [31], but for a given choice of dynamical process, the generic states should all yield a size close to a given value, as in any statistical system.

7.9 What happens to a particle that falls into a black hole?

This is a dynamical question. Our constructions have for the most part concerned the structure of stationary states of the hole, so we will have to make simple computations with these states and make some guesses about what might happen in dynamical situations.

We should start with the simplest case: a single quantum falling into the extremal 2-charge hole. One thing that we can do is can study the process in the dual CFT, and use the results there to infer something about the evolution in the gravity picture. The CFT dual to the D1-D5 geometry is the orbifold CFT, but this CFT is not at the 'orbifold point' when we are at typical value of the moduli in the gravity theory. But for the moment let us ignore the 'twist operator deformation' that takes the CFT away from the orbifold point (this is like using the free Yang-Mills to guess the evolution in the $AdS_5 \times S^5$ dual.)

For the evolution under these conditions an simple study was made in [**?**]. A quantum was thrown into a D1-D5 state. As the quantum entered the throat, a pair of left and right excitations were created in the dual CFT. The CFT state has many component strings (of different lengths), and there is some amplitude for each component string to be excited. As the quantum proceeds down the throat, the excitations in the dual CFT separate more and more. If all the component strings were equal, the excitations would go around the component strings, recollide, and emerge as radiation. In the dual CFT, the quantum would re-emerge from the throat. This was the computation performed in [71], where exact equality of re-emergence times was shown.

But now our interest is in a generic state, where the component strings are of *unequal* lengths. Once the excitations in the CFT separate by about half the length of the component string, we have the following situation. On some component strings, the excitations are more than half-way around, so in the gravity picture the quantum should be heading out of the throat. On longer component strings the excitations are not even half-way around, and the quantum should be heading deeper into the throat. So *where* is the quantum?

We can check that this 'confusion' about the location of the quantum happens when the quantum reaches the boundary of the typical fuzzball. Thus we find that when a quantum enters the fuzzball, the idea of position itself becomes confused. For more details, see [**?**].

Of course the next thing to do is to add to this computation the effect of the twist operator which takes us from the orbifold point to the actual gravity dual. Hopefully, by performing many such studies we will get a more complete answer to what happens to objects that fall into a black hole.

7.10 What happens to a generic fuzzball state when a quantum falls in?

When we make coherent states for the fuzzball, then we 'put many quanta in the same state', and get a classical background. A test quantum, if it is light enough, travels on this background without affecting the background severely. In a generic state the background is not classical, and the test quantum can make the state of the hole change through backreaction to the presence of the quantum. Thus for generic black hole states we can expect that the infalling quantum will change the black hole state as it falls in, and one should not write a factored state describing the hole plus matter; the full state should be quite entangled. For estimates on the backreaction for different 2-charge states, see [71].

7.11 Can a collapsing shell change to a fuzzball?

This is an instance of a very time dependent process, and again we have to see how we could reach the stationary states of the system after 'ergodization'. One possibility was noted in [**?**]. We normally think that something macroscopic like a shell of matter would behave classically. A fuzzball is also macroscopic, and a very different configuration. Can the shell tunnel to a fuzzball? In principle yes, but the amplitude will be very small, since the action for tunneling would be very large

$$\mathcal{A}_{tunnel} \sim e^{-\frac{1}{G}\int R} \sim e^{-\alpha G M^2} \tag{13}$$

where $\alpha = O(1)$ and we have used the length scale $r \sim GM$ to estimate the Einstein action for tunneling between two configurations that have length and mass scales set to those of the black hole. We would normally ignore such a small tunneling amplitude, but the special point about a black hole is that there are a large *number* of fuzzball configurations that we can tunnel to. The number of configurations is

$$\mathcal{N} \sim e^{S_{bek}} \sim e^{GM^2} \tag{14}$$

Thus we can model the situation by the following toy quantum mechanical problem. We have a quantum trapped in a potential well. The quantum can tunnel to any neighbouring well with a very small amplitude, but there is a very large number of wells. If we then ask what happens to the quantum, we find that the amplitude for it to remain in its initial state vanishes very fast – the wavefunction becomes a linear superposition of states in all the wells. In a similar manner the black hole may be a surprisingly quantum system; even though it is macroscopic in size, it is also characterized by a correspondingly high degeneracy.

