Questions on various proposals regarding the black hole information paradox

Abstract

The quantum theory of black holes has led to many puzzles. Resolving these puzzles should illuminate the deep mysteries of gravitational physics. But many solutions which have been proposed have led to confusions of their own: what exactly is the proposal, what are the implicit assumptions, and what are the final conclusions?

The questions below seek to clarify these issues. It is hoped that those who believe in various ideas about black holes can explain the proposals in simple terms, so that the whole community can understand what exactly is being claimed and how.

Anyone with input on these questions, or anyone having further questions along similar lines, can send their feedback at mathur.16@osu.edu, and we will try to incorporate that in further iterations of these questions.

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1 What is ER=EPR?

The phrase 'ER=EPR' suggests that entanglement between two distant states somehow generates a 'bridge' between their locations in a theory of gravity. But is this possible?

1.0.1 Is ER=EPR a new notion in the fundamental, full theory of quantum gravity?

Consider 10-dimensional IIA string theory for concreteness. Imagine two D0 branes, placed a large distance L apart. We can entangle the spins of the D0 branes, just the way we entangle the spins of two electrons in quantum mechanics to make a singlet.

Question: Is there any effect of this entanglement, in this exact theory of quantum gravity, that would be different from the entanglement story which we are all familiar with in usual quantum mechanics?

1.0.2 Does ER=EPR violate the linearity of quantum theory?

Suppose one were to say that the answer to question 1.0.1 above is 'Yes'. Then the postulate seems to violate the linearity of quantum theory in the following way:

(i) Take the first D0 to be spin up, the second D0 spin down: $|\psi\rangle = |\uparrow\rangle_1 |\downarrow\rangle_2$. There is no entanglement, so no novel effect. Let the future evolution of this state be $|\psi(t)\rangle$.

(ii) Take the first D0 to be spin down, the second D0 spin up: $|\chi\rangle = |\downarrow\rangle_1 |\uparrow\rangle_2$. There is no entanglement, so no novel effect. Let the future evolution of this state be $|\chi(t)\rangle$.

(iii) We can now make an entangled state

$$\frac{1}{\sqrt{2}}\Big(|\psi\rangle + |\chi\rangle\Big) \tag{1}$$

By linearity of quantum mechanics, this state will have to evolve to

$$\frac{1}{\sqrt{2}}\Big(|\psi(t)\rangle + |\chi(t)\rangle\Big) \tag{2}$$

So how can entanglement ever give any new effects? Or are we prepared to violate the linearity of quantum mechanics? (Note that all statements are in our exact theory of gravity which we have assumed for concreteness to be string theory.)

1.0.3 ER=EPR for black holes

Suppose one were to say that the answer to question 1.0.1 is 'No'. Then consider the following.

Instead of the two well-separated D0 branes, we could take two well separated bound states. Each bound state has 100 D1 branes, 100 D5 branes and 100 units of P charge. With string coupling $g = 0.1$, this makes two Strominger-Vafa type black holes (since $gN \gg 1$).

Question: Presumable the answer to question 1.0.1 will remain 'No' for this setting. Then how can we get any ER=EPR for black holes?

Question: van Raamsdonk [13] had argued that entangling two regions creates a connecting spacetime bridge between them. In [14] it was argued that this cannot be correct. How do we get around the arguments in [14]?

1.0.4 ER=EPR for a box of gas?

Suppose we say that there is NO effect like ER=EPR in the exact gravity theory, but such an effect appears in the low energy approximation. Compare two systems:

(i) Two D1D5P brane bound states (as in 1.0.3) at a large separation L

(ii) Two boxes of gas at a large separation L.

Question: As noted in the section on 'bit models', there is no difference between the bit models describing a complicated brane bound state and a box of gas. So if ER=EPR is true, then wont we get this effect when considering the effective dynamics for two well separated boxes of gas?

