TOPIC VII

ADS/CFT DUALITY

The conjecture of AdS/CFT duality marked an important step in the development of string theory. Quantum gravity is expected to be a very complicated theory. String theory provides a consistent framework for quantizing gravity, but this framework has been mostly perturbative: it gives explicit answers when we have a few strings or branes interacting with each other. What complications lie beyond this limited domain?

The conjecture of AdS/CFT duality says that the entire dynamics of string theory can be given an alternate description: one in which we just have a standard quantum field theory, with *no* gravity. In the simplest case, this field theory is just a gauge theory, given by the Yang-Mills Lagrangian. It would seem that at one stroke we have banished all the dark mysteries of quantum gravity: since gauge theories can be completely solved in principle – perhaps by putting the theory on a computer – we have, at least in principle, obtained an understanding of all of quantum gravity.

At first it would appear that we can also use this duality to resolve the black hole information paradox. After all the black hole is just another object in string theory, and so it should be possible to recast questions about black holes as questions in ordinary quantum field theory. But here we run up against several difficulties. Firstly, while AdS/CFT duality says that there is a one-to-one and onto map from string theory states to field theory states, it does not actually give us this map. Secondly, the dual field theory turns out to be a 'large N' theory, so we have to understand the dynamics of a large number of degrees of freedom. Lastly, and perhaps most importantly, this large N theory needs to be studied at strong coupling – a domain where the perturbative treatment using Feynman diagrams gives little useful information. Thus while the field theory should presumably know about the physics of the black hole, this knowledge is locked up in the complexity of a large N strongly coupled gauge theory. In consequence the field theory description of black holes turns out, in many ways, to be harder to analyze than the original gravity description.

As it turns out, AdS/CFT duality does answer an important question about black holes, but this question is different from the question: "What is the resolution of the information paradox?". Suppose we ask: "Can there be a consistent theory of quantum gravity that looks like ordinary gravity at low curvatures, but in which we do not run into inconsistencies from black holes?" Here AdS/CFT duality does give an answer. We can see explicitly that the field theory gives an adequate dual description of the regime where gravitational effects are *weak*. This weak gravity regime is not the regime of black holes. But we can simply define the full gravity theory as whatever we get as the dual of the field theory. Then there can be no violation of quantum mechanics in this gravity theory, since there is no such violation in the field theory.

We will see that AdS/CFT also does not allow the possibility of remnants. Thus we have made some progress. If we define string theory as the dual of a field theory, then we rule out the information loss initially suggested by Hawking, and also the remnant scenario favored by many relativists.

But the information paradox poses a somewhat different question; one that arises as a conflict arising from two different observations:

(a) When we look at the gravity theory in the domain where we do make black holes, we find the 'no hair theorems' which say that the black hole will suck everything in and leave the vacuum around its horizon.

(b) Pair creation around this vacuum horizon then creates an ever growing entanglement between then radiation and the remaining hole, and this is what leads to a problem at the endpoint of evaporation.

So we ask: can AdS/CFT duality say anything about (a) and (b)?

It turns out that the idea of AdS/CFT duality by itself cannot tell us anything about how this problem is to be resolved. In fact all of the following possibilities have been suggested, even after many years of work with the duality:

(i) The black hole has its traditional vacuum horizon to leading order, but subtle quantum gravity effects slightly modify the state of the emitted radiation quanta; these small corrections remove the entanglement between the radiation and the hole, and encode the information of the initial matter into the outgoing radiation.

(ii) The no-hair theorem is violated in string theory, as in the case with fuzzballs; thus the traditional black hole does not form, and this resolves the puzzle.

(iii) The black hole has its traditional vacuum horizon, but novel nonlocal effects arise when black holes form. For example, tiny wormholes can link the hole to the radiation quanta near infinity. These effects modify the usual entanglement relation between the hole and the emitted radiation, and somehow resolve the paradox.

(iv) AdS/CFT duality itself breaks down at the threshold of black hole formation. Thus black holes have a traditional horizon, but the interior of the black hole spacetime is not captured by the field theory description.

Possibility (i) was suggested by Maldacena in [?] in 2001. But as we will see shortly, it can be proved that small changes to the state of the emitted radiation *cannot* remove the entanglement, so we now know that this is not a possible resolution of the puzzle. Possibility (ii) is, of course, the resolution we have proposed for the paradox. But the fact that string theory states do not form horizons was first discovered by studying the heuristic properties of intersecting branes, some six months before the advent of AdS/CFT. Thus while AdS/CFT has been a very useful tool in the further development of the fuzzball paradigm, we cannot deduce this paradigm from the duality. Possibility (iii) was suggested in 2013 by Maldacena and Susskind as an alternative to the fuzzball resolution; we will describe it in a little more detail later. Possibility (iv) was discussed in 2014, again as an alternative where the traditional vacuum horizon would be preserved.

Thus the duality itself does not suggest an answer to the information puzzle. It is difficult to extract useful information from AdS/CFT because the field theory does not easily access the regime of energies where black holes form, and also because the notion of locality in the gravity description becomes hard to recover in the field theory description. Nevertheless we will see that the idea of AdS/CFT will be very valuable in two ways. First, it will help us to argue that the fuzzball solutions we construct in the gravity description are directly related to the class of all black hole states, since it is easier to characterize the full set of states in the field theory. Secondly, the duality suggests the idea of *fuzzball complementarity*, which will address the question of what happens when we fall onto a fuzzball; we will come to this later.

Let us now study the remarkable conjecture of AdS/CFT duality.

Lecture notes 1

Creating 'space'

When we study interactions caused by electromagnetic, weak or strong forces, we use flat spacetime. The interactions can be described, at least for weak couplings, by Feynman diagrams. One particle emits a 'gauge boson'; this boson travels to the second particle which absorbs it.

We have described gravity, on the other hand by changing spacetime from flat to curved. On this curved spacetime, we say that gravity is no longer a force; the curvature of spacetime encodes all the effects of gravity. The advantage of this perspective is, of course, that it automatically builds in the equivalence principle – the equality of inertial and gravitational masses, something that has no analog in the other fundamental interactions.

But if we wanted to, we should be able treat gravity in the same way as we did the other forces: use flat spacetime, and describe the gravitational interaction by Feynman diagrams involving the exchange of gravitons. It in interesting to compare this treatment with the treatment using curved spacetime. Consider a star of mass M. In the treatment using flat spacetime, the gravity of the star would be described by virtual gravitons emitted and reabsorbed by the star. In the treatment using curved spacetime, spacetime *stretches*, so the star has somehow generated some extra space due to its gravity. What is the mathematical relation between a Feynman diagram and the notion of creating extra space?

Let us start with the weak field approximation, where one particle (mass m) scatters off the gravitational field of a second particle (mass M). The leading order Feynman diagram for this is given in fig.**??**(a). The free action for the particle can be written (in a certain gauge) as

$$-m\int d\tau \frac{1}{2}\frac{dx^\mu}{d\tau}\frac{dx^\nu}{d\tau}\eta_{\mu\nu} \tag{1.1}$$

The gravitational coupling has the form

$$-\lambda h_{\mu\nu}T^{\mu\nu} \tag{1.2}$$

with

$$T^{\mu\nu} = mU^\mu U^\nu, \quad U^\mu = \frac{dx^\mu}{d\tau} \tag{1.3}$$

The particle M produces the graviton $h_{\mu\nu}$ through the coupling (1.2). The particle m then gets a total action

$$S = -m\int d\tau \frac{1}{2}\frac{dx^\mu}{d\tau}\frac{dx^\nu}{d\tau}(\eta_{\mu\nu} + 2\lambda h_{\mu\nu}) \tag{1.4}$$

Now defining a new metric

$$g_{\mu\nu} \equiv \eta_{\mu\nu} + 2\lambda h_{\mu\nu} \tag{1.5}$$

we get the dynamics of the particle in a form where we have no interactions but a new metric

$$S = -m \int d\tau \frac{1}{2} \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau} g_{\mu\nu} \tag{1.6}$$

This is the way we generate curved space; the 'stretching' of the space under gravity is a rewriting of the Feynman diagrams giving graviton exchange.

The above analysis describes the effect of the gravitational field of M to linear order; i.e., in the situation where $h_{\mu\nu} \ll 1$. But gravity is really a nonlinear theory, where gravitons can themselves give rise to new gravitons. These nonlinear terms are described by Feynman diagrams like those in fig.**??**. Consider the region around M where gravity is strong; i.e., $|h_{\mu\nu}| \sim 1$. Let us focus of the gravitational field created by M, in the absence of the test mass m in the above computation. Then in our 'flat spacetime' description using graviton propagators, we get a 'cobweb' of gravitons surrounding the mass M, as depicted in fig.**??**.

In the description using curves spacetime, on the other hand, we do not have any such propagators; we just have the extra space that has been created by stretching of spacetime under the influence of the mass M. Let us now ask a question of dynamics. On this 'extra spacetime' we can imagine a gravitational wave; this is a deformation of this extra space, which evolves in time in a certain fashion. What is the description of this wave in the flat spacetime picture with propagators?

It is plausible that in the flat spacetime picture the wave is a collective oscillation mode of the cobweb of propagators. The quantum state of the gravitational field – as encoded in these propagators – must allow a simple low energy dynamics that corresponds to simple wave propagation in the curved space description.

While this flat spacetime picture appears plausible, it may also appear to be fairly useless; after all, it seems very hard to get an accurate description of the cobweb of propagators, and even harder to study its low energy collective excitations. What we will now see is that in string theory this flat spacetime picture actually has an elegant characterization – it will be just a Yang-Mills theory, with its standard Lagrangian. The Yang-Mills theory will have a large gauge group; in the simplest case, this group is $SU(N)$, where N is, very roughly speaking, the mass M that created the gravitational field, measured in planck units. If we wish to get a large region of 'extra space' created by M, then we will need $N \gg 1$, so we have a gauge theory with a very large number of degrees of freedom. In addition we will see that this gauge theory will be at large effective coupling, so that the Feynman diagrams of relevance have a very large number of loops. These features make it difficult to extract information from this gauge theory. But as we will see, the existence of this 'dual' field theory description has an enormous conceptual significance, since it tells us that in principle, using enough patience, we can uncover all the mysteries of gravity using no new fundamental ideas beyond those already present in ordinary quantum field theory.

1.1 Open closed duality

In string theory we make black holes by putting together the elementary particles of the theory. The most useful objects to start with have been D-branes, so let us take another look at the properties of these branes. In particular, we will note a relation called 'open-closed duality', which will set the stage for the idea of AdS/CFT duality below.

1.1.1 Matter dynamics versus gravitational dynamics

Let us start with the standard model of today, where we have four fundamental interactions: electromagnetic, weak strong, and gravitational.

With this model, consider a ball of copper and a ball of iron, both with the same mass M. The internal structure of these balls is different: the nuclei have different masses, and the electrons have different orbits. These differences are mainly due to three of the fundamental interactions: electromagnetic, weak and strong.

Each ball also creates a deformation of the spacetime around it, but this deformation is created by the fourth interaction – gravity. Since gravity is caused by an interaction that is separate from the other interactions, we do not have a truly unified theory. In our example, the balls have the same mass M, so they produce approximately the same gravitational field, even though they are very different objects under the other fundamental interactions. In fact we can imagine a nonunified theory where the two balls can have exactly the same energy distribution – and therefore the same gravitational field – and yet be different under the other fundamental forces.

String theory on the other hand is a unified theory, so gravity cannot be separated from all other interactions. We will now see that in string theory there is a direct relation between the internal structure of an object, and the gravitational field that it creates.

Consider a set of D-branes as shown in fig.**??**(a). We have seen that the excitations of these branes are given by adding open strings whose ends are attached to these D-branes. Thus studying the general dynamics of all such open string states gives a complete description of the internal dynamics of our system.

Now consider the gravitational interaction between these branes and another D-brane some distance away. In a field theory description, this interaction is caused by exchanging a graviton – the first brane emits the graviton, while the second absorbs it. This is depicted in fig.**??**(b), where the graviton is depicted like a pointlike particle creating a 1-dimensional worldline.

But in string theory the graviton is really an oscillation mode of a closed string, so the graviton exchange should be drawn as in fig.**??**(c). Now we have a loop of string, whose worldline becomes a cylinder; one end of this cylinder lies on the first D-brane and one on the second D-brane.

Now we notice something interesting: there are two ways to view this cylindrical world sheet:

(i) As a closed string emitted by the first brane, and absorbed by the second brane.

(ii) As a pair of open strings connecting the two branes. This is depicted in more detail in fig.. At an early time, there are no opens strings between the branes. At some time a pair of coincident open strings gets created. These open strings separate and exist for a while, and then they come together again to annihilate.

Thus the 'tree level' closed string exchange can be viewed as a '1-loop' open string diagram. This is called 'open-closed duality'. The term 'duality' signifies the following. We can *either* view the interaction as the exchange of a closed string *or* as a 1-loop effect of an open string, but we should not add both contributions because that would be double counting. In this way we see that the degrees of freedom that describe the internal dynamics of the D-branes cannot really be separated from the degrees of freedom that give rise to the gravitation field of the D-branes.

1.1.2 Open-closed duality at large N

In spite of the unification noted above, in many situations we find an *approximate* split between the internal dynamics of the system and the gravitational field it produces. Consider a small number of D-branes placed close to each other, and restrict attention to low energy excitations. Then we have two kinds of effects:

(i) The internal excitations of the D-branes. These will be described by open strings attached to the D-branes, but because we are looking at low energy excitations, the open strings will be in their ground state or in a state close to the ground state. Thus the length of these open strings will be $\sim l_s$, the string length.

(ii) The gravitational field created by the D-branes. Consider the effect of this gravity at distances

$$r \gg l_s \tag{1.7}$$

from the branes. The gravitational field is described by a cylindrical world sheet of the kind depicted in fig.**??**. We have seen that we can regard this cylinder as a 1-loop diagram of open strings. Because of (1.7), these open strings will be very long and therefore very energetic. The gravitational field is however still a low energy effect; this is possible because the cylinder is very *thin* – the open strings are heavy but they live for a very short time, making a very small loop. This small loop describes the graviton propagating from one brane to the other, and the graviton is a low energy state of the *closed* string.

Thus we see that the low energy internal dynamics of the D-branes are given by small *open* strings, while the long distance gravitational field of the branes is given by small *closed* strings. What we will see now is that under certain conditions, these two different limits merge; that will be the domain where we get AdSCFT duality.

1.1.3 Merging the two limits

In the limit (1.7) the gravitational field of a single D-brane is weak; thus we would be in the linear regime of gravity $|h_{\mu\nu}| \ll 1$. To reach the nonlinear regime, we would have to look in the immediate vicinity of the brane $r \sim l_s$. But we can increase the size of the region where gravity is nonlinear by taking a large number N of D-branes. In fig.**??** we depict this region on nonlinearity.

In this nonlinear region, spacetime 'stretches' in this region, generating 'new space'. We had seen that the description in terms of curved spacetime can be recast using using flat spacetime, with a cobweb of graviton propagators surrounding the mass which generated the curvature. Since we are now considering string theory, the graviton in this description should be replaced by a general mode of the closed string; these modes give gravitons, fields like $B_{\mu\nu}$, and also massive particles described by higher excitation modes of the closed string. Thus our 'cobweb' of propagators has become, if anything, even more complicated than it was in a simple theory of quantum gravity.

With string theory we do have a new tool: using open-closed duality, we can look at this cobweb of closed strings as a cobweb made of *open* strings. On the face of it this seems to make the description even more complicated. After all, each closed string propagator is a loop of open strings. The open strings in this loop would appear to be very long, having length $\sim R$, the size of nonlinear gravity region we are considering. Thus one might expect that they would have very high levels of excitation above the ground state. The number of highly excited states of the string is very large indeed, so we seem to be looking at a very complicate mess of stringy physics.

But at this point we get a dramatic simplification from the fact that the number of D-branes is very large: we took $N \gg 1$ to make the region of nonlinear gravity large: $R \gg l_s$. Each end of an open string has to end on a D-brane, so for each open string we have N^2 possible ways of choosing the endpoints. This factor N^2 gives a large 'phase space' of possible quantum states.

This large phase space has an interesting consequence. Suppose we have an open string in some excited state, having energy E. This open string has N^2 possible choices of endpoints. Now suppose it breaks up into 10 smaller open strings, with energy $\sim E/10$ each. Now each of these lower energy strings can have their own choice of endpoints; in fact there will be $(N^2)^{10} = N^{20}$ such choices. This is a much larger phase space than we had with the single open string. The quantum wavefunction of the strings thus tends to spread towards the configurations where we have as many open strings as possible, with each open string having as low an energy as possible. In other words, we will have a very large number of open strings, but each open string will be in its ground state.

But we had seen in section **??** that the dynamics of open strings in their ground state is very simple – it is just given by Yang-Mills gauge theory. We thus begin to see the outlines of AdS/CFT duality. We have two descriptions of the physics created by large number N of D-branes:

(i) We can look at the curved spacetime around these D-branes. In particular we can focus on the region where the gravitational field is nonlinear. We will see below that the metric in this region described anti-de Sitter spacetime (AdS). The low energy excitations in this AdS region are given by gravitational waves on AdS, but we also have all higher energy string and brane states that can live on this AdS space. Thus we have the full complexity of string theory on anti-de Sitter space.

(ii) We can look at $U(N)$ Yang-Mills theory, living on flat spacetime. This is a field theory, with no gravity. This field theory will be at strong coupling however, so it will not be easy to analyze. The field theory will be supersymmetric, possessing $\mathcal{N} = 4$ supersymmetry. We will see below that such a theory has conformal symmetry, so it is a CFT - a conformal field theory. We will also see that this CFT should be thought of as living at the 'boundary' of the AdS space which emerged in the gravity description.

The AdS/CFT duality conjecture says that (i) and (ii) are two descriptions of the same physics. Thus there must be one-to-one and onto map between the states in (i) and the states in (ii), and the evolution of states in (i) must agree with the corresponding evolution in (ii).

Let us now see how this works in more detail.

1.1.4 AdS/CFT with D3 branes

Bibliography