1 The contrast between he fuzzball and wormhole paradigms

Below we explain briefly what fuzzballs are, and how the fuzzball paradigm resolves the information puzzle.

1.1 What problem are we trying to solve?

The Hawking puzzle arises because (i) the semiclassical description of the hole appears to be valid around the horizon because curvatures are low (ii) entangled pairs are created at such a horizon, leading to an ever-rising Page curve. he small corrections theorem makes this puzzle into a rigorous result: if (i) semiclassical dynamics holds around the horizon to leading order and (ii) there are no relevant nonlocal Hamiltonian interactions between the hole and its faraway radiation, then the Page curve cannot come down

A fuzzball is an object which does not have a semiclassical horizon; i.e., the local low energy dynamics near the black hole surface departs by order unity from low energy semiclassical dynamics. The fuzzball conjecture says that all microstates of all holes are fuzzballs. Evidence towards this conjecture is found by using the full gravity theory – string theory and constructing microstates of the hole. In all cases where it has been possible to construct the microstate, one has found a fuzzball rather than the traditional hole with horizon. The generic fuzzball is expected to be a quantum mess, but sometimes simple supergravity solutions can be found when the dual CFT state has 'many excitations in the same mode'. The fuzzball radiates from its surface like a normal body, so there is no information paradox. Gibbons and Warner explained how the structure of fuzzballs in string theory bypasses various no-hair theorems of usual 3+1 gravity.

By contrast, in the wormhole paradigm one *assumes* that in the exact quantum gravity theory the black hole radiates like a piece of coal; this is called the central dogma. One then tries to show that this description as a piece of coal is somehow *compatible* with the semiclassical picture of the hole with horizon. Both these aspects are very puzzling to the fuzzball folks. Why are we *assuming* that the black hole in the exact theory is like a piece of coal? After all, the whole point of the information puzzle was to *show* how quantum gravitational effects changed the semiclassical hole to something that radiated like a piece of coal. The second part is equally confusing. The small corrections theorem says that one *cannot* make the piece-of-coal behavior compatible with a smooth horizon. The only way out is to have nonlocal effects between the hole and infinity, so are the wormhole people postulating such nonlocal dynamics in their exact quantum gravity theory? the issue has been confused further by claims that different ways of taking low energy limits can yield such nonlocality, but how this can happen has not been clear to the fuzzball community.

1.2 The essential quantum gravity effect

The essential question in the information paradox is: Why does the Hawking's argument break down when all it uses is dynamics around then horizon which is a region of low curvature?

In the fuzzball paradigm this question is answered by the dynamical effects of the fuzzballs in the process of gravitational collapse. The gravitational vacuum has an important component arising from virtual fuzzballs: the natural suppression of such fluctuations for large size fuzzballs is offset by the large degeneracy of such fuzzballs. The formation of closed trapped surfaces results in a stretching of these ‘extended size’ virtual objects, leading to a breakdown of semiclassical dynamics and a transition over the crossing time scale to on-shell fuzzballs.

By contrast in the wormhole paradigm it is generally argued that new effects somehow appear at the Page time. For example a new Quantum Extremal Surface develops at this Page time. In the proposed resolution of the ‘bags-of-gold’ problem it is argued that once the entropy inside the hole becomes of order the Bekenstein entropy, small overlaps between semiclassical states prevent a further rise of entropy; thus this change also happens after a time of order the black hole evaporation time. (By contrast, in the fuzzball paradigm, the bags-of gold geometry never forms because the collapsing star turns into fuzzballs much before the Page time.)

The essential physical argument in the wormhole paradigm can be traced to the idea that there is a subleading saddle point in gravitational dynamics which is missed in the leading order Hawking calculation. A subleading saddle point, by definition, produces small corrections to leading order dynamic. Thus one is forced to look for resolutions of the puzzle through effects that are small, and which therefore must act over long timescales like the Page time. As mentioned above, all such resolutions conflict with the small corrections theorem, and it would b e great to understand in detail what the proposals are really saying.

2 Wormhole terms from path integrals?

First we recall the computation as averages over states, then we convert these to path integrals.