1.0.5 What is the meaning of wormholes?

In the paper [9], Maldacena and Susskind pictured thin wormhole tubes connecting the two entangled members of each Hawking pair; these wormholes were then imagined to join up near the horizon giving a 'squid diagram', which would somehow help to resolve the problem of growing entanglement while maintaining a smooth horizon.

Question: Consider the thin wormhole connecting the two members of the entangled pair:

(i) Is this wormhole simply a depiction of the entanglement, without any dynamical effects? If so, we can equally well *not* use this depiction, keeping the original Hawking picture of entanglement. Then how have we learnt anything about the information puzzle?

(ii) Does the thin wormhole imply a novel dynamical effect connecting the inside of the hole to its radiation? In that case, won't we violate the linearity of quantum mechanics as in question 1.0.2?

2 Locality in gravity

Many people are implicitly assuming that gravity has some kind on nonlocality across large distances, of a kind not seen in QFTs. Here 'large distances' means that we are not talking about any nonlocalities at planck or string distances, where local operators can be hard to define. We also do not mean the nonlocal nature of entanglement, which happens in any QFT. We mean real dynamical effects that connect well-separated systems.

2.0.1 Nonlocal Hamiltonian in the exact theory

Let the exact theory be IIB string theory in 9+1dimensions. Is there any hamiltonian interaction in the exact theory which does not fall off with the separatuon L between two brane bound states, in the limit $L \to \infty$?

2.0.2 Path integrals in gravity

Consider a set of 10^4 branes at position $x = 0$, making a black hole. Consider another set of 10^4 branes at $x = L$, with $L \to \infty$. We can write a path integral of the degrees of freedom of the exact gravity theory (string theory) to describe the black hole at $x = 0$; and we can also write a path integral describing the hole at $x = L$.

Question: In this exact gravity theory, is there any significant contribution from paths that connect the two brane bound states?

If so

(i) Why?

(ii) Won't such terms lead to an ensemble averaged theory, as in the old work on wormholes in gravity?

2.0.3 Semiclassical approximation

Suppose the exact path integral does not have any significant paths connecting the objects at $x = 0$ and $x = L$, in the limit $L \to \infty$. We now take a low energy limit of the gravity theory.

Question: Will some paths now emerge connecting the locations $x = 0$ and $x = L$?

If so, how? (This does not happen for a QFT.)

2.0.4 Dot products vs amplitudes

In [8] it was argued that some computations (e.g. [5]) are using dot products in gravity that look different from the usual dot product: they are using what look like transition amplitudes in place of dot products.

Question: Suppose we fix a gauge and write down the Wheeler-de Wit wave function for a low energy gravity state. Why should the dot product on the Hilbert space be conceptually any different from the dot product for a normal field theory?

3 The small corrections theorem

Suppose we say

(A) In the exact quantum gravity theory, the black hole is like a piece of coal; thus the Page curve goes up and comes down the way it does for any normal body radiating from its surface.

(B) There is some way of taking a low energy approximation so that usual low energy semiclassical dynamics is obtained to leading order around the horizon.

The small correction theorem says that it is not possible to have both (A) and (B). Yet some people have tried to say that (A) and (B) are both true. In this section, we explore this issue.

3.1 The meaning of the low energy approximation

Let us begin with defining the low energy limit more precisely. In normal physics, the low energy approximation is made as follows:

(i) The exact system is described by a Hilbert space $\mathcal{H}_{exact}$, with states ψ_i^{exact}. These states evolve by a Hamiltonian $\hat{H}_{exact}$, on which we have observables $\hat{O}_{exact}$. Equivalently, we have a path integral given through an action S_{exact}; the space of paths can involve nontrivial topologies where appropriate.