7.12 What happens to evolution along the 'good slices' in a black hole geometry?

Consider the traditional geometry of a black hole. We can slice it with spacelike slices in such a way that everything looks regular: the intrinsic and extrinsic curvatures are small everywhere, and all matter on the slices is low energy. Further, the slices capture the infalling matter making the hole, as well as the Hawking pairs that are created. Since the entire evolution seems to be in a semiclassical gravity regime, it looks that we cannot avoid generating the entangled pairs of Hawking radiation, and that these pairs will have no information about the infalling matter.

In [49] the following observation was made. If we make our slices have all the nice properties listed above, then these slices turn out to necessarily have the following behavior: they *stretch* a lot during the evolution. This stretching happens because we need to move the slices up near infinity to capture the Hawing radiation, but we need to keep the slice 'down' near the origin because we do not want the inner edge of the slice to approach the singularity.

In general relativity we can of course bend and stretch our spacelike slices any way we like; this is after all the 'many fingered time' in the language of Wheeler. But in the present case the stretching is order the radius of the hole times a power of $(\frac{M}{m_{pl}})^2$. It is possible that stretching by such factors takes us outside the domain of classical relativity, and new phenomena can arise.

It was noted in [49] that the information problem can be bypassed if we say that classical physics breaks down if we stretch the slice to a length bigger than $\sim \frac{V}{G}$, where V is the volume of the region which is stretched. By itself this appears to be just a postulate pulled out of thin air, but in [71] it was found that for the 2-charge D1-D5 system, a similar 'maximal stretching length' appeared: the maximum depth to which we could stretch the throat of a 2-charge geometry was $\frac{V}{2G}$.

Thus it may be that what we think of as smooth manifolds in general relativity are in realty only low energy effective descriptions of a large but finite number of degrees of freedom. In that case it should not be possible to bend and stretch these manifolds by arbitrary amounts, and the 'good slices' break down at just the place where they start creating the information paradox. (For more details on this argument, see [50].)

8 General comments on the fuzzball proposal

8.1 What are some of the objections that have been raised about fuzzballs?

Let me list here some of the objections and confusions that I have heard about fuzzballs.

- *Are 2-charge holes really black holes?*

  Early work on fuzzballs concerned the 2-charge system. 2-charge holes had been studied for before, in the work of [3]. But it was not clear if these were 'good' examples of black holes, since it was unclear if they has a good horizon.

  This changed after the work with higher derivative corrections, when it was realized that when the R^2 terms are added to the action we get a Bekenstein-Wald [24] entropy that exactly agrees with the count of microstates [77]. I would think that today this objection has vanished; it would be hard to argue that 2-charge extremal holes are in some way essentially different from 3-charge extremal holes.

- *Are fuzzballs classical or quantum?*

  As we discussed in section **??** above, the generic fuzzball is very quantum. The fuzzballs we initially construct and display are classical, and this happens because we have chosen to 'put many quanta in the same state'. As in any quantum system, this makes the state behave classically, and in the cases we studied, allows it to be well approximated by a classical geometry. We use these classical geometries to judge the size and nature of the generic state, and to understand why the state of the hole can change all the way up to the horizon.

  I think that the explicit presentation of classical metrics led some people to believe that the fuzzball conjecture was saying that all black hole states were classical supergravity metrics. This was a confusion, and should be clarified by reading the discussion of quantum effects in section **??** and looking at the papers referred to there.