2.1 Averaging states

Take two system which are entangled as

$$|\Psi\rangle = \sum_{i,a} C_{ia}|i\rangle|a\rangle \tag{1}$$

with

$$\sum_{i,a} |C_{i,a}|^2 = 1 \tag{2}$$

Tracing out the first system gives the density matrix for the second as

$$\rho_B = \sum_{i,a,a'} C_{ia} C^*_{ia'} |a\rangle\langle a'| \tag{3}$$

Thus

$$Tr[\rho_B^2] = \sum_{i,j,a,a'} C_{ia} C^*_{ia'} C_{ja'} C^*_{ja} \tag{4}$$

For a given choice of C_{ia}, this is difficult to evaluate. But we can compute the average $\langle Tr[\rho_B^2]\rangle$, using

$$\langle C_{ia} C^*_{i'a'}\rangle = D\delta_{ii'}\delta_{aa'} \tag{5}$$

where D is a constant. To find D, we set $i = i'$ and $a = a'$, and sum over i, a. This gives

$$1 = Dmn \tag{6}$$

where $i = 1, \dots m$ and $a = 1, \dots n$. Thus $D = \frac{1}{mn}$. We now compute

$$\langle Tr[\rho_B^2]\rangle = \sum_{i,j,a,a'} \langle C_{ia} C^*_{ia'} C_{ja'} C^*_{ja}\rangle \tag{7}$$

$$= \frac{1}{m^2n^2} \sum_{i,j,a,a'} [\delta_{ii}\delta_{aa'}\delta_{jj}\delta_{a'a} + \delta_{ij}\delta_{aa}\delta_{ij}\delta_{a'a'}] \tag{8}$$

$$= \frac{1}{m^2n^2}[m^2n + n^2m] = \frac{1}{m} + \frac{1}{n} \tag{9}$$

Thus we see that $\langle Tr[\rho_B^2]\rangle$ can never become too small; the smaller of m, n givers this quantity by the above expression. What was interesting in this computation was that there was a term with Wick contraction between the two different copies of the system (which we can call a 'wormhole' connecting the two copies).

2.2 Converting to path integrals

We had

$$\langle C_{ia} C^*_{i'a'}\rangle = D\delta_{ii'}\delta_{aa'} \tag{10}$$

We assume that the two systems are not interacting, so that

$$E = E_1 + E_2 \tag{11}$$

We then write

$$C_{ia} = De^{-\frac{\beta E_i}{2}} e^{-\beta\frac{E_a}{2}} e^{i\theta_{ia}} \tag{12}$$

where D is a constant and θ_{ia} is a random phase. To fix D we write

$$1=\sum_i\sum_a C_{ia}C^*_{ia}=|D|^2e^{-\beta E_i}e^{-\beta E_a}=|D|^2Z_1Z_2 \tag{13}$$

where

$$Z_1=\sum_i e^{-\beta E_i},\quad Z_a=\sum_a e^{-\beta E_a} \tag{14}$$

Thus

$$D=Z_1^{\frac{1}{2}}Z_2^{\frac{1}{2}} \tag{15}$$

We finally obtain the following

$$C_{ia}C^*_{i'a'}=|D|^2e^{-\frac{\beta E_i}{2}}e^{-\beta\frac{E_a}{2}}e^{-\frac{\beta E_{i'}}{2}}e^{-\beta\frac{E_{a'}}{2}}e^{i\theta_{ia}}e^{-i\theta_{i'a'}} \tag{16}$$

We have

$$\langle e^{i\theta_{ia}}e^{-i\theta_{i'a'}}\rangle=\delta_{ii'}\delta_{aa'} \tag{17}$$

Thus

$$\langle C_{ia}C^*_{i'a'}\rangle=|D|^2e^{-\frac{\beta E_i}{2}}e^{-\beta\frac{E_a}{2}}e^{-\frac{\beta E_{i'}}{2}}e^{-\beta\frac{E_{a'}}{2}}\delta_{ii'}\delta_{aa'}=|D|^2e^{-\beta E_i}e^{-\beta E_{a'}}\delta_{ii'}\delta_{aa'} \tag{18}$$