(ii) The semiclassical approximation is a map to a low energy Hilbert space $\mathcal{H}_{sc}$, with states ψ_i^{sc}. These states evolve by a Hamiltonian $\hat{H}_{sc}$, on which we have observables $\hat{O}_{sc}$. Equivalently, we have a path integral given through an action S_{sc}; the space of paths can involve nontrivial topologies where appropriate. The operators of interest $\hat{O}_{sc}$ are, typically, a small (low energy) subset of the full set of operators $\hat{O}_{exact}$.

(iii) There is a map $\mathcal{H}_{sc} \rightarrow \mathcal{H}_{exact}$ with

$$|\psi_i\rangle_{sc} \rightarrow |\psi_a\rangle_{exact}, \quad \hat{O}_{sc} \rightarrow \hat{O}_{exact} \tag{3}$$

such that the evolution between the exact states is well approximated by the semiclassical evolution between the approximate states

$${}_{exact}\langle\psi_j|e^{-i\hat{H}_{exact}t}|\psi_i\rangle_{exact} \approx {}_{sc}\langle\psi_b|e^{-i\hat{H}_{sc}t}|\psi_a\rangle_{sc} \tag{4}$$

and a similar approximation holds for expectation values

$${}_{exact}\langle\psi_j|\hat{O}_{exact}|\psi_i\rangle_{exact} \approx {}_{sc}\langle\psi_b|\hat{O}_{sc}|\psi_a\rangle_{sc} \tag{5}$$

3.2 Questions

3.2.1 Nature of semiclassical approximation in gravity

Question: Is the semiclassical approximation in gravity of the same kind as above or is it fundamentally different?

If it is different, why? In usual string theory, it is assumed that the low energy limit of interactions below the string scale gives supergravity, just like in the standard way of getting semiclassical approximations described above. The high energy string excitations automatically become irrelevant to low energy processes, just like GUTS particles automatically become irrelevant at the scale of QCD. Are we taking a semiclassical approximation that is fundamentally different from this?

3.2.2 Low energy limits of Ensemble averaged theories

If we take an ensemble averaged theory like the SYK model, then we can recast its correlation functions using surfaces weighted with the action of JT gravity.

Question: Is this emergence of JT gravity supposed to be the actual semiclassical limit of the full quantum gravity theory in which the SYK states are quantum states?

Or are these JT gravity surfaces a completely different mathematical artifact unrelated to the semiclassical approximation of the exact gravity theory?

Question: If these JT gravity surfaces emerge in a low energy limit of the exact gravity theory, do we always need the fundamental theory to be an ensemble averaged theory? Only such ensemble averaged theories have surfaces connecting two disconnected field theory copies; the connecting surface arises from the quenched correlations of coupling constants. If we extend this picture to nonensemble averaged theories, we get the 'factorization puzzle': we find $\langle Z^2\rangle \neq \langle Z\rangle^2$.

3.2.3 Relating quanta inside and outside the black hole

The semiclassical Hawking radiation process produces entangled pairs, with one member (called b below) inside the hole and the other (called c below) outside. After the Page time, one finds that the entanglement between the $\{b\}$ and the $\{c\}$ is larger than the Bekenstein entropy of the hole; this is one way of stating of the information puzzle.

In some approaches (e.g, [12], and the so-called $A = R_b$ models) one keeps the semiclassical picture of the hole. But then one says that after the Page time, the interior of the semiclassical hole is somehow 'described' by the radiation bits at infinity.

Question: What exactly does this mean?

Question: The information puzzle arises because the radiated b quanta are entangled with the c quanta in the hole. In the $A = R_b$ models one says that the entanglement partners c are in the radiation (after the Page time), so the entanglement of the radiation with the hole is decreasing. But how can this result in a smooth horizon? A smooth horizon has a certain local dynamics from slice to slice, and this dynamics is not reproduced if we take the emerging b quanta to have their partners far away instead of being just inside the horizon.