- *Is it likely that quantum corrections invalidate the fuzzball picture for the 2-charge hole?*

  No, this would be very hard to do. Let us recall the main issue. The paradigm change that we are after is that the infinite throat of the extremal hole gets replaced by a long but finite throat. Thus there is information about the state of the hole at the end of the throat. This is as opposed to the conventional picture where the infinite throat has a horizon at the end with the vacuum state $|0\rangle$ in the vicinity of this horizon; such a structure would have *no* information about the state of the hole in the throat or in the vicinity of the horizon. Once we have found that leading order computations give 'caps' instead of a horizon, the 'boot is on the other leg', as discussed above. To invalidate the fuzzball proposal one would now need to show that any higher order corrections to the cap changes the structure back to an infinite throat with a horizon.

It has been checked that when these corrections are applied to simple coherent fuzzball states of the fuzzball, the corrections are *bounded*; this is important because a correction that diverges at some point could lead to a significant change in structure. Thus to invalidate fuzzballs one would have to show that these (bounded) corrections change the 'cap' back to an infinite throat. To the best of my knowledge, nobody has advanced any argument saying that such a change would indeed happen, but if one does want to argue against the proposal then one should try to see if one can get the quantum corrections to 'remove the cap'. The fuzzball paradigm would just say that the cap gets becomes fuzzy and gets quantum corrections, but does not change to an infinite throat.

- *When a shell collapses through its horizon, it seems hard to imagine that something suddenly changes. Why should we get a fuzzball?*

  This question relates to the dynamics of fuzzballs, while we have constructed time independent states of the hole. But in the section on conjectures about dynamics we had noted that there were two very different time scales in the problem. The collapse happens on the crossing time scale, but to become a fuzzball the hole has to 'ergodize' to a generic configuration, and it can take upto 30% of the Hawking evaporation time to do this. For more details, see the above sections on conjectures on dynamics.

8.2 Is there a simple abstract reason that we have fuzzballs?

It is possible that the fuzzball phenomenon is just an expression of phase space. The characteristic feature of a black hole is that it has a very large entropy. This means that it has a large number of states, which are all orthogonal to each other.

How can we construct such a large number of orthogonal states? In a simple quantum mechanical system like a harmonic oscillator, the ground state has a certain spatial extent and no nodes. The next state must be orthogonal to this; it has one node and spreads out a little more. Continuing in this fashion, we see that if we wish to make many orthogonal states, then they will spread out over quite a large region.

In the traditional black hole we do not see the different microstates of the hole, so we have the 'entropy puzzle': where are the $e^{S_{bek}}$ states of the hole? In a full string construction, we will necessarily see all the states, and these will have to be orthogonal to each other. So the generic state might need to have a typical spread, which will grow with the entropy of the system. From this view it might be that black holes are just high degeneracy bound states which are as densely packed as possible given the number of states that they have.

The relation to phase space may be seen most explicitly in the 2-charge system in the NS1-P duality frame. Each fourier mode of the string is a harmonic oscillator. Suppose we require that when the string carries its oscillations, it stays within a distance r of its central axis. If r is very small, then we are allowed very few states: since the total momentum is fixed, the only way to carry this momentum without large transverse displacements is to put it in a few very high frequency modes, and this gives very little entropy. If we allow a slightly larger r, then we will get more states, and so on, till we reach the typical r for a generic state. The entropy will not increase much for larger r, so this r will set the size of the 2-charge fuzzball. It would be interesting see if such a phase space argument can explain why the surface area of the fuzzball satisfies a Bekenstein type relation with the entropy of the fuzzball.

8.3 What happens to the 'no hair' theorem?

This was not really a theorem in the strict sense of the word, but it does hold for most matter fields around most horizons. If we require regularity of the matter field near the horizon, then it is usually not possible to support a nontrivial time independent field configuration outside the hole, so there is no 'hair'.

With the fuzzball structure, the horizon does not form. For the extremal hole for example, as we go down the throat, we encounter a 'quantum fuzzy cap' before we come to a horizon. This cap is not spherically symmetric, and holds data about the state of the hole. But because there was no horizon, there was no problem with the existence of this 'hair' describing the state.

8.4 What is the significance of the traditional black hole geometry?

One might ask the following question. If we solve the low energy gravity equations, we encounter the traditional black hole solution, which has no information in the vicinity of the horizon (the matter state is $|0\rangle$). Where happens to this state in the fuzzball paradigm?