Note that we assume that the average is such that the coefficients C_{ia} are random variables that have no higher point correlations, so we can do Wick contractions in Thus we get

$$\langle Tr[\rho_B^2]\rangle=\sum_{i,j,a,a'}\langle C_{ia}C^*_{ia'}C_{ja'}C^*_{ja}\rangle \tag{19}$$

$$=|D|^4\sum_{i,j,a,a'}[e^{-\beta E_i}e^{-\beta E_a}\delta_{ii}\delta_{aa'}e^{-\beta E_j}e^{-\beta E_a}\delta_{jj}\delta_{a'a}+e^{-\beta E_i}e^{-\beta E_a}\delta_{ij}\delta_{aa}e^{-\beta E_i}e^{-\beta E_{a'}}\delta_{ij}\delta_{a'a'}] \tag{20}$$

$$=|D|^4\Big(\sum_i e^{-\beta E_i}\Big)\Big(\sum_j e^{-\beta E_j}\Big)\Big(\sum_a e^{-2\beta E_a}\Big)+|D|^4\Big(\sum_i e^{-2\beta E_i}\Big)\Big(\sum_a e^{-\beta E_a}\Big)\Big(\sum_{a'} e^{-\beta E_{a'}}\Big) \tag{21}$$

$$=\frac{1}{(Z_1(\beta))^2(Z_2(\beta))^2}\left((Z_1(\beta))^2(Z_2(2\beta))+Z_1(2\beta)(Z_2(\beta))^2\right) \tag{22}$$

$$=\frac{Z_2(2\beta)}{(Z_2(\beta))^2}+\frac{Z_1(2\beta)}{(Z_1(\beta))^2} \tag{23}$$

We can now write these. as path integrals in the usual Feynman way. We can then 'fill-in' the cylinders with a cap to write the path integral as. Gibbons-Hawking path-integral.

But in this case, what have we done? We have assumed that the Gibbons-Hawking entropy is the entropy of the black hole, so we have not addressed the information puzzle. But we have also not computed the entanglement of any given gravity state, since we have done an averaging in the process.

3 Is the QES prescription compatible with the linearity of quantum theory?

We will look at the set up which gives the entropy of a black hole using the QES prescription. Looking at the prescription with states near the black hole state, we will note a conflict with the linearity of quantum mechanics.

3.1 The set-up

In fig.1 we depict the JT gravity description of a black hole that has evaporated past the Page time. The part to the right of the AdS boundary is flat spacetime. The interior of AdS has an Island I; this has quanta (drawn in green) which are entangled with radiation quanta R (drawn in blue). We now say that quantum gravity effects add an extra ingredient not present in J-T gravity; i.e., the QES prescription. This prescription says that we should compute the generalized entropy of the black hole

$$S^{gen} = \frac{A}{4G} + S_{ent} \tag{24}$$

Here $\frac{A}{4G}$ is a schematic expression for the analogue of the area term in JT gravity, and S_{ent} is the entanglement entropy of all quanta in the region between the AdS boundary and the QES surface (i.e., in the segment marked in yellow). One is then required to extremize over possible positions of the QES, and then compute S^{gen} at this extremum. The result gives the entropy of the CFT, which we identify with the entropy of the 'black hole' in the AdS.

Figure 1: The set up before we add any quanta

3.2 Adding a pair of quanta

We now consider three cases:

(i) In fig.2 we show the state $|\psi_1\rangle$ where we have added two quanta depicted in purple; one just to the right of the original QES, and one far out in the flat space region. Both quanta have spin up; thus they are not entangled. We compute the QES for this geometry, find the corresponding S^{gen}, and denote its value as S_0. Thus we have for this state $|\psi_1\rangle$:

$$S_1^{CFT} = S_1^{BH} = S_1^{gen} = S_0 \tag{25}$$

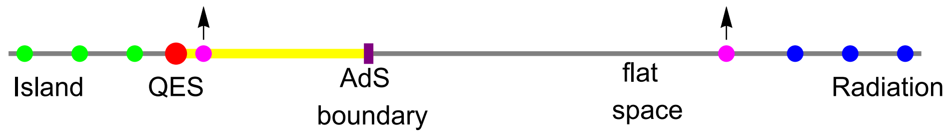

Figure 2: The state $|\psi_1\rangle$

(ii) In fig.3 we show the state $|\psi_2\rangle$ which is constructed just like $|\psi_1\rangle$, but where the spins of both the purple quanta are down. By symmetry, all entropies must be the same as those is case (i):

$$S_2^{CFT} = S_2^{BH} = S_2^{gen} = S_0 \tag{26}$$

Figure 3: The state $|\psi_2\rangle$

(iii) Now we come to the crucial step. We take the linear combination of the above two states

$$|\psi_3\rangle = \frac{1}{\sqrt{2}}(|\psi_1\rangle + |\psi_2\rangle) \tag{27}$$