4 The Page curve

Some people believe that a newly discovered saddle point in the gravitational path integral makes the Page curve come down. However, it has also been argued that this conclusion is false: there is no such saddle in the correct path integral computing entanglement entropy, and the claimed saddle is computing something other than the Renyi entropy that it is supposed to compute [8]. The discussion of this section pertains to this issue.

4.1 Separating different assumptions

Before we come to specific questions, it is helpful to list arguments which sound roughly similar but which might actually correspond to very different physics assumptions:

(i) In the work of Marolf and Maxfield [3, 2], it is assumed that the *exact* quantum gravity theory has a nonlocal interaction: wormholes from any point of space can connect any other point of space. It is known from earlier work of Coleman [1] and others that this situation leads to an ensemble averaged theory. One can discuss the observable effects of such ensemble averaging (and some recent papers do), but it is generally agreed that string theory is not an ensemble averaged theory.

(ii) In the analysis of Pennington,Shenker, Stanford, Yang (PSSY) [5] one starts with the density matrix ρ of two entangled systems, and then assumes the same averaging of the entries of ρ that one assumes for a box of gas. For the box of gas, this averaging shows that the entanglement between two systems A and B is limited by the size of the smaller of the two systems (this is the usual Page curve). The relevant traces are written diagrammatically for the CFT description. The corresponding gravity diagrams are obtained by 'filling in' the diagrams of the CFT, in the manner of Gibbons-Hawking. Thus in this approach one assumes that the system under study is a normal system with a fixed number of bits (unlike the semiclassical black hole).

A similar analysis in Lorentzian signature by Liu and Vardhan [16] also starts with a normal pair of systems A and B each with a finite number of bits, and obtains the Page curve after suitable averaging of phases in the evolution. There does not appear to be any special property of quantum gravity used in this analysis. One then applies this averaging to black holes, and uses the Gibbons Hawking cigar to write the entropies on the gravity side.

(iii) In the proposal of Penington [6], it is argued that gravity is a novel theory, in the following sense. Distant systems are not connected with each other in the exact path. integral, but such a connection appears in the low energy effective path integral. The replica wormhole in this approach appears as a special property of quantum gravity.

(iv) In the SYK model, wormholes connect two copies of a system, because the Hamiltonian is ensemble averaged. Such wormholes lead to the 'factorizing problem', which is resolved for a non-ensemble averaged theory by including 'half-wormholes' which undo the linkage between the two copies [7].

Averaged systems also appear in approaches where one takes an average over the coupling constants of many CFTs to get a dual gravity theory.

The above four approaches appear to have different underlying assumptions, but they appear superficially similar since they all argue for wormhole connections between systems that are not expected in usual physics. The questions below aim to understand these assumptions.

4.2 Questions

4.2.1 Difference between wormholes in the exact theory and wormholes arising from averaging

It would seem that assumption of having wormholes in the exact quantum gravity theory, used by Marolf and Maxfield in [2, 3], is different from the assumptions used in the other approaches (e.g. PSSY). The Marolf-Maxfield papers assume that in the exact quantum gravity theory we can have baby universes, and the quantum statistics of such baby universes causes a conceptual problem with measuring the outcome of many copies of the same experiment. This does not seem to be the case in the other approaches mentioned above (in some of these other cases, the wormholes emerge only in some low energy approximation but are not present in the exact quantum gravity theory).

Question: Is it correct that these are two completely different approaches?

4.2.2 Changing the definition of Renyi entropy

Maxfield [4] has made a bit model of Penington's Island proposal, which seeks to clarify the proposal. In this model, if we compute the actual Renyi entropy, we find a monotonically rising Page curve. But the Renyi entropy is replaced by a modified quantity, which gives a normal Page curve that goes up and then comes down.

Question: Since the actual Renyi entropies are rising, why dont we just conclude that the Page curve is monotonically rising?

It is argued that the semiclassical approximation in gravity is different from that in other QFTs. In gravity this approximation couples different replica copies, which are not coupled in the exact theory. This coupling does not happen in other QFTs.