Let us first note that this traditional metric has a singularity somewhere inside the horizon. So we cannot rigorously argue that the state must exist in the full quantum theory. If a geometry was smooth, then it is reasonable to expect that quantum corrections will only distort it somewhat, but that the state would be there in the final tally of states. With a singularity we have to be more careful. Some singularities are allowed. For example in electrostatics, the singular potential $\phi = \frac{e}{r}$ is allowed, since the electron generates it. The potential $\phi = \frac{1.1e}{r}$ is not allowed since there is no source with that charge in the theory. Thus once there is a singularity, we have to go back to the full theory and see what to do.

The 2-charge extremal case is very illustrative here, so let us see the situation there. Let us work in the duality frame NS1-P. We have seen that the NS1 carries the P charge as travelling waves. Since there is no longitudinal mode for the NS1, we have to choose some transverse direction to bend the string in. This breaks the symmetry either in the angular directions, or in the compact T^4 directions. Thus the microstates do not have the symmetry which we use in writing the naive black hole metric: for the naive metric we assume spherical symmetry in the noncompact directions and translation invariance in the compact directions. Thus for the coherent state geometries constructed in [71] we find no spherical symmetry; each geometry had a 'cap' of a different shape. The traditional spherically symmetric geometry did not appear in the collection of geometries.

What then can we do with this traditional geometry? It would seem that if we 'average' over the different microstates in some way, then we should get a spherically symmetric structure. There is of course no canonical way to average over different metrics (there is no way to say which point in one geometry 'corresponds' to which point in another geometry, and one can also change the different metrics by different coordinate transformations). In the Lorentzian problem, all we have is one microstate out of all the possible ones, and this will not have spherical symmetry.

So how do we consider the collection of all microstates at the same time? One thing that we *can* do is to compute the thermal partition function at some temperature T. This requires us go to Euclidean time, and compute a 1-loop path integral where all the microstates of the system will run along the loop. Thus this 1-loop partition function at least manages to look at all the microstates as a collection.

Now let us imagine that this path integral has a saddle point. Thus we suppose that there is a Euclidean geometry that evaluates the entire path integral to good accuracy. This saddle will be spherically symmetric, and given what we know about the Euclidean computation of entropy, it will be the traditional *Euclidean* black hole geometry. This geometry is like a 'cigar', and has no horizon or interior of the black hole.

Thus we did find a possible role for the traditional black hole geometry, but the *Euclidean* one. What happens if we continue this geometry back to Lorentzian signature? We can formally do that, but the Lorentzian geometry so obtained will have no significance: it is not an actual microstate, and it is not a saddle point that can represent all microstates. So this Lorenzian geometry has no existence, except as a formal, singular solution of the classical equations. Working with this Lorentzian solution creates all the black hole paradoxes, since Hawking pairs get created at the horizon where there is no information. We should instead discard this solution, and keep the actual fuzzball microstates for the Lorentzian states, and the Euclidean 'cigar' as a saddle point for the thermal path integral.

9 Questions from the Paris conference, May 2008

In the summer of May 2008 a workshop was held at Paris to discuss what progress has been achieved on the information paradox in the 30 years that the paradox has existed. Listed below are some topics that came up during the discussions on fuzzballs, and which may not have been sufficiently addressed in the sections above.

9.1 How significant is it if one establishes the fuzzball picture for extremal holes?

Extremal holes face the information paradox just like non-extremal ones, so one would learn something significant if one understands how the information paradox is resolved for these holes.

The study of black holes in string theory usually starts with extremal holes, then moves to near extremal holes, and finally towards general black holes. In general relativity, there was a feeling that extremal holes might be different from other holes in some basic way: while they had a horizon, it has been argued that their entropy should be *zero*. But from string theory we have learnt that extremal holes have entropy just like nonextremal holes, and one should regard extremal holes as a smooth limit of the family of nonextremal holes.

The extremal hole has an infinite throat. Does this mean that something falling in never crosses the horizon, and we never have to worry about the information paradox?