The purple quantum in the AdS region was described by the CFT since it was to the right of the QES. The superposition (27) says that the entanglement entropy of the CFT has increased by $\ln 2$:

$$S^{linearity} = S_3^{CFT} = S_3^{BH} = S_0 + \ln 2 \tag{28}$$

where we have assumed the normal change of entanglement entropies that we get on taking a linear superposition of the states $|\psi_1\rangle$ and $|\psi_2\rangle$.

On the other hand, the QES prescription gives something different. We are required to first check if moving the QES surface to a different location can give a lower value of S^{gen}. For example if we move the QES to the right of the purple quantum then we will not get the contribution $\ln 2$ to S_{ent}. We may have to pay a slightly higher cost in the value of $\frac{A}{4G}$, but we assume that we still get a lower S^{gen} by moving the surface this way. (After all, this is exactly like the movement of the QES which makes the Page curve goes down after the Page time.) We will therefore find

$$S_0 < S^{QES\ prescription} < S^{linearity} \tag{29}$$

This new QES location is depicted in fig.4.

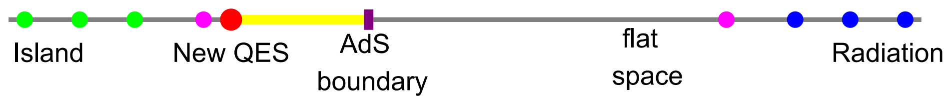

Figure 4: The location of the new QES when the two purple quanta are entangled.

3.3 The conflict

The relations (28) and (29) give

$$S^{QES\ prescription} < S^{linearity} \tag{30}$$

This is the conflict.

3.4 A second example

Consider a black hole geometry created by a shock wave of mass M in the AdS region. The AdS region is joined to an infinite flat spacetime as in the above discussion.

A Cauchy slice through this geometry looks as follows (fig.5):

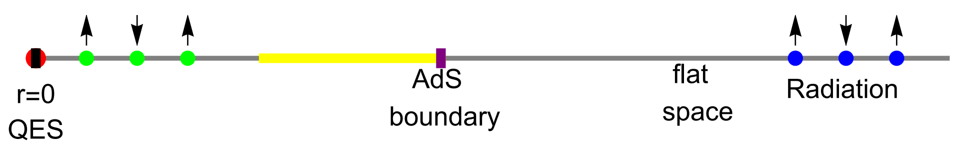

Figure 5: An unentangled state. The quantity S^{gen} is minimized for the QES position $r = 0$, where the $\frac{A}{4G}$ term vanishes.

We have N quanta in the AdS region, and N quanta in the far away flat space region. The quanta in the AdS region each have a spin which can be up or down; let us call these possibilities 0 and 1 respectively. Thus the state in the AdS region has a form 01001110101. . . .. Let the state in the flat space region have the same spins. Let us denote the overall state as

$$|\psi\rangle_{01001110101....} \tag{31}$$

This state is not entangled between the AdS and flat regions. Thus in the expression

$$S^{gen} = \frac{A}{4G} + S_{ent} \tag{32}$$

we have $S_{ent} = 0$. The quantity S^{gen} is then minimized by taking the QES at the origin, so that $\frac{A}{4G} = 0$. By our hypothesis all quanta in the region between the AdS boundary and the QES are described by the state of the CFT. Thus the spins in a state like (31) are directly captured by the state of the CFT. Let the corresponding CFT state be denoted as

$$|\psi\rangle^{CFT}_{01001110101....} \tag{33}$$

The overall state (31) has the form

$$|\psi\rangle_{01001110101....} = |\psi\rangle^{CFT}_{01001110101....} \times |\psi\rangle^{radiation}_{01001110101....} \tag{34}$$