But by the AdS/CFT correspondence, each CFT state is dual to an exact gravity state. This correspondence is just open-closed duality.

Question: How can we get an interaction between different replicas in the semiclassical limit of the gravity theory when there is no such interaction between replicas in the CFT description?

4.2.3 Connected vs disconnected systems

In a QFT, we find entanglement between the field values at neighboring points for the following reason: (i) The gradient term in the Hamiltonian couples the harmonic oscillators describing the field $\phi(x)$ at these two points (ii) We ask for the ground state (or some other low energy state), and in this state the two coupled oscillators have an entangled state.

The reverse however is not true: If we entangle the spins of two electrons that are separated by a large distance L, then this does not imply a Hamiltonian interaction between the two electrons.

In entanglement wedge reconstruction in AdS/CFT, the CFT fields at different boundary points are coupled to each other by the CFT Hamiltonian, and similarly the bulk points are coupled to each other Suppose we look at the vacuum state on both sides of the duality. The entanglement on the CFT side is reproduced by the RT surface on the bulk side, and subleading corrections arise from the entanglements of bulk fields across the RT surface.

But now consider the black hole and its far away radiation. These two systems are in an entangled state, but there is no interaction between the hole and the far away radiation.

Question: Why should any RT kind of argument apply in this situation?

4.2.4 Islands and the Page curve

Suppose the Island prescription is a meaningful way of understanding why the Page curve comes down.

Question: How does this prescription bypass the small corrections theorem, which says that the Page curve cannot come down in the exact quantum gravity theory if there exist any low energy approximation which yields a smooth horizon to leading order?

4.2.5 Relation to Bekenstein's black hole entropy conjecture

The work of Bekenstein had led to a general belief that any region bounded by an area A could never have entropy more than the entropy of a black hole in that region:

$$S \le \frac{A}{4G} \tag{6}$$

If an object has entropy $S = \log N$ (N is the number of possible microstates) then the maximal entanglement of this body with its surroundings is S. Thus (6) is equivalent to saying that the entanglement entropy of the region bounded by area A satisfies

$$S_{ent} \le \frac{A}{4G} \tag{7}$$

The Island conjecture says that the entanglement of a black hole with its radiation is

$$S_{ent} \leq \min\left(\frac{A}{4G}, S_{rad}\right) \tag{8}$$

where S_{rad} is the entropy of the radiation.

Question: Is this conjecture not simply a restatement of (7), since it just says that the radiation cannot entangle with the hole more than the area entropy of the hole?

It would of course be significant if one could *prove* (8). But this cannot be done without actually constructing the black hole microstates, which is not something that is being attempted in the Island proposal. There is thus a potential circularity in the argument. Consider a Schwinger process between two charged spherical shells placed at the center of AdS. Electron positron pairs are created, with one set (say the electrons) heading to the outer plate. The outer plate can be a mesh which allows the electrons to reach infinity and be collected. In this Schwinger effect the Page curve keeps going up monotonically.

Question: How will the Island conjecture work here? Why won't we get an Island after some time, which will imply that the Page curve comes down? We can try to say that we get no Island since the whole Schwinger apparatus must be big; thus $A/4G$ computed from the area A bounding the apparatus will always be too large to matter. But this is a circular argument: to know that any such apparatus must be too big we must first know about all the states in the gravity theory that can fit inside an area A,

4.2.6 Which entanglement is involved in the derivation of the Island conjecture?

The Island idea seems to have its origins in the RT and HRT prescriptions, where (i) The minimal surface can jump to a different minimal surface when the size of the boundary entangling regions is changed (ii) It appears plausible that the bulk entropy computations should involve not only the contribution $\frac{A}{4G}$ from the RT surface but also the bulk entanglement $S_{ent,bulk}$ across this RT surface.

But these computations relate (a) The entanglement entropy of two boundary regions to (b) The generalized entropy $S_{gen} = \frac{A}{4G}|S_{ent,bulk}$ from the bulk.

