Supplementary information by J. Huston McCulloch for

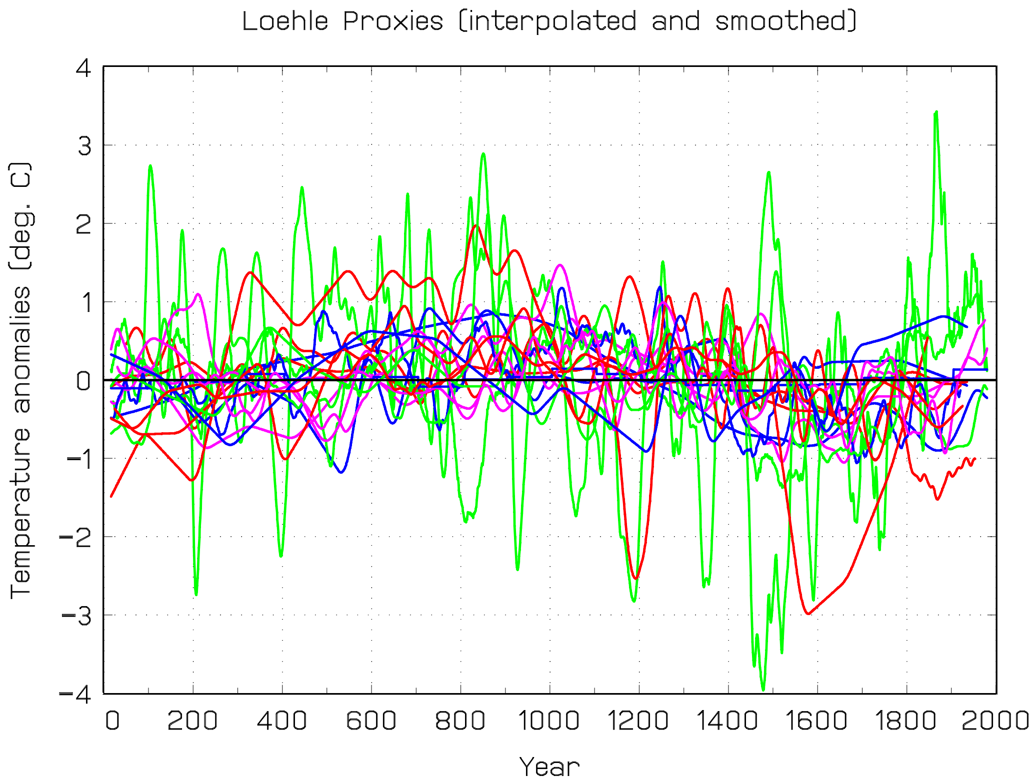
"Correction to: A 2000-Year Global Temperature Reconstruction Based on Non-Tree Ring Proxies," by Craig Loehle and J. Huston McCulloch

Jan. 10, 2008

Loehle and McCulloch (LM, submitted to *Energy and Environment* as a correction to Loehle 2007) construct a Global Temperature Reconstruction for 16AD – 1935 AD based on 18 peer-reviewed published non-treering proxy series. These are the same 18 series used by Loehle (2007), with the dating of 4 of the series corrected by a 50-year shift. Each series was newly interpolated and smoothed with a 29 year moving average by CL over the period 1AD – 1980AD, to the extent available. Each series was converted to bimillennial anomalies by subtracting out its own mean. The global temperature reconstruction is the unweighed average of these anomalies. Because the number of available series drops abruptly from 11 to 8 in 1935, i.e. to less than half the maximum number of series, the reconstruction was terminated in 1935. The 18 smoothed series and their residuals about the global average are individually graphed at the end of this note. The smoothed series, as used in the reconstruction, are online via <http://www.econ.ohio-state.edu/jhm/agw/> . Links to the raw data have been compiled by Stephen McIntyre of climateaudit.org, at <http://www.climateaudit.org/?p=2393>.