The same is true of any of the 2^N states that come from different possible choices of the spins in the AdS region; of course in each case the spins in the flat space region are taken to be the same as the spins in the AdS region. Let us call the set of these states like (31) as

$$|\psi\rangle_k, \quad k = 1, \dots 2^N \tag{35}$$

For each k we have

$$|\psi\rangle_k = |\psi\rangle^{CFT}_k \times |\psi\rangle^{radiation}_k, \quad k = 1, \dots 2^N \tag{36}$$

Now take the state

$$|\psi\rangle_{entangled} = \left(2^N\right)^{-\frac{1}{2}} \sum_{k=1}^{2^N} |\psi_k\rangle \tag{37}$$

The state (37) is entangled between the CFT and flat space regions. By the linearity of quantum mechanics, this entanglement is

$$S^{linearity}_{ent} = \ln[2^N] = N \ln 2 \tag{38}$$

But the QES prescription gives a different answer when we choose N appropriately. Recall that the geometry in the AdS region is created by a shock wave carrying energy M. Suppose this black hole radiates away after emitting N_{total} quanta. The Page time corresponds to the radiation having $\frac{N_{total}}{2}$ quanta. We choose

$$\frac{N_{total}}{2} < N < N_{total} \tag{39}$$

so that the number of quanta in our radiation region corresponds to the number expected at some point after the Page time. At such a point the QES surface will be nontrivial as in fig.6:

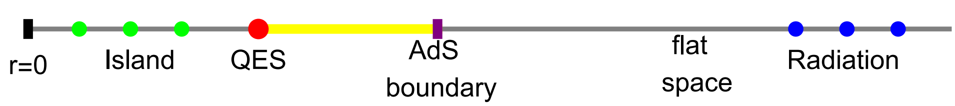

Figure 6: Superpositions over states of the type in fig.5. Now S^{gen} can extremized for a nontrivial position for the QES where $\frac{A}{4G} > 0$; in fact this will the case when the number of quanta exceeds the number radiated by the Page time.

The entanglement between the CFT and the everything else will be, under the QES proposal

$$S_{ent}^{QES} < \frac{N_{total}}{2}\ln 2 < N \ln 2 \tag{40}$$

since we want the entanglement to start decreasing after the Page time.

The relations (38) and (40) give

$$S_{ent}^{QES} < S_{ent}^{linearity} \tag{41}$$

which is the conflict between the QES prescription and linearity.

4 What is the AEHPV model trying to do?

The AEHPV model tries to argue that the semiclassical hole which creates Hawking pairs is somehow compatible with the black hole in the exact theory behaving like a piece of coal. To do this the authors write a map between the exact and approximate theories. Below we write down what seems to us to be this map reduced to its absolute basics; we then note that such a map seems to tell us nothing, as it can be made between any system and any smaller system.

4.1 The overall idea

We have two different quantum mechanical systems:

(i) One model, which we call U, which is the usual unitary evaporation of a normal body like a piece of coal. Here the Page curve goes up and comes down as expected on general grounds. This is supposed to be the description of the black hole in the full quantum gravity theory.

(ii) A second model, which we call H. This model describes the semiclassical Hawking evaporation process, where entangled pairs (b_i, c_i) are produced and the b_i escape to infinity. The state of each entangled pair is

$$|\psi\rangle_{pair} = \frac{1}{\sqrt{2}}\left(|0\rangle_b|0\rangle_c + |1\rangle_b|1\rangle_c\right) \tag{42}$$

(iii) We make a map V from the H Hilbert space to the U Hilbert space. Take a basis $|h_i\rangle$ in the H Hilbert space and a basis $|u_a\rangle$ in the U Hilbert space. Then a state in the H space given by

$$|\psi\rangle = \sum_i d_i |h_i\rangle \tag{43}$$

maps under V to a state in U given by

$$|\chi\rangle = \sum_a c_a |u_a\rangle \tag{44}$$

where

$$c_a = \sum_i V_{ai} d_i \tag{45}$$

In general we think of H having a larger Hilbert space than U, so this map V will project down to a space U of smaller dimension than the dimension of H.

(iv) The idea is that the H model will give a good approximation for appropriate quantities in the U model. In this way we can hope to show how a semiclassical Hawking computation can be consistent with a unitary model U in some the exact quantum gravity theory.