Question: The Island prescription however says that the entanglement between two *bulk* regions is governed by the same generalized entropy. How do we make this jump from a relation between boundary and bulk computations to a statement about just a bulk theory?

Question: Sometimes one argues for double holography, where the black hole and the radiation can be thought of as both living on some boundary, whose bulk is a theory with yet higher dimension. But there is a difficulty with this. In the boundary CFT above, the points along the boundary were connected by Hamiltonian interactions, and the entanglements in the vacuum state of the boundary arose as a consequence of these Hamiltonian interactions. On the other hand, the black hole and its radiation are not connected by a Hamiltonian interaction. So how is any double holography emerging? Or is the higher dimensional theory involved in double holography creating a nonlocal Hamiltonian interaction between the black hole and its radiation?

Question: The Hawking puzzle arises because of the low energy dynamics around the horizon creates entangled pairs. Is the higher dimensional theory involved in double holography supposed to reproduce this dynamics? If yes, then how can the entanglement come down? If no, then what is the relevance of this doubly holographic depiction?

4.2.7 How does the CGHS model work?

The CGHS model, and its evaporating extension, the RST model, are simple models of 1+1 dimensional quantum gravity which can be explicitly solved. The Page curve is found to keep going up monotonically. But the Island prescription applied to this model shows that the Page curve comes down [17].

Question: Why does this not imply that the Island prescription is just incorrect?

It is sometimes said that even though the Island prescription applied to the RST model gives an answer opposite to the the answer from the exact solution of the model, the answer of the Island prescription is 'correct' in that it gives the answer we would get if we improved the RST model to a more complete theory of quantum gravity.

Question: What does this mean, given that the Island computation is not invoking any step which has any details of this more complete theory?

5 Bit models

Consider a box of gas with N atoms, having spin $\frac{1}{2}$ each. There is a small hole in the box through which the atoms can escape. At first the entanglement of the radiated atoms with the box will increase, since a radiated atom A_R will in general be entangled with some linear combination of states in the box. For simplicity let A_R be entangled just with one atom A_B in the box:

$$|\psi\rangle_{pair} = \frac{1}{\sqrt{2}}\Big(|0\rangle_{A_R}|0\rangle_{A_B} + |1\rangle_{A_R}|1\rangle_{A_B}\Big) \tag{9}$$

Thus the entanglement rises by $\log 2$ upon emission of A_R. But at some later time (typically after half the evaporation) the atom A_B will also be radiated out, and the entanglement of the radiation of the box will then *decrease* by $\log 2$.

By contrast, in Hawking's computation new pairs are continuously pulled out of the vacuum, in an entangled state

$$|\psi\rangle_{pair} = \frac{1}{\sqrt{2}}\Big(|0\rangle_b|0\rangle_c + |1\rangle_b|1\rangle_c\Big) \tag{10}$$

where b is the radiated quantum and c is its negative energy partner in the hole. Thus the entanglement keeps rising.

5.0.1 Black holes vs box of gas

Some papers start of with the goal of saying something about black holes. But then they start working with a model that is like a box of gas (9) instead of (10). If we assuming a normal body like a box of gas to start with, how can these papers say anything nontrivial about black holes?

Question: As an example, consider the Hayden-Preskill protocol, where a diary is thrown into a hole, and the information is argued to be reemitted quickly if the diary os thrown in after the Page time. Is this a statement where the black hole is already assumed to be a normal body like a box of gas rather than a black hole with (10)?

5.0.2 The number of bits

Sometimes people say that even if we model a black hole with bits behaving as (9), there is still a difference from a box of gas because the black hole has a very large Hilbert space.

