Fuzzballs, Firewalls and all that ...

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1 Early history

1.1 What is a black hole?

Black holes are probably the most fascinating objects in the Universe. They result from the attractive nature of gravity, which pulls any object towards any other object. A large ball of matter thus tends to compress under its own gravitational attraction. Remarkably, if this ball is large enough, the other fundamental forces – electromagnetic, weak and strong – are unable to resist the compression, and we get a *runaway collapse*. As the ball shrinks to smaller sizes, the pull of gravity becomes stronger and stronger, till the entire ball ends up in a point. Imagine the mass of the sun, compressed to a dot!

But even more fascinating has been the story of how black holes relate to the ideas of quantum theory. Gravity, quantum mechanics, thermodynamics and information theory all come together in resolving a profound puzzle, found by Stephen Hawking in 1975 [1].

1.2 Hawking's puzzle

Einstein taught us that a particle of mass m has an intrinsic energy $E = mc^2$. But if this particle is placed in a gravitational field, then it also has a potential energy, which is *negative*. The crucial point is now the following. If we lower a particle into a black hole, then the potential energy of gravity is sufficiently negative to outweigh the intrinsic energy mc^2, resulting in a *net negative* energy for the particle. This leads to a strange situation: we can create a pair of particles: one inside the hole with negative energy, and one outside with positive energy, without violating overall energy conservation.

A black hole does not start with such a pair of particles, but what Hawking found was that quantum mechanical fluctuations automatically create such pairs. The outer member of the pair floats off to infinity, as 'Hawking radiation'. The inner member falls to the center of the hole, reducing the hole's mass since it has negative energy. The black hole evaporates away, though the process is extremely slow – a solar mass hole would disappear in 10^{63} years, much longer than the age of the Universe!

The problem comes when we look at the nature of the created pair. The outer particle could be an electron; by charge conservation, the inner particle must then be a positron. But equally, the outer particle could be a positron and the inner particle an electron – quantum fluctuations in fact produce a state that is a superposition of these two possibilities. Thus the outer particle has no state by itself; it is an electron *if* the inner particle is a positron, and it is a positron *if* the inner particle is an electron. Such states are called 'entangled states', and are common in quantum theory.

The problem comes when we recall that the black hole is evaporating away. When the hole disappears, all that is inside it disappears as well. But then what state do we assign to a particle that emerged in the radiation left outside? This particle had a state that was conditional on the state of the particle in the hole, but the latter particle has vanished from the Universe! The radiation is left with no well-defined state at all, so quantum theory seems to have broken down. This is Hawking's puzzle; it came to be called the "black hole information paradox" [1].

1.3 The no-hair theorem

Other things also burn away to radiation – for example a piece of coal can burn away to photons – but we don't face a violation of quantum theory. The difference is that the radiation from the coal is produced from the atoms on the surface of the coal, while with black holes the radiation quanta were pulled out from the *vacuum*. If we could alter the black hole so that we had a normal surface at the horizon, instead of just a vacuum region, then we would evade Hawking's paradox.

But such an alteration is hard to obtain, for a simple reason. Anything placed at the horizon gets sucked into the black hole by the gravitational field of the hole, and the region around the horizon returns to its vacuum state. Efforts to add structure at the horizon proved so frustrating that Wheeler coined the phrase: "Black holes have no hair"; i.e., black holes are 'bald', with no structure at the horizon. Making some reasonable looking assumptions, people even managed to rigorously prove this claim; these proofs were called 'no-hair theorems'.

The information paradox is thus a combination of *two* things: (a) The no-hair theorems which say that the region around the horizon must be the vacuum and (b) Hawking's computation which shows that that this vacuum creates entangled pairs; these lead to a violation of quantum theory when the black hole evaporates away.

We will now see that the fuzzball paradigm resolved this problem by invalidating (a); the fuzzball construction gives structure at the horizon that does not 'fall in'. Other attempts to solve the problem attack (b), typically by modifying some tenet of quantum theory.