Question: Here U was supposed to describe the black hole in the full quantum gravity theory. But the above steps (and the steps that will follow) will be the same if U describes a piece of coal instead of the exact black hole. We will not use any property of the quantum gravity theory (e.g. string theory) anywhere. Then how can any such map (24) tell us anything about the black hole puzzle?

4.2 The map V at different times

(i) Let the system U evolve unitarily in a sequence of time steps labeled by an integer $t = 0, 1, 2, \ldots$. The system H also evolves in the same time steps. The map V at timestep t will be called V_t.

(ii) Let the evolution in the system U, from time step t to $t+1$, be given by a map $R(t, t+1)$. In symbols

$$|\chi_{t+1}\rangle = R(t, t+1)|\chi_t\rangle \tag{46}$$

In components, consider a state $|\chi_t\rangle$ in U at time t

$$|\chi_t\rangle = \sum_a c_a |u_a\rangle \tag{47}$$

This will evolve to a state $|\chi_{t+1}\rangle$

$$|\chi_{t+1}\rangle = \sum_a c'_a |u_a\rangle \tag{48}$$

where

$$c'_a = \sum_b [R(t, t+1)]_{ab}\ c_b \tag{49}$$

(ii) Similarly, Let the evolution in the system H, from time step t to $t+1$, be given by a map $R^H(t, t+1)$. In symbols

$$|\chi_{t+1}\rangle = R^H(t, t+1)|\chi_t\rangle \tag{50}$$

In components, consider a state $|\psi_t\rangle$ in H at time t

$$|\psi_t\rangle = \sum_i d_i|h_i\rangle \tag{51}$$

This will evolve to a state $|\psi_{t+1}\rangle$

$$|\psi_{t+1}\rangle = \sum_i d'_i|h_i\rangle \tag{52}$$

where

$$d'_i = \sum_j [R(t, t+1)]_{ij}\ d_j \tag{53}$$

(iii) We make the map V_t at different times in such a way that the evolution in the system U matches on to the evolution in the system H. In symbols

$$V_{t+1}\ R^H(t, t+1) = R^U(t, t+1)\ V_t \tag{54}$$

We assume that R^H is invertible. Then multiplying both sides of the above equation by $(R^H)^{-1}$ we get

$$V_{t+1} = R^U(t, t+1)\ V_t\ (R^H(t, t+1))^{-1} \tag{55}$$

In components, this means that we should take

$$[V_{t+1}]_{ai}\ =\ \sum_{b,j} [R^U(t, t+1)]_{ab}[V_t]_{bj}[(R^H(t, t+1))^{-1}]_{ji} \tag{56}$$

With such a map V_{t+1}, we will find, by construction, that we get the same state if we first apply V_t and then evolve by R^U, or first evolve by R^H and then map by V_{t+1}.

Question: The above steps could have been done for any two systems H and U, with H having dimension equal to or larger than U. So how can all this tell us anything about black holes?

Question: The above worry about the black hole in the exact theory and the coal can be made more detailed. For example, U could be a 1 Kg piece of coal, and H could be a 10 Kg block of wood. The Page curve of the coal will come down after its halfway evaporation point, while the Page curve of the wood would still be rising at this time. This looks very similar to the system U being the black hole in the exact theory and H being the semiclassical hole. So how can making such a map help us to reconcile the semiclassical picture of the black hole with the exact description of the hole?

4.3 A first model

(A) First we make the unitary model U. Consider a box of gas containing N atoms, all in their excited state, which we assume to be a spin singlet. In the first step of radiation, one atom radiates a photon (thus coming down to its ground state). The emitted photon is entangled with the atom left behind: if the photon is spin up, the atom left behind is spin down and vice versa. With suitable definitions of states, we can write the entangled state of the photon and atom as

$$\frac{1}{\sqrt{2}}(|0\rangle_\gamma|0\rangle_a + |1\rangle_\gamma|1\rangle_a) \tag{57}$$

where γ is the photon and a is the atom.