A black hole with mass $M = 10^{10}\, m_p$ should exhibit all the essential aspects the quantum dynamics of black holes, since $M \gg m_p$. Such a hole entropy $S_{bek} \sim 10^{20}$. Modeling it as a normal body would imply

$$N_{bh} \sim 10^{20} \tag{11}$$

bits. On the other hand a box containing 1 gm of Hydrogen atoms has

$$N_{gas} \sim 10^{23} \tag{12}$$

bits.

Question: Given the above, how can we use any argument of the size of the Hilbert space to distinguish black holes from normal bodies?

5.0.3 Fast scrambling

Suppose we represent the black hole by a finite number of bits with behavior as in (9) rather than (10). Some people might still say that the black hole is different from a normal system because it has *fast scrambling*, which a piece of coal might not have.

But this does not appear to be the case: as far as bit models go, normal systems can have fast scrambling too. In the bit model the only relevant scales are (i) the number of bots N_{bits} and (ii) the number of interactions N_{int} between the bits before the next bit is emitted. Fast scrambling means that N_{int} is large enough that the system samples a large fraction of phase space before the next emission, so that the system reaches in a generic state before its next emission. For the box of gas with N atoms, this is easily achieved by making the hole in the box sufficiently small. Then an atom interacts with other atoms in the box many times before escaping, and we have fast scrambling.

Question: So how can any bit model with the behavior (9) say anything useful about black holes? Whatever properties we extract from such a model, wont they also hold for a box of gas with a small hole?

5.0.4 The AEHPV map

In [18] a bit model is used to argue that the radiation from a black hole that behaves like a piece of coal (where the Page curve comes down) can be mapped, to a good approximation, to the Hawking process where the Page curve keeps going *up*.

Question: By what was noted above about bit models, this map can be written for any box of gas as well. What is the significance if the semiclasssical horizon dynamics that would then emerge for the box of gas? Is there any physical significance of such a map?

6 Type I vs Type II vs Type III algebras

There has been interest recently in theories described by von Neumann algebras of Types I, II and III. It has also been argued that this issue may have relevance to understanding black holes. All quantum systems with a finite dimensional Hilbert space are of Type I,

6.0.1 Finite vs infinite systems

We study QCD using a finite lattice with $N_{lattice}$; it is then expected that the difference between actual observables and those computed by the lattice will go to zero as $N_{lattice} \to \infty$.

Question: Suppose someone says that there is some novel effect that occurs when the system had a Type II or type III algebra, but not when the system has a Type I algebra. Then how can this effect ever be reproduced on a computer, which always makes a finite lattice approximation to any differential equation?

If the effect cannot be so reproduced by. lattice approximation, what is the meaning of the effect?

Question: Usually any effect that is 'only present for an infinite dimensional system' is also present as an approximation in a finite dimensional approximation; its just a question of orders of limits. If this is the case here, what is the effect and what is the relevant order of limits?

6.0.2 The relevance to black holes

Suppose we look at the wave equation for a black hole in Schwarzschild coordinates. The redshift diverges at the horizon, and so any energy eigenmode has an infinite number of oscillations outside the horizon. As a consequence, we have a continuous spectrum of energies.

By contrast, if we place a wall at some location $2M + \epsilon$, then the spectrum for a black hole (placed in a box) becomes discrete.

Question: Is this a situation where we have moved from a Type III algebra to a Type I algebra?

If so, what can we learn from this story of different algebras? After all, we can consider Minkowski space (with no black hole) and look at a Rindler patch. The Rindler patch will exhibit a continuous spectrum of field modes, even if the Minkowski space was placed in a finite box and thus given a discrete spectrum. Thus if we just look at the usual semiclassical dynamics around the horizon, how do we ever bring in anything relevant to black holes as against just studying Minkowski space is Rindler coordinates?