#	Short name	First obs.	Last obs.	$\hat{V}_j$	$\hat{S}_j$
1	Dahl-Jensen	35	1980	0.151	0.388
2	Gajewski	50	1953	0.214	0.463
3	Cronin	20	1980	1.702	1.305
4	Keigwin	16	1910	0.224	0.474

5	Nyberg	148	1935	0.217	0.466
6	Korhola	16	1935	0.038	0.196
7	Tan	16	1970	0.334	0.578
8	Yang	16	1975	0.187	0.432
9	Magnini	16	1920	0.149	0.386
10	deMenocal	16	1847	1.306	1.143
11	Holmgren	16	1980	0.481	0.693
12	Calvo	16	1945	0.074	0.273
13	Viau	16	1980	0.059	0.242
14	Stott MD982181	24	1921	0.228	0.477
15	Stott MD982176	16	1795	0.148	0.384
16	Ge	30	1980	0.172	0.415
17	Farmer	16	1935	0.270	0.519
18	Kim	16	1925	0.079	0.280


Loehle Proxies (interpolated and smoothed)
Temperature anomalies (deg. C)
4
3
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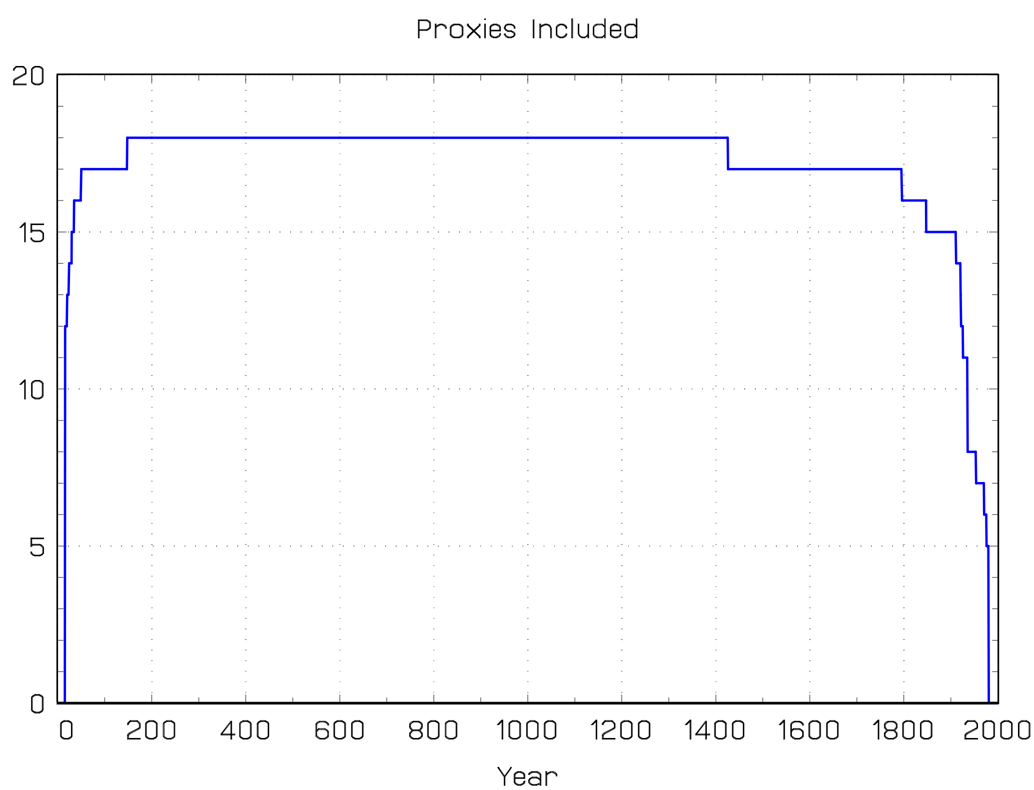
200
400
600
800
1000
1200
1400
1600
1800
2000
Year


Proxies Included
20
15
10
5
0
0
200
400
600
800
1000
1200
1400
1600
1800
2000
Year

As noted in LM, standard errors and confidence intervals are somewhat complicated by the presence of cross-sectional heteroskedasticity (unequal variances) in the data. The variance about the global mean temperature of Calvo et al. (2002), for example, is almost 7 times as great as that of Viau et al. (2006). Because of this heteroskedasticity, conventional local, or pointwise variance estimates will not have their customary χ^2 distribution, and hence the Student t distribution (see e.g. Casella and Berger 2002) will not provide accurate critical values to form confidence intervals. Instead, the LM standard errors exploit the panel (pooled cross section/time series) structure of the data to estimate each series' variance consistently from its time series residuals.

It is assumed in LM that the demeaned temperature reconstruction X_{jt} from proxy j at time t provides an unbiased observation on global mean temperature anomaly μ_t at time t:

$$X_{jt} = \mu_t + \varepsilon_{jt},$$

The errors ε_{jt}, which contain both the proxy's error in measuring true local temperature anomaly and the true local temperature anomaly's deviation from the global temperature anomaly, are assumed to be normally distributed with mean 0 and proxy-specific variance V_j, and to be independent across proxies at each point in time. As in Loehle (2007), μ_t is estimated by the equally weighted mean

$$m_t = \frac{1}{n_t} \sum_{j \in J(t)} X_{jt} \; ,$$

where $J(t)$ is the set of proxies that are active at time t and n_t is the number of such proxies (n_t = 18 for most dates in LM, as plotted above). Under the maintained assumption of independence, the variance of m_t is therefore

$$Var(m_t) = \frac{1}{n_t^2} \sum_{j \in J(t)} V_j \ .$$

The proxy-specific variances V_j are consistently estimated over the time-series dimension, with a conservative adjustment for degrees of freedom, by

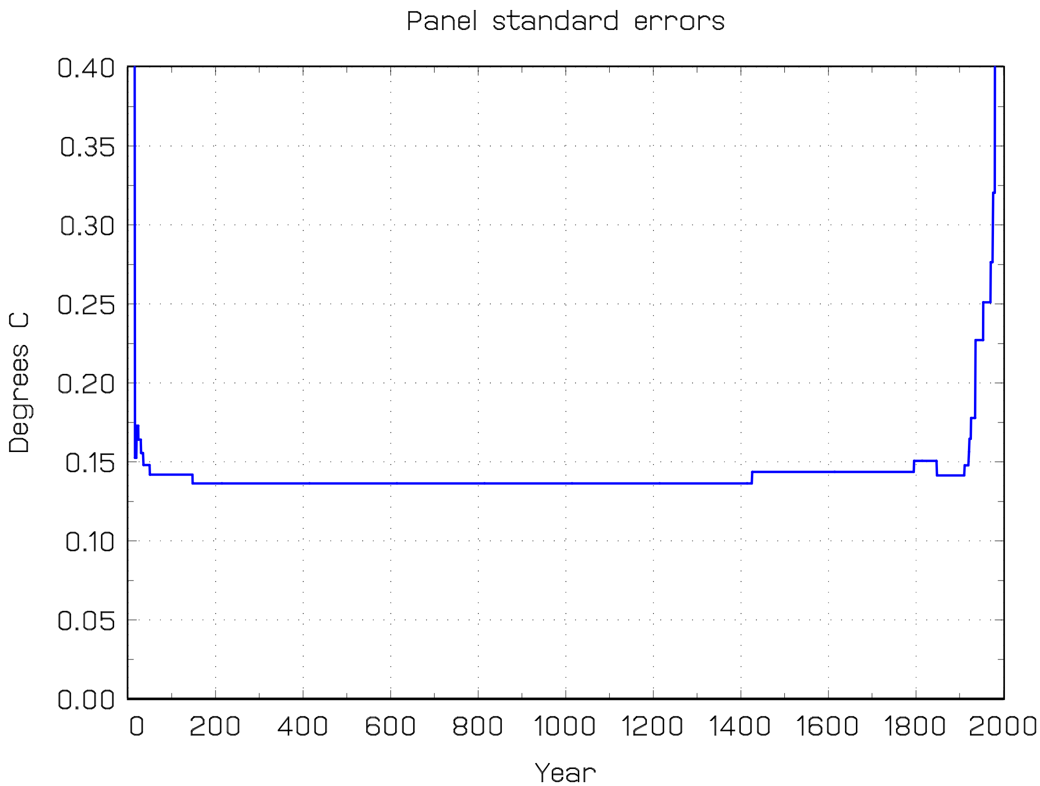
$$\hat{V}_j = \frac{1}{N_j} \sum_{t \in T(j)} (X_{tj} - m_j)^2 \frac{n_t}{n_t - 1}, \qquad (1)$$

where $T(j)$ is the set of dates for which proxy j is active, and N_j is the number of such dates. These panel variance estimates and the corresponding standard deviation estimates $\hat{S}_j = \hat{V}_j^{1/2}$ are tabulated in Table 1 above.

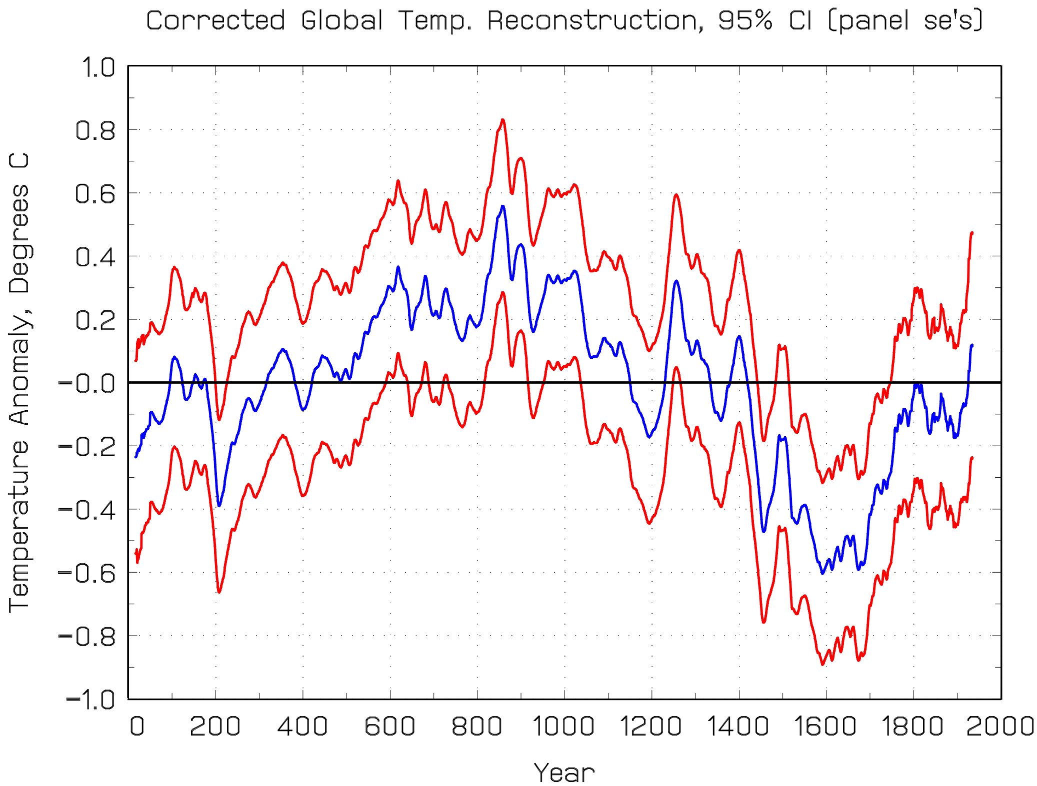
The heteroskedasticity-adjusted standard error of m_t is then

$$s_t = \frac{1}{n_t} \left(\sum_{j \in J(t)} \hat{V}_j \right)^{1/2} .$$

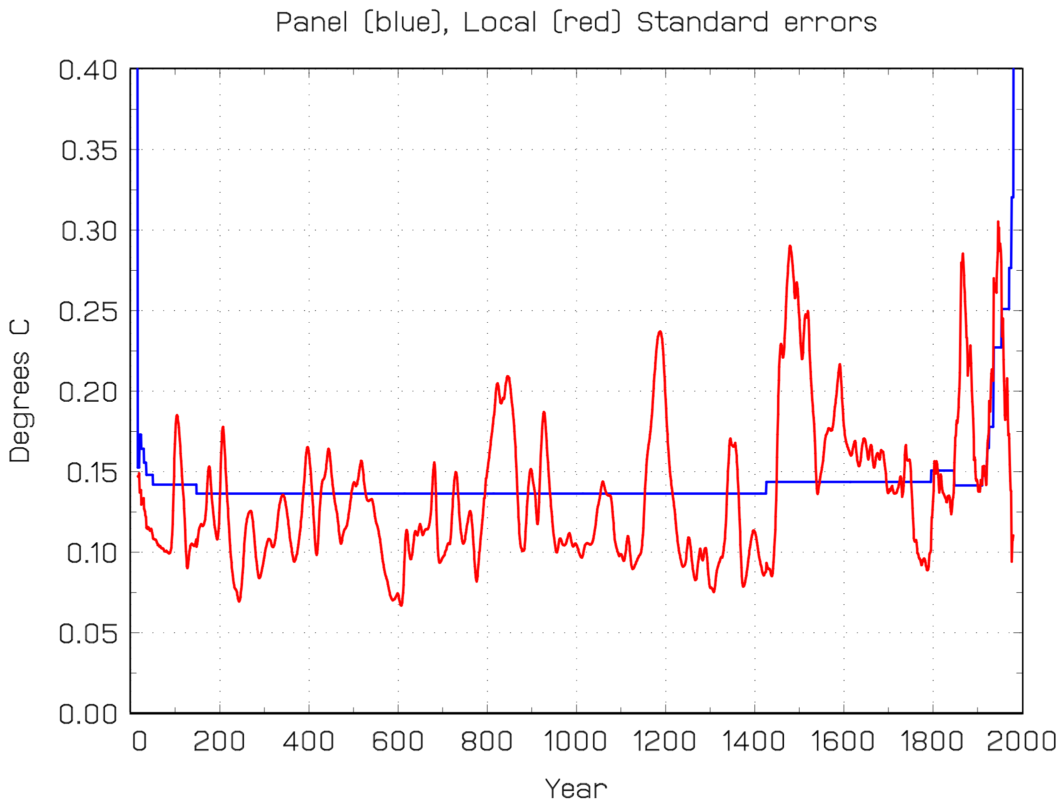
During 148 – 1425 AD all 18 proxies are active and s_t is constant at 0.136 °C. The standard errors increase gradually as proxies drop out, rising to 0.178 °C in 1935 when only 11 proxies are still active. The red line in the following graph shows s_t for the entire period 16 AD – 1980 AD. It may be seen that at both ends, as the number of included series drops off, the standard error increases inordinately. Accordingly, the published reconstruction only includes the period 16AD - 1935 AD, when at least half of the 18 series are active.



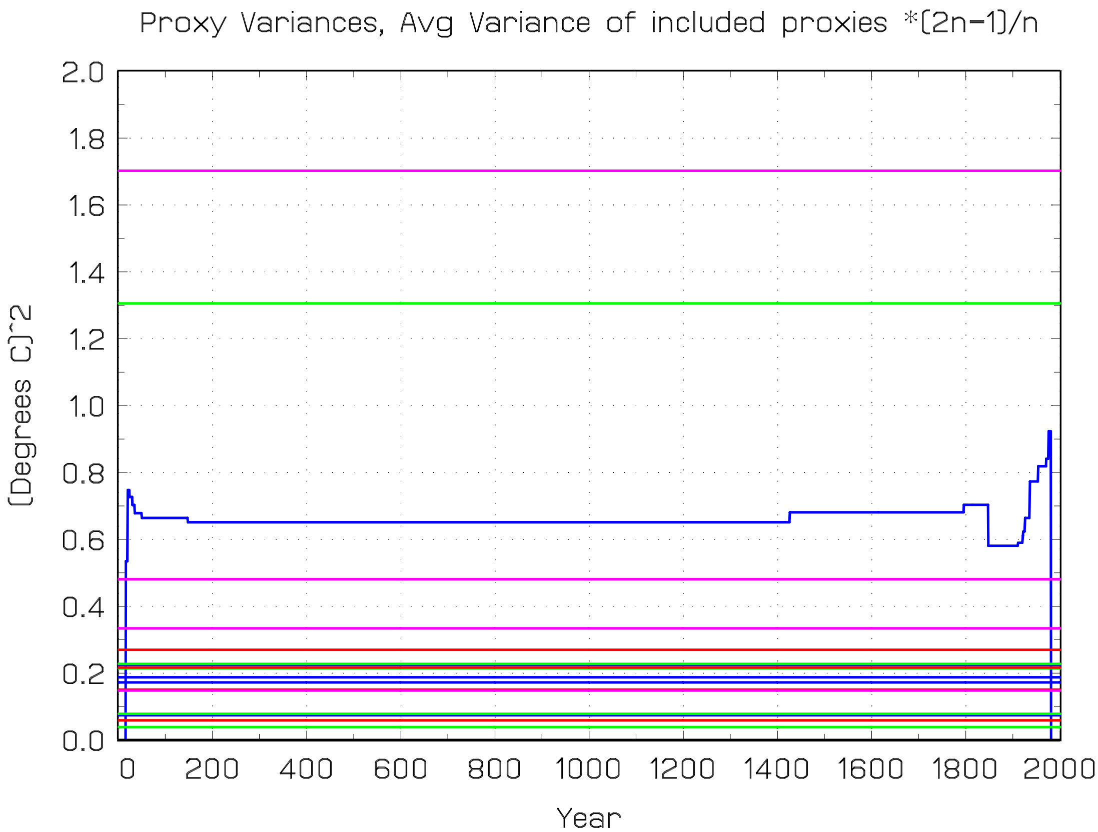
Although each V_j is estimated with almost 2000 points in time, the 29-year running mean implies that effectively at most only about 60 of these are independent. Assuming approximately 60 degrees of freedom, a 95% confidence interval extends $2.00s_t$ above and below m_t. The means, standard errors, and CI bounds for this graph are tabulated at <http://www.econ.ohio-state.edu/jhm/agw/> and (Loehle's site).



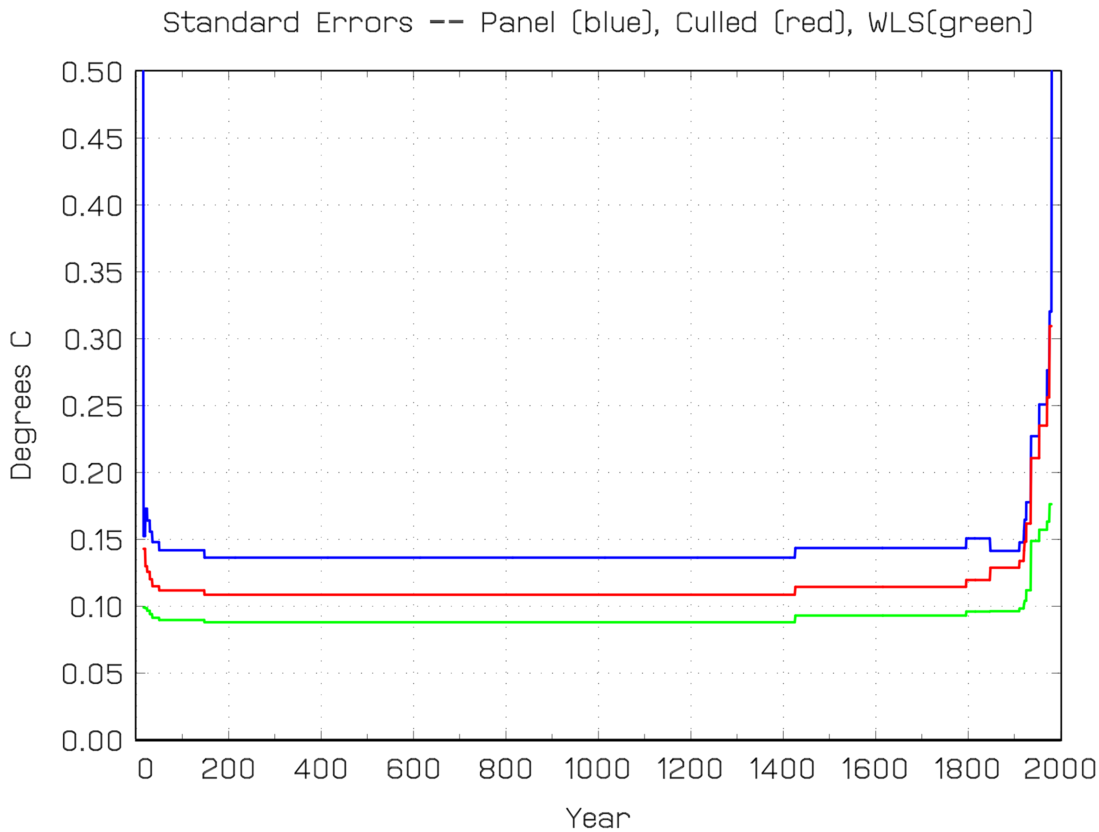
The following graph shows, in red, conventional "local" standard errors that are computed for each point in time using only that date's squared residuals. As noted above, since the errors for the 18 series have very unequal variances, these standard errors are not governed by the conventional chi-squared distribution, and hence the standard Student t critical values will be invalid for t-statistics computed from them. The "panel" standard errors shown in blue and reported in LM are far less erratic, and more nearly conform to the assumptions required to use the standard Student t critical values. When the number of proxies is constant, it can easily be shown that the square of the constant panel standard error is simply the average of the squares of the erratic local standard errors, since the two measures use exactly the same squared errors, simply combined in a different fashion.



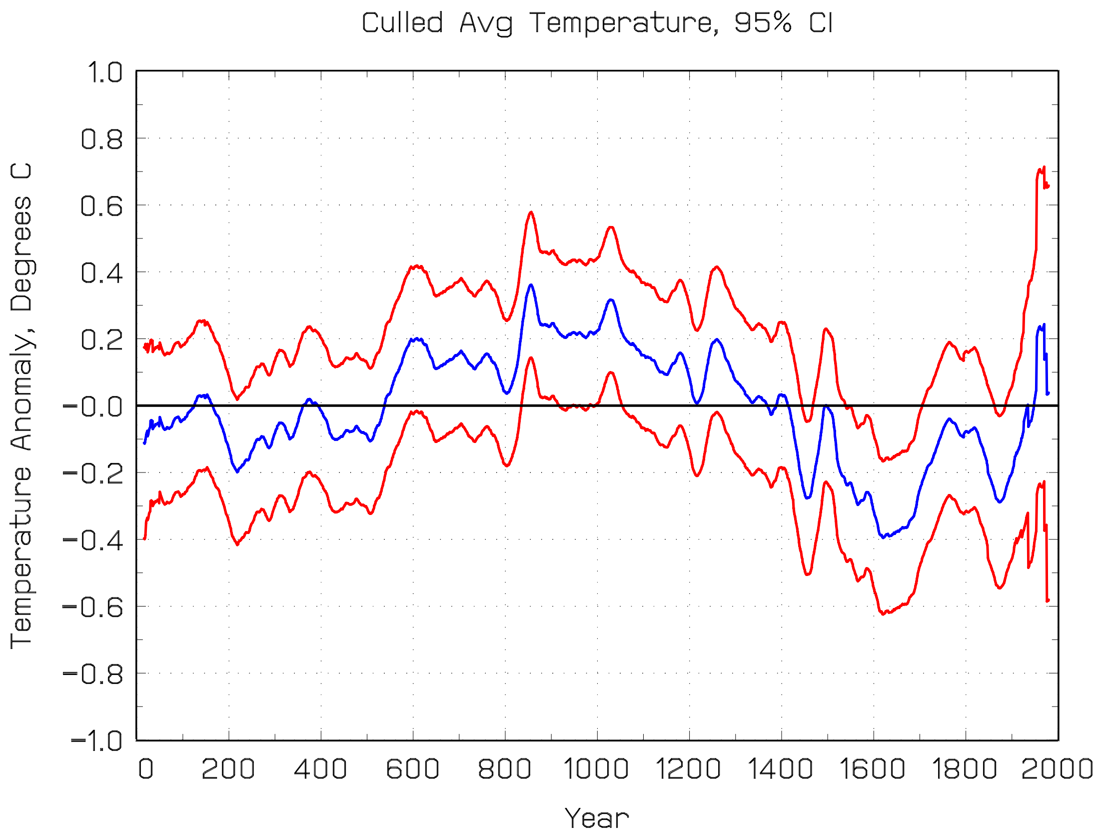
If all proxies had equal variance, the equally weighted or "ordinary least squares" average of the proxies would provide the optimal estimator of the true population mean, and omitting any proxy would increase the standard error of the equally weighted average. However, it can easily be shown that if any proxy variance V_j is greater than the average of the all the available proxy variances times the factor $(2n_t\text{-}1)/n_t$, omitting that proxy actually reduces the standard error of the equally weighted proxy average. The following graph compares the 18 proxy variances (the horizontal lines) to this value (in blue), which changes somewhat with the identity of the included proxies. It may be seen that two of the proxies, which turn out to be #3 Cronin and #10 deMenocal, are actually so noisy as to be detrimental to the unweighted proxy average.



The following graph shows the full sample or OLS standard errors (in blue), along with the standard errors that result when these two series are culled, or omitted from the sample (in red). (The third curve, in green, will be discussed below.)

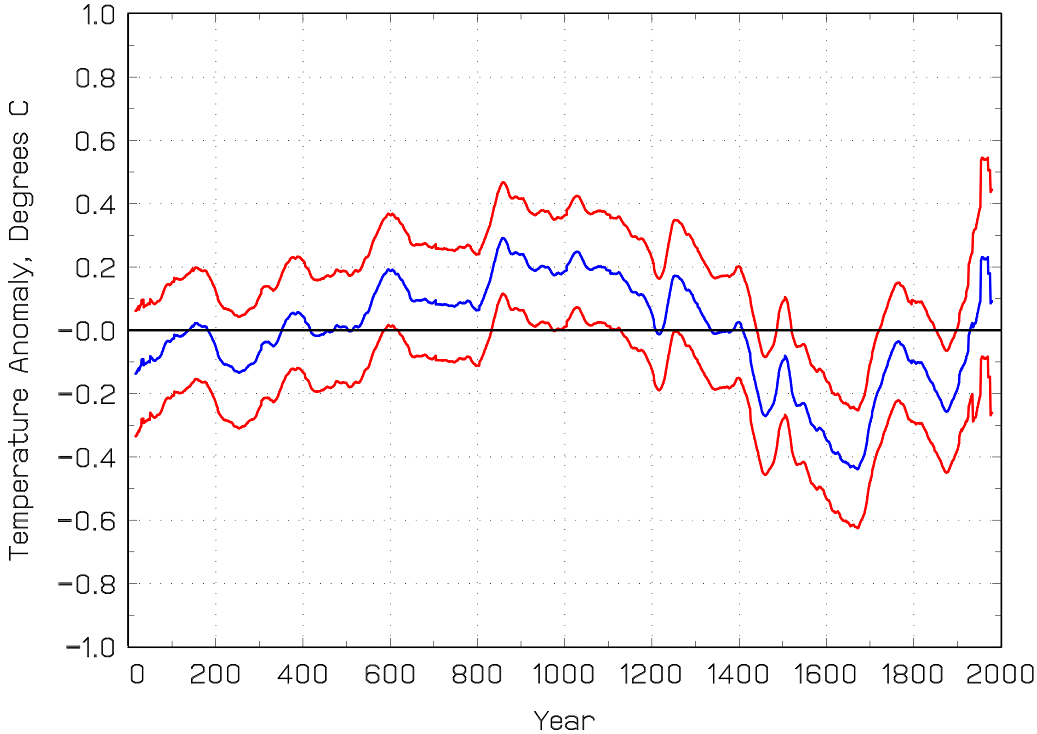


The next graph shows the culled average, along with 95% confidence intervals. It may be seen that omitting these two series happens to somewhat attenuate the shape of the reconstruction. Nevertheless, the MWP and LIA are still significant (or right at the margin of significance) over essentially the same period as with the full sample. Culling the sample in this manner does not constitute "data mining," "cherry picking," or "lemon dropping" (reverse cherry picking), however, since it is done according to the objectively computed time-series variance of the proxy in question, and without regard to the sign of its impact.

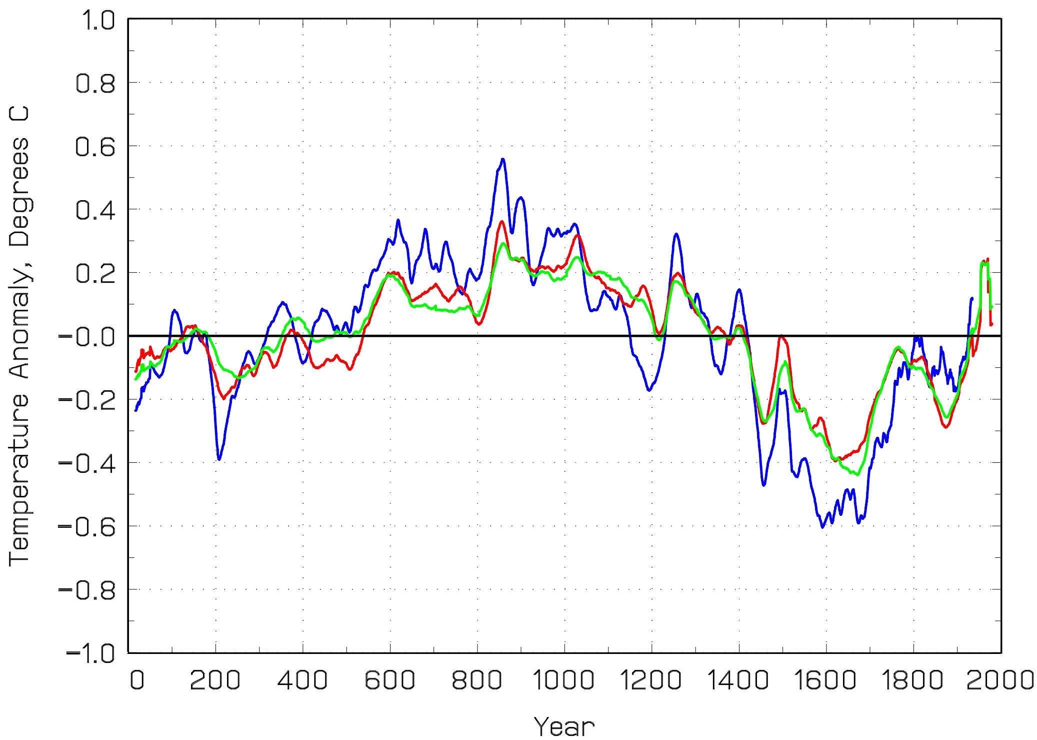


The standard error of the estimate of the mean can be even further reduced with Weighted Least Squares estimates, which optimally weight each proxy in inverse proportion to its variance. In WLS, it pays to include every proxy, though the ones with the highest variances receive very little weight and make very little improvement to the weighted average. The WLS standard errors are shown in green two graphs above. The WLS point estimates, below, are similar to the culled estimates. This is not surprising, since culling is just a crude form of weighting. Again, the MWP and LIA are significant over essentially the same period as with the unweighted full sample estimates.

WLS Avg Temperature, 95% CI



Avg Temperature -- Unweighted (blue), Culled (red), WLS(green)