(B) At each step of emission, another atom deexcites, so the entanglement of the radiation with the remaining gas after $n < N$ steps is

$$S_{ent} = n \log 2 \tag{58}$$

(C) After N steps of emission, we assume that the deexcited atoms start to drift off to infinity one by one. (For simplicity, let us assume that they drift out in the same order as the order in which they emitted photons.) The entanglement of the radiated matter with the remaining gas goes down with each step of emission, reaching $S_{ent} = 0$ after all the N atoms have left.

This is of course just a simple model for the Page curve of a normal body.

(D) Now we consider the Hawking model H. We take this to consist of (b_i, c_i) entangled pairs, with $i = 1, 2, \ldots M$, with $M \gg N$. (In the Hawking process these pairs are created one by one, but we might as well consider the Hilbert space where the pairs that we will use are all present at all times.)

(E) Now we make the map V, First consider the first half of the evaporation process of the gas; i.e., steps $1, \ldots N$ where the photons get emitted by the atoms. After the first emission, the map V is

$$|0\rangle_{b_1} \to |0\rangle_{\gamma_1}, \quad |1\rangle_{b_1} \to |1\rangle_{\gamma_1}, \quad |0\rangle_{c_1} \to |0\rangle_{a_1}, \quad |1\rangle_{c_1} \to |1\rangle_{a_1} \tag{59}$$

After the first two emissions, the map V is

$$|0\rangle_{b_1} \to |0\rangle_{\gamma_1}, \quad |1\rangle_{b_1} \to |1\rangle_{\gamma_1}, \quad |0\rangle_{c_1} \to |0\rangle_{a_1}, \quad |1\rangle_{c_1} \to |1\rangle_{a_1} \tag{60}$$

$$|0\rangle_{b_2} \to |0\rangle_{\gamma_2}, \quad |1\rangle_{b_2} \to |1\rangle_{\gamma_2}, \quad |0\rangle_{c_2} \to |0\rangle_{a_2}, \quad |1\rangle_{c_2} \to |1\rangle_{a_2} \tag{61}$$

After N steps the map is

$$|0\rangle_{b_1} \to |0\rangle_{\gamma_1}, \quad |1\rangle_{b_1} \to |1\rangle_{\gamma_1}, \quad |0\rangle_{c_1} \to |0\rangle_{a_1}, \quad |1\rangle_{c_1} \to |1\rangle_{a_1} \tag{62}$$

$$|0\rangle_{b_2} \to |0\rangle_{\gamma_2}, \quad |1\rangle_{b_2} \to |1\rangle_{\gamma_2}, \quad |0\rangle_{c_2} \to |0\rangle_{a_2}, \quad |1\rangle_{c_2} \to |1\rangle_{a_2} \tag{63}$$

$$\ldots\ldots \tag{64}$$

$$|0\rangle_{b_N} \to |0\rangle_{\gamma_N}, \quad |1\rangle_{b_N} \to |1\rangle_{\gamma_N}, \quad |0\rangle_{c_N} \to |0\rangle_{a_N}, \quad |1\rangle_{c_N} \to |1\rangle_{a_N} \tag{65}$$

Now consider the second half of the evaporation of the gas, where the atoms a_i come out one by one. At step $N+1$ the map is

$$|0\rangle_{b_1} \to |0\rangle_{\gamma_1}, \quad |1\rangle_{b_1} \to |1\rangle_{\gamma_1}, \quad |0\rangle_{c_1} \to 0, \quad |1\rangle_{c_1} \to 0 \tag{66}$$

$$|0\rangle_{b_2} \to |0\rangle_{\gamma_2}, \quad |1\rangle_{b_2} \to |1\rangle_{\gamma_2}, \quad |0\rangle_{c_2} \to |0\rangle_{a_2}, \quad |1\rangle_{c_2} \to |1\rangle_{a_2} \tag{67}$$

$$\ldots\ldots \tag{68}$$

$$|0\rangle_{b_N} \to |0\rangle_{\gamma_N}, \quad |1\rangle_{b_N} \to |1\rangle_{\gamma_N}, \quad |0\rangle_{c_N} \to |0\rangle_{a_N}, \quad |1\rangle_{c_N} \to |1\rangle_{a_N} \tag{69}$$

$$|0\rangle_{b_{N+1}} \to |0\rangle_{a_1}, \quad |1\rangle_{b_{N+1}} \to |1\rangle_{a_1}, \quad |0\rangle_{c_{N+1}} \to 0, \quad |1\rangle_{c_{N+1}} \to 0 \tag{70}$$

In this map note the following:

(i) the emitted quantum b_{N+1} in the H model is mapped to a_1 in the U model; recall that the atom a_1 is entangled with the first photon γ_1 which was emitted.