6.0.3 Type III vs Type II through quantum fluctuations

Recently Witten [15] and others have studied the divergences of quantum fields near a horizon, and noted that these divergences can be softened if one takes into account the fact that the mass M of the hole fluctuates a little, and the matter state is correspondingly different for each M. Mathematically, one says that the overall state should be written as a semi-direct-product of the matter state with a state on an auxiliary variable which captures the values of M; this changes the trace properties of the von-Neumann algebra from Type III to Type II. The goal below is to understand the physics behind this mathematical formalism.

In the early -nineties, the following issue came up in connections to ‘t Hooft’s claim that high energy modes could be relevant at the horizon:

(i) Consider the Schwarzschild frame near the horizon, which is Rindler physics. If we fix the metric, then the energy of Rindler modes diverges at the horizon, since the redshift goes to infinity.

(ii) But the horizon location fluctuates a little. If a Rindler quantum is absorbed it moves the horizon outwards and if a Rindler quantum is emitted then the horizon goes inwards.

(iii) Let us try to find the energy density of matter fluctuations at a point just outside the horizon. We specify this point by letting it be at a given proper distance s from a fixed boundary at some large distance R. We choose s so that we are at looking at a point very close to the horizon, but as mentioned above, the horizon will fluctuate a little around this position.

(iv) Suppose the horizon moves outwards a little under the fluctuation. Then the point s will fall inside the hole, and so will not contribute to the energy outside the hole.

(v) Suppose the horizon moves inwards a little under the fluctuation. Then the point at s will be a little further outside the horizon, where the Rindler energy is not so high.

(vi) Thus we find that the horizon fluctuations smooth out the divergence in the Rindler energy, in a way that makes this energy not divergent at the horizon. Note that the Rindler Hamiltonian is the modular Hamiltonian to leading order for the state near the horizon, so we can replace the word ‘energy’ in the above steps by ‘entanglement’. More generally, the matter state near the horizon has large fluctuations due to the horizon not being at a precise position since the mass M is not exact in the full wavefunctional.

Question: Is the smoothing of entanglement (through the Type III to Type II transition noted by Witten) just the above phenomenon in more formal language?

In the discussion of the nineties it was found that the smoothing of divergences had nothing to do with the actual puzzles posed by black holes; these fluctuations were simply arising in Schwarzschild coordinates which are bad near the horizon. The fluctuations disappear when using ‘good slices’ to describe the hole.

Question: Is there some new physical input being added in the current set of papers which might lead to a nontrivial statement about black hole physics?

Question: Consider a mirror with a constant acceleration; this is also known to generate a Rindler type state on one side. The mirror position has small fluctuations; will these also smooth out the Rindler divergences in the same way? If so, why is there any relation of these fluctuations to the gravitational physics of the black hole?

7 Ensemble averaged theories

The original AdS/CFT duality is supposed to be an exact duality: if you put the CFT in a computer, and the string theory on a computer, you will get identical correlation functions under the AdS/CFT map. This is because the duality is essentially open-closed duality in the limit of a large number of colors and a small string coupling.

But some recent approaches (e.g. [11]) have investigated more approximate dualities where low energy gravity emerges as a description of some kind of as an average over CFTs. In this section we wish to explore which approaches are using such an approximate duality, and exactly what these averaged dualities can tell us about black holes.

7.0.1 What information do we lose upon averaging?

In usual statistical mechanics, we can replace the given microstate by an ensemble of microstates, and still get reasonable accurate answers for simple (few-point) observables.

But the entanglement entropy S_{ent} for a system is a very delicate computation, requiring very high-point correlation functions for its measurement.

Question: Exactly what kind of averaging allows this quantity to be still found from the ensemble average? What are the explicit steps which replace the actual black hole microstate by an averaged state and how is the entanglement preserved in this process?

7.0.2 What can we learn from spectral functions in ensemble averaged theories?

In a field theory we can compute spectral functions, which tell us about the distribution of eigenvalues of the Hamiltonian. The SYK model is an ensemble averaged theory, and the spectral functions for this theory have been computed [10].

Question: How are these computations related to the low energy limit of string theory?

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