No, this would be incorrect. Infalling quanta cross the horizon in a finite proper time, just as for Schwarzschild black holes. The infalling object creates a horizon behind it (in the classical picture) , and this happens in a finite time even as measured from infinity. Pair creation at this horizon leads to information loss, just as in other black holes.

9.2 Is it possible for extremal holes to be fuzzballs just because we can spread BPS particles in a ball without having the ball collapse?

No, this is not the case. It is true that in the classical limit we can place a set of BPS particles at any set of locations, and make a BPS configuration. But such a configuration is not really BPS when quantum effects are taken into account. If we localize a particle of mass m to within a distance D, then we get a momentum uncertainty $\Delta p \sim \frac{1}{D}$, which causes an extra energy over the BPS mass $\Delta E \sim \frac{\Delta p^2}{2m}$. Thus with quantum effects, we cannot make a BPS state where quanta are just placed at given locations. If we make such a state, the particle wavefunctions would spread, according to the usual rules of quantum mechanics, and the particles will disperse to infinity.

Fuzzballs are *not* made by distributing BPS particles in a ball shaped region. They are true bound states of the charges. Distributing BPS particles like dust in a ball corresponds to having them on the 'coulomb branch', where they are separated from each other. The bound states describe the quanta on the 'higgs branch' where they cannot be separated from each other. One can count the exact BPS ground states of the branes, and find the entropy of the hole; these states are exact bound states in the sense that they have mass=charge to all orders in the coupling, not just at some leading classical order.

When branes make a fuzzball, they bind together, dissolve into fluxes, and create 'dipole charges' which are held apart at fixed distances by 'push-pull' forces caused by the fluxes between the charges. This separation of the dipole charges gives the fuzzball its size. Thus the bound states of branes is not at all like branes scattered around in a ball shaped region.

In particular, if we add a small amount of energy to a dust cloud of branes, then the system will tend to collapse under its self-attraction. But for the fuzzball geometries, we can look at small non-BPS excitations, which appear as modes localized in the 'cap' region of the geometry. For all cases where this computation has been done, the energy of such modes agrees exactly with the energy gap expected in the dual CFT. The fuzzball does not collapse; instead the localized mode leaks out of the throat and escapes to infinity as 'Hawking radiation' at exactly the rate predicted by the CFT computation. Thus we again see that fuzzball geometries are not just a distribution of BPS branes, and it is nontrivial to construct them even for extremal holes.

9.3 How compact can a non-extremal fuzzball be without collapsing into a black hole?

First, we note that the fuzzball proposal does not require that the typical fuzzball be of horizon size. All we need is that the horizon not exist, but the fuzzball can be much larger than horizon size. For 2-charge extremal states we the size of the typical fuzzball depends on how we define 'size': by the definition in [76] it is order horizon size, while by the definition used in [31] it is larger than horizon size.

Let us see of there is some reason to believe that the typical fuzzball *cannot* be horizon size. There is a theorem (given in the textbook by Wald) that for 3+1 dimensions, spherically symmetric stars cannot have a radius smaller than $9M/4$ if the geometry has a timelike Killing vector. If the star tries to be smaller, then the pressure at the origin diverges.

It is interesting that this result cannot be applied as such to place a lower limit on the size of fuzzballs. For one thing, the fuzzballs are not spherically symmetric. But more important, all the non-extremal fuzzballs that have been constructed are such that there is no timelike killing vector. The geometry is either time dependent, or if it is time independent then there is an ergoregion where the killing vector becomes spacelike. This is important, because in either case there will be no time-independent vacuum, and there will be pair creation. In [28] it was found that this pair creation in ergoregions gave the Hawking radiation from the nonextremal geometry being studied. (These geometries were constructed in [26], and their radiation was computed in [27].) Thus we expect that this absence of a Killing vector will be a generic feature of all non-extremal fuzzball geometries, and so the above quoted limit on the radius of stars cannot be used to constrain the allowed sizes of fuzzballs. Of course the generic fuzzball will be very quantum, and in that case it is unclear how classical theorems should be used for them.

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