(ii) The quantum c_1 is now mapped to zero (in earlier steps it was earlier mapped to a_1)

(iii) The quantum c_{N+1} is mapped to zero.

Note the entanglement in the H model has increased at the $N+1$ step (by $\log 2$) while the entanglement in the U model has decreased by $\log 2$.

We proceed in this way till the gas has completely evaporated. At the last step labeled $2N$ the map is

$$|0\rangle_{b_1} \to |0\rangle_{\gamma_1}, \quad |1\rangle_{b_1} \to |1\rangle_{\gamma_1}, \quad |0\rangle_{c_1} \to 0, \quad |1\rangle_{c_1} \to 0 \tag{71}$$

$$\ldots \tag{72}$$

$$|0\rangle_{b_N} \to |0\rangle_{\gamma_N}, \quad |1\rangle_{b_N} \to |1\rangle_{\gamma_N}, \quad |0\rangle_{c_N} \to 0, \quad |1\rangle_{c_N} \to 0 \tag{73}$$

$$|0\rangle_{b_{N+1}} \to |0\rangle_{a_1}, \quad |1\rangle_{b_{N+1}} \to |1\rangle_{a_1}, \quad |0\rangle_{c_{N+1}} \to 0, \quad |1\rangle_{c_{N+1}} \to 0 \tag{74}$$

$$\ldots \tag{75}$$

$$|0\rangle_{b_{2N}} \to |0\rangle_{a_N}, \quad |1\rangle_{b_{2N}} \to |1\rangle_{a_N}, \quad |0\rangle_{c_{2N}} \to 0, \quad |1\rangle_{c_{2N}} \to 0 \tag{76}$$

Question: This map V is just a map between two different systems doing different things. How can such a map tell us anything about the information paradox which is the conflict between the H model (forced by semiclassical dynamics) and the U model (which models a piece of coal) ?

4.4 Complexity of the map

It sometimes argued that the evolution in the black hole is very complicated and chaotic, and that this will lead to new features for the black hole bit model that are not present for the bit model of a piece of coal. But this is not the case:

(A) The radiation from a normal body is characterized by

(i) The number N of quanta emitted

(ii) How much the bits in the body 'scramble' before emitting the next quantum; let us denote this by a parameter n_{sc} which we can define more precisely when needed.

(iii) The mean energy E of the quanta

All these parameters can be controlled at will by taking different parameters for the body. In particular, we can get an arbitrarily large amount of scrambling between emissions by taking the following model. We have a box of gas containing N atoms. In the box we make a tiny aperture of size δ; the atoms escape from the box slowly as the 'radiation' from the system. As δ is made smaller and smaller, we get more and more scrambling between emissions. Thus such a simple model for a gas in a box can give arbitrary values of N, E, n_{sc}.

(B) A bit model of the black hole is characterized by

(i') The number of bits N

(ii') How much the bits in the hole 'scramble' before emitting the next quantum; let us denote this by a parameter n_{sc} which we can define more precisely when needed.

The energy of the quanta E does not appear in a bit model, either for a normal body or for a black hole. We can however get any value of N, E by taking a large number N_3 of $D3$ branes wrapped on a T^3 of volume V, and exciting this with N quanta. As we take N_3 larger and larger, the energy of the emitted quanta decreases.

(C) It is sometimes said that a black hole is a very complicated object, and this leads to novel phenomenon involving complexity. But this is not the case. A black hole of 100 planck masses should exhibit all the physics of black holes to a good approximation, and it has only $100^2 = 10^4$ bits. A mole of gas in a box has $\sim 10^{23}$ bits.

Question: Thus we see that any unitary bit model of the black hole can be reproduced by the bit model of a box of gas with a small aperture. So how can invoking any complexity arguments tell us anything about black holes from a bit model? Whatever conclusions one gets for the black hole will also hold for the box of gas.