1.4 The entropy of black holes

In 1972 Bekenstein [2] had noticed an odd fact: for consistency of thermodynamics, a black hole must be assigned an entropy S equal to its area measured in planck units: $S = A/4G$. By the rules of normal thermodynamics, this implies that the hole should have

$$N = e^S \tag{1}$$

different possible states. For a solar mass hole, this implies

$$10^{10^{77}} \tag{2}$$

states! This number is vastly larger than the states that normal matter would have. Worse, the 'no-hair' theorem implies a unique shape for the hole, so the hole seems to have only *one* state. So what did Bekenstein's entropy mean?

In 1996 Strominger and Vafa [3] made an interesting discovery when looking at states in string theory. First they imagined that the force of gravity is switched off. Then we just have the normal matter of string theory – strings and branes – and we can count all the states allowed for some total mass M. When gravity is switched back on, we expect a black hole, and do not know how to count states. But now we can use Bekenstein's formula and infer a number of states from the area of the horizon. The zero gravity calculation and the black hole entropy gave numbers that exactly agreed, indicating that Bekenstein's entropy was not fundamentally different from the entropy of normal systems.

But we still had a picture of the hole where the region around the horizon was the vacuum, and so we had no resolution of the information paradox. To resolve the paradox, one has to look at the *structure* of the states in string theory. This is what the fuzzball program did.

2 The fuzzball paradigm

2.1 Fuzzballs: beginnings

In 1997 I was trying to extend some work I had done with Sumit Das a year earlier, which had shown that the string states studied by Strominger and Vafa radiated energy exactly as *black holes* should [4]. In the process something curious emerged. With gravity switched off, string states had a certain size, which could be computed and held no surprises. But remarkably, when gravity was switched on, the string states *expanded* instead of getting compressed under their self-gravity [5]. In fact the size of the string state was always of the same order as the expected horizon size of the black hole, suggesting that no true horizon forms at all! This curious behavior resulted from a property of strings and branes called 'fractionation', and suggested that states in string theory might behave very differently from what had been expected.

2.2 The structre of fuzzballs

The natural next step was to actually construct the states of the hole, starting with the simplest states. In work with Oleg Lunin, the states of the simplest black hole – called the 2-charge hole – were constructed, and it was found that *none* of these states had a horizon; thus their surface was more like a planet than like the traditional picture of a black hole [6]. Many other people joined this program; in particular my students Saxena, and Srivastava, and postdoc Giusto. Soon an interesting picture emerged for the states of black holes in string theory [7]. Let us understand this picture schematically with the help of fig.1.

In fig.1(a) we simplify the black hole by showing only 1 dimension instead of 3. Then the ball representing the hole looks like a line, with the singularity in the middle and points on either side marking the horizon. Next, we recall that string theory has extra dimensions, which are believed to be curled up into small circles. We draw one of these circles, and the space then looks like a drinking straw, as depicted in fig.1(b).

The idea of extra dimensions is an old one, but because the size of the corresponding circle is so small, it was always assumed that its presence would introduce no new physics at the horizon. With such an assumption, one would get the picture depicted in fig.1(b); we still have the mass concentrated in the center, with horizon as indicated. If this was indeed all that happened with extra dimensions, we would have solved nothing, since we still have a vacuum around the horizon.

But there is a completely different structure possible with a drinking straw, shown in fig.1(c),(d), and described as follows:

(i) First, cut out and discard a middle section from the straw – the part inside the horizon.

(ii) Now close up the open ends of the straw, by adding hemispherical 'caps'. This gives a smooth space, which has no horizon or singularity.

(iii) The mass M at the center has now vanished. But the curvature of the 'caps' contributes energy, and these caps also contain other energy carrying objects of string theory (strings, branes, fluxes). This energy E makes up the mass M which the hole should have, through the relation $E = mc^2$.

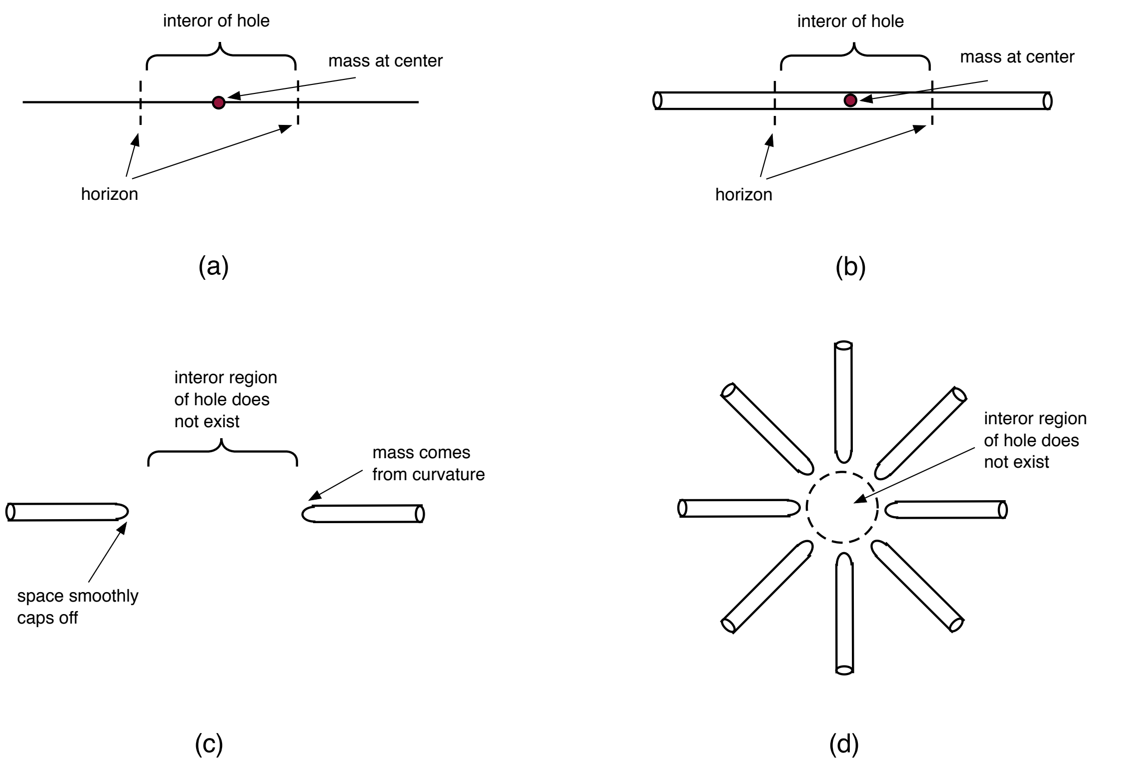

Figure 1: (a) A 1-dimensional slice through the traditional picture of the black hole. The mass M is at the center, and the horizon shows up as two marks on either side of this central mass. (b) A similar depiction of the traditional hole, in a space with an extra compact dimension. (c) The actual solution in string theory is very different: the interior region of space is missing – the space 'caps off' just outside the horizon on either end. There is no central mass; its energy is nor carried by the curvature at the 'caps'. (d) The same as (c), now with all dimensions shown, The space ends just outside the region where the horizon would have been.

(iv) Returning to the full 3-dimensional problem, we get the picture in fig.1(d); there is a somewhat different shape to the cap in different directions, and all the possible choices for these caps give all the states of the black hole.

We see that there is nontrivial structure just outside the place where the horizon would have been, but this structure does not 'fall in' – there is no place for it to fall into, since spacetime ends outside the horizon.[1] The black hole now has a surface just like any other star or planet, exactly what was needed to solve the information paradox! In fact the radiation from the surface of a simple set of fuzzballs was explicitly computed [9]. In a work with Borun Chowdhury, I found that the energy spectrum of this radiation *exactly* agreed with the predicted Hawking radiation for these special states. But the *details* of the emitted quanta were very different: they were not entangled in the way that Hawking had feared, and therefore there was no information paradox [10].

3 The Bena-Warner microstate program

Gravity is a nonlinear theory, and such fuzzball solutions were in general very hard to construct. Remarkably, in 2005 there appeared a pair of papers – one by Bena and Warner, and one by Berglund, Gimon and Levi – which reduced the nonlinear equations for fuzzballs to a set of *linear* equations. Suddenly it became possible to obtain a plethora of microstates with no horizons, and Bena and Warner developed a sequence of tools to make and analyze such solutions [11].

But what about the 'no-hair theorems', which argued that such solutions should not be possible? It turned out that these theorems had been proved after making certain assumptions, which appeared natural for a theory in 3 space dimensions. But in a nice paper, Gibbons and Warner [12] showed that when the 3-dimensional theory was derived as an effective theory from a 10-dimensional theory like string theory, these assumptions were *violated*. Thus black holes had hair after all, and the fuzzball constructions showed how to find examples of this hair.

The program of Bena, Warner and their many students and collaborators has gone very far in convincing people that black holes can indeed have hair. People realized that they had, in the past, just missed this vast set of solutions which correspond to the actual states of the hole. This program has recently obtained even larger classes of solutions, with a structure called a 'superstratum' [13].

It is interesting to see the progress in the construction of horizonless microstates:

(a) All states of the simplest black hole – the 2-charge extremal hole – have been constructed.

[1] The structure here is roughly analogous to an old construction of Witten called a 'bubble of nothing'; there too, an interior region of space is missing, and space ends smoothly at the surface of this 'bubble'.

(b) For the standard Strominger-Vafa black hole, the most basic states were constructed first. But people asked: are all these states truly states at the horizon, or could some of these be states that have structure far away from the horizon? In response to this question, states away from the horizon were identified, and it was shown that the remaining states that had been constructed indeed were located at the position where the horizon would have traditionally appeared. In the program of Bena and Warner, large families of states were constructed that appeared at the right 'depth' to correspond to states that live just outside the horizon.

(c) Generic black hole states should break all symmetries of the background. There was a steady progression in this direction, and now one can see in principle how all such symmetries will be broken by horizonless solutions; this constructions uses the idea of 'supertubes' placed inside known microstate structures.

(d) Some people were concerned that the microstates constructed for extremal holes may not belong to a fundamental set of states counted by something called an 'index'. But the states constructed in [13], for example, are in fact those counted by the index. The Strominger-Vafa black hole was defined in 4+1 dimensions, but it has proved equally easy to make horizonless microstates for 3+1 dimensions. Here too, the states are seen to belong to the required index.

(e) It is important that the fuzzball construction extends to nonextremal states as well. A family near extremal states were constructed in [14]. States with *no* charge but maximal rotation were constructed in [15].

This steady progress has brought us to the point where it does not make sense, in my opinion, to deny that the resolution of the information paradox in string theory must be through the fuzzball microstate construction.

4 The small corrections theorem

In spite of this concrete success, some people were still not convinced that one needed an alteration of the horizon; the traditional picture with a vacuum horizon was too well entrenched in people's minds. But if one had the traditional horizon, then what was the solution of Hawking's puzzle? In answer to this question, there emerged a belief that Hawking's initial argument might be *incorrect*. After all, Hawking had computed the entanglement of the created pairs of particles to a first approximation. Everyone would agree that there could be small corrections to this entanglement, say of order ϵ, with $\epsilon \ll 1$. Normally a small correction should not change an argument, but a black hole emits a *lot* of particles before it evaporates. Could it not be that these small corrections accumulate through the process of evaporation, and being so numerous, alter Hawking's conclusion?

It should be noted that no one actually showed how these small corrections would get us out of Hawking's puzzle. But if one did not accept fuzzballs, then one had to look to some such way out of the problem. In fact in 2004, based on a similar argument, Hawking retracted his belief that his argument implied information loss. But almost no one else understood this retraction; after all Hawing had shown no explicit way out of the entanglement problem.

In 2009 I derived a theorem which showed that *no* set of small corrections could resolve Hawking's entanglement puzzle [16]. The power of this very general proof came from a powerful theorem in quantum information theory: the strong subadditivity of entanglement entropy. The conclusion in [16] was that "*One needs order unity corrections at the horizon to resolve the information paradox.*". This theorem, in a sense, completed the fuzzball argument: One now knows that one *must* have structure at the horizon, and fuzzballs have provided concrete constructions of this structure. Steve Avery later made a nice extension of the argument to more general emission patterns [17].

The theorem made only one assumption: namely, that once the emitted radiation quanta went far away from the hole, then they had 'normal' physics; that is, they were no longer influenced by the hole. As we will note below, two groups: Papadodimas and Raju [18], and Maldacena and Susskind [19] have assumed a violation of this condition, and made models where the horizon was still a vacuum. We can thus state the fuzzball proposal, and how it differs from other proposals to resolve the information paradox:

The fuzzball proposal: *The no hair theorem is violated, so that there is an order unity change to evolution at the horizon; the fuzzball construction gives explicit examples to show how this can happen. Thus the hole burns away just like a piece of coal; no new physical principle needs to be invoked.*

By contrast, the proposals of Papadodimas and Raju, and of Maldacena and Susskind, say that the burning of coal and the evaporation of a black hole are fundamentally different. For the coal, the space of states of the radiation far away is distinct from the space of states of the coal. But for the black hole, the states allowed for the radiation are *identified* with the states allowed for the hole left behind. Thus a new physical principle is being proposed to deal with black hole evaporation. New principles were also proposed in [20] where the initial value formulation of quantum mechanics was altered, or in 'traditional complementarity' (discussed below) by Susskind where different observers were allowed to see different states (and thus duplicate information).

5 Why is the semiclassical approximation violated?

There is a last question that many people asked: given that black holes are so big, why do effects of string theory so completely alter their structure at the horizon? After all, the horizon radius is 3 Km for a solar mass black hole, while string theory effects are normally expected at the planck scale, which is 10^{-33} cm.

This has of course been the central question with the information paradox: it was always hoped that quantum gravity would help solve the problem, but given the length scales involved, how could it ever help?

The answer to this puzzle was given in [21], where it was argued that there is a profound cancellation between two effects; one large and one small. Suppose a star is collapsing to make a black hole. Classically, nothing seems to impede this process. But quantum mechanically, there is a very tiny probability for the star to transition to a *fuzzball* state. The reason why this very small transition probability is relevant is the following: the number of possible fuzzballs is a very *large* number; given by

$$N = e^S \tag{3}$$

where $S = \frac{A}{4G}$ is the Bekenstein entropy. Even though there is a tiny probability to transition to any one fuzzball state, we must multiply this tiny number with the number of possible fuzzball states that we can transition to, to get the *overall* probability of transition. In [21] it was argued that these two numbers – one large and one small – are such that they can cancel. Recently, in a work done with Per Kraus, it was shown that they do in fact cancel *exactly*.

We thus see a remarkable confluence of factors. Black holes were characterized by an abnormally large entropy, but we did not know what to do with this fact, since we did not know what kind of states this entropy counted. Now that we can see these states explicitly as fuzzballs, we find that they solve the other feature of black holes: the apparent paradox with entanglement. The large number of fuzzball states overwhelms the classical physics of the black hole, and saves us from Hawking's paradox.

6 Fuzzball complementarity

The fuzzball proposal solves the information paradox. But we can ask a different question: what does an observer feel if he falls into the black hole? In the traditional hole, he would pass uneventfully through the horizon, since the region around the horizon is the vacuum. In a fuzzball, on the other hand, spacetime ends just outside the horizon, so he cannot proceed any further. So why did the fuzzball people not immediately say: the fuzzball surface is like a 'brick wall' or like a 'firewall'? The reason is that we have an interesting possibility called *Fuzzball complementarity*: Even though the infalling object must stop at

Figure 2: (a) In the traditional hole, a particle just falls in; quantum mechanically, this behavior is described by some frequencies. (b) In the fuzzball, there is no place to fall in, but the fuzzball surface vibrates with some frequencies. The fuzzball complementarity conjecture says that these frequencies agree with those in (a) in the limit $E \gg T$; thus freely infalling objects suffer a behavior that 'mimics' free infall.

the fuzzball surface, the dynamics of the impact could *mimic* infall through a horizon [22]. To see what this means, consider a simple example.

In one room, we place a piano; and in a second room, we place an electronic keyboard. The piano produces sounds through vibrations of strings; the different notes correspond to different frequencies of vibration of the strings. The electronic keyboard produces its frequencies by oscillations of electric currents in electronic circuits, a very different mechanism! But the set of allowed frequencies is approximately the same in the two cases, so it is hard to tell if the instrument playing in a room is a piano or an electronic keyboard. We say that the keyboard 'effectively mimics' the piano, though it is a completely different physical object.

In quantum theory, any dynamical system is described as a set of allowed frequencies of oscillation.[2] We can choose to excite any combination of these frequencies, just like we can press any set of keys on a piano. Each choice of excited frequencies corresponds to a different dynamical behavior of the quantum system, and all allowed dynamical behaviors are obtained in this way.

Now we can understand the idea of fuzzball complementarity. The traditional black hole has a simple dynamics: things fall in. This infall motion is described by a set of frequencies. The fuzzball is quite different: its surface oscillates when hit, again with some set of frequencies. Suppose these two sets of frequencies were approximately the same? In that case, oscillating the fuzzball surface would mimic infall!

There is one very important point here, however. The traditional black hole has just one state – it has no 'hair' – but the whole point of fuzzballs is that there is a different fuzzball for each one of the e^S states of the black hole! So the fuzzballs *cannot* all have

[2] These frequencies are called 'energy levels', since energy E is related to frequency ν by $E = h\nu$.

the same frequency spectrum, and so they cannot all mimic the traditional hole perferctly. The conjecture of fuzzball complementarity actually says the following:

(i) We know that the radiation from the black hole emerges in quanta that have a very low energy; this energy is of order T, the temperature of the hole. When we focus on this low energy $E \sim T$ physics, all fuzzballs appear different, as they should. The radiation therefore differs in its details for different fuzzballs, and that is why it is able to encode the information of the corresponding black hole state.

(ii) But now consider a particle that falls in freely from far away. Because of the gravitational field of the hole, this particle speeds up, and impacts the fuzzball surface hard, with energy $E \gg T$. The resulting large oscillations of the fuzzball surface are very similar between different fuzzballs, and they have approximately the same spectrum of frequencies. The fuzzball complementarity conjecture says that *this spectrum of fuzzball oscillation frequencies under $E \gg T$ impacts is the same as the spectrum of frequencies describing infall into the traditional hole.*

Note that all dualities in string theory work in a similar way, though they do not involve an approximation like $E \gg T$. For example in AdS/CFT duality [23], we can either consider (i) The spread of a collection of gauge fields on a 3-dimensional surface or (ii) infall of a particle in 4-dimensional anti-de Sitter space. The two situations are completely different, but because they have the same frequency spectrum, one situation can be mapped exactly onto the other.

What is good about the fuzzball complementarity conjecture is that it solves the information paradox (which deals with $E \sim T$ emission), while also allowing us to keep the intuition of the traditional hole for typical infall processes (which naturally have $E \gg T$).

7 The firewall claim

In 2012 four authors – Almheiri, Marolf, Polchinski and Sully (called AMPS in what follows) put out a claim: “there is a firewall at the horizon of a black hole” [24]. Few people accepted the claim, but it did lead to a large amount on confusion: what had the authors proved, and how? Their proof seemed to have just a few lines, where they looked at Hawking radiation using the tools of the ‘small correction theorem’ referred to above in section 4. How can this tell us that there is a firewall at the horizon?

We will first explain what AMPS did and did not do. We will then note that AMPS made an extra assumption in their analysis, but this assumption is in conflict with what is known as the Bekenstein limit on entropy. If one removes the problematic assumption, the AMPS argument does not work. This problem with their argument was explained in [25].

Let us first clarify what AMPS did *not* do:

(i) AMPS did not give any mechanism by which a firewall (or any other structure) can stand at the horizon (without falling in). They point to fuzzballs as a possible construction of such structure in string theory, but they are not confining themselves to any particular theory of gravity.

(ii) AMPS did not claim that they have proved the need for structure at the horizon: this was done by the small correction theorem that I proved in 2009 [16], and they cite this theorem for the nature of the tools they will use.

Now we can say what they *did* do. The small corrections theorem had proved that there must be order unity changes to the low energy physics around the horizon. AMPS added one extra assumption:

Assumption: The outside of the hole can be described by traditional 'effective field theory' all the way upto the horizon. In particular, if a particle is falling onto the hole, then it will feel only the traditionally known effects till it reaches the horizon.

With this assumption, they used the bit model of [16] to argue that *there can be no effective description of the black hole interior which will mimic the vacuum.*

Thus what AMPS were addressing was the idea of complementarity. *If* someone gives a structure at the horizon that solves the information problem, and *if* this new structure involved no changes to the physics outside the horizon, *then* the interior of the hole cannot have the same effective dynamics as the vacuum.

7.1 What kind of complementarity were AMPS addressing?

The AMPS paper caused quite some confusion because it was not clear what was being assumed and what was being proved. Some thought AMPS were proving there was structure at the horizon, but they had no construction of any such structure; that was the job of the fuzzball program. Others thought AMPS had proved that there must be changes at the horizon, but this had already been proved by the small corrections theorem. What AMPS were actually seeking to do was address the old idea of complementarity – *if* in one description information somehow reflects off the horizon, then (with their extra assumption), one cannot have a complementary description where the new physics of the black hole interior can be mapped into the physics of the vacuum. Thus their paper was correctly titled: "Complementarity or firewalls?": *if* someone finds structure at the horizon, then will a person falling on this just get destroyed (feel a firewall), or is there a possible effective complementary description giving the vacuum?

But two years before this AMPS paper, we had already conjectured a kind of complementarity – fuzzball complementarity [22]. How did this square with what AMPS were saying now?

Together with David Turton, I looked at the AMPS paper in detail. Almost immediately one thing became clear. In the idea of fuzzball complementarity, the effective or 'complementary' description was only obtained as an approximation for impacts with energy $E \gg T$. But AMPS were not trying to make any approximation that separated high and low energies! On the contrary, they talked of lowering an observer gently to the horizon, and scooping up $E \sim T$ Hawking radiation quanta.

Thus AMPS were addressing what I will call 'traditional complementarity', an idea discussed by Susskind in 1993 [26]. In those days no one knew how to construct any structure at the horizon. So Susskind proposed that black holes have a novel kind of physics: observers outside and inside the hole see *two different copies of reality*. Thus information will be outside in the outer observer's description, and fall inside the hole in an infalling observer's description.

There were obvious difficulties with the 'traditional compelmentarity' proposal: normal quantum theory has just one wavefunction Ψ, not a different wavefunction for different observers. And since the 'no -hair' theorem had not been broken in 1993, there was no way to really satisfy the needs of the outer observer, who needs that information be reflected back to him from the horizon (instead of falling inside).

What AMPS did was formulate 'traditional complementarity' in a more rigorous fashion, and then prove that it was not possible. As noted above, they did not look for an approximate complementarity in the limit $E \gg T$, so they were not addressing fuzzball compelmentarity. But could their arguments be extended anyway to say something about fuzzball complementarity? Turton and I took a deeper look at their paper and found something very curious: their underlying assumption, stated above, would probably not be good in *any* theory of gravity.

7.2 The conflict of AMPS with the Bekenstein limit

Bekenstein's argument says that the entropy of a black hole is given by $S = A/4G$, the area of the horizon measured in planck units. Since then it has been commonly accepted that:

It is not possible to have an entropy larger than $\frac{A}{4G}$ *inside a surface of area* A.

AMPS had assumed that the incoming particle notices nothing new till it hits the 'stretched horizon'.[3]. Let this infalling particle be a 'bit' having have one unit of entropy.

[3](The stretched horizon is a surface drawn a planck distance from the horizon, demarcating what should be called the 'outside' and 'inside' of the hole

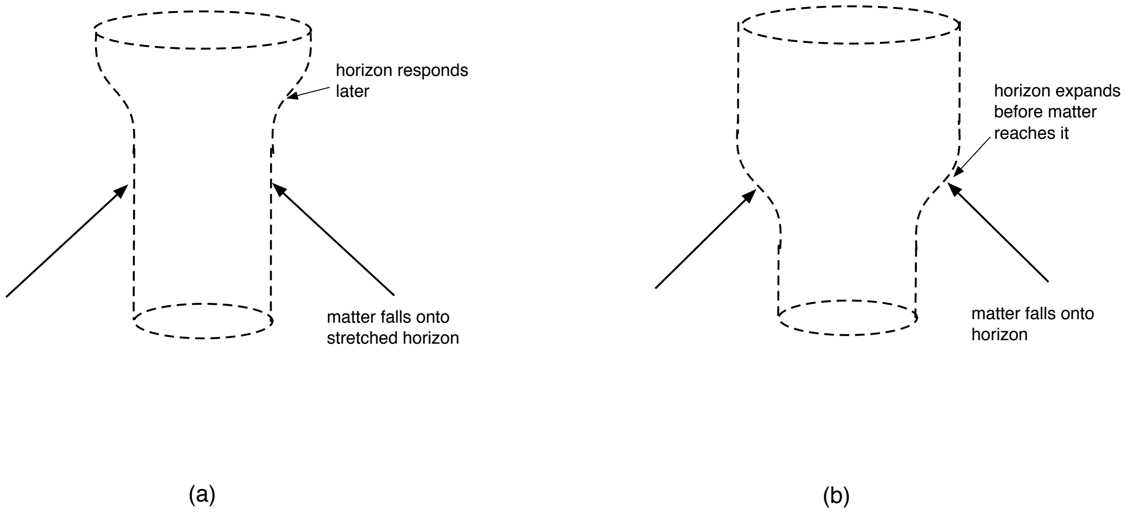

Figure 3: (a) AMPS assumed that the boundary of the hole – as defined by the 'stretched horizon' – responds only after infalling matter reaches it; this assumption violates the Bekenstein limit. (b) In standard general relativity, it is known that the horizon expands *before* matter reaches it; that is why the Bekenstein limit is not violated.

If it lands on the stretched horizon *before* the stretched horizon responds, then for a moment we have

$$\frac{A}{4} + 1 \tag{4}$$

units of entropy on a surface of area A, which is forbidden by the Bekenstein theorem.

One might ask: does this problem not always happen when anything falls into a black hole? The answer is no, because as is well known, the horizon expands in area *before* it absorbs the incoming particle. We depict this in fig.3. Thus normally we always expect the Bekenstein limit to be obeyed.

With fuzzballs, the fuzzball surface is expected to track the 'apparent horizon', so it is also expected to 'tunnel out' to meet the incoming particle. This involves the tunneling mechanism discussed above.

But AMPS assumed that the boundary of the hole – the stretched horizon – did not expand until it was hit by the incoming particle. This violates the Bekenstein limit. In [25] it was shown that if we drop this assumption that AMPS make, then their argument does not work.

8 Summary

The fuzzball program has given a comprehensive resolution of the information paradox. This program has four aspects:

(A) The fuzzball construction. This gives explicit examples of states that have structure at the horizon which does not 'fall into the hole' . We also understand why this construction bypassed the no-hair theorems (sections 2.2,3).

(B) The 'small corrections theorem'. This says that there must be an order unity change in physics at the horizon for all states of the black hole, if we are to avoid the information paradox without invoking new physics (section 4).

(C) An argument saying how the semiclassical approximation got violated at scales as large as the black hole horizon. The small amplitude for transitioning to a fuzzball state is cancelled by the large number of states e^S that we can transition to (section 5).

(D) The conjecture of fuzzball complementarity. This recovers the classical intuition of 'free infall' in an 'effective' or 'complementary' description for high energy processes $E \gg T$ (section 6).

The firewall argument is quite different. It

(A') *Assumes* that some (unspecified) construction solves the information paradox by modifying the horizon; they agree that fuzzballs could be an example of such a construction, but are not limited to any particular theory of gravity.

(B') It uses the bit model of (B) above (and the same tool of strong subadditivity), but adds an extra *assumption* that no effects other than the traditional semiclassical ones show up until a particle reaches the horizon. AMPS then argue that *the physics of the black hole interior cannot be mapped to vacuum physics.*

(C') The difficulties with this argument are as follows:

(i) The AMPS argument made no attempt to take a limit $E \gg T$ in getting complementarity. Thus they were addressing 'traditional complementarity' (the conjecture studied by Susskind), and not fuzzball complementarity. Susskind's conjecture needed new physics anyway, so it was not clear how to make it work.

(ii) More seriously, AMPS assumed that the 'stretched horizon' would not respond till an infalling particle reached it. This violates the Bekenstein limit which says that one cannot have entropy more than $\frac{A}{4G}$ inside an area A. Thus it is unclear how the AMPS argument would work in *any* theory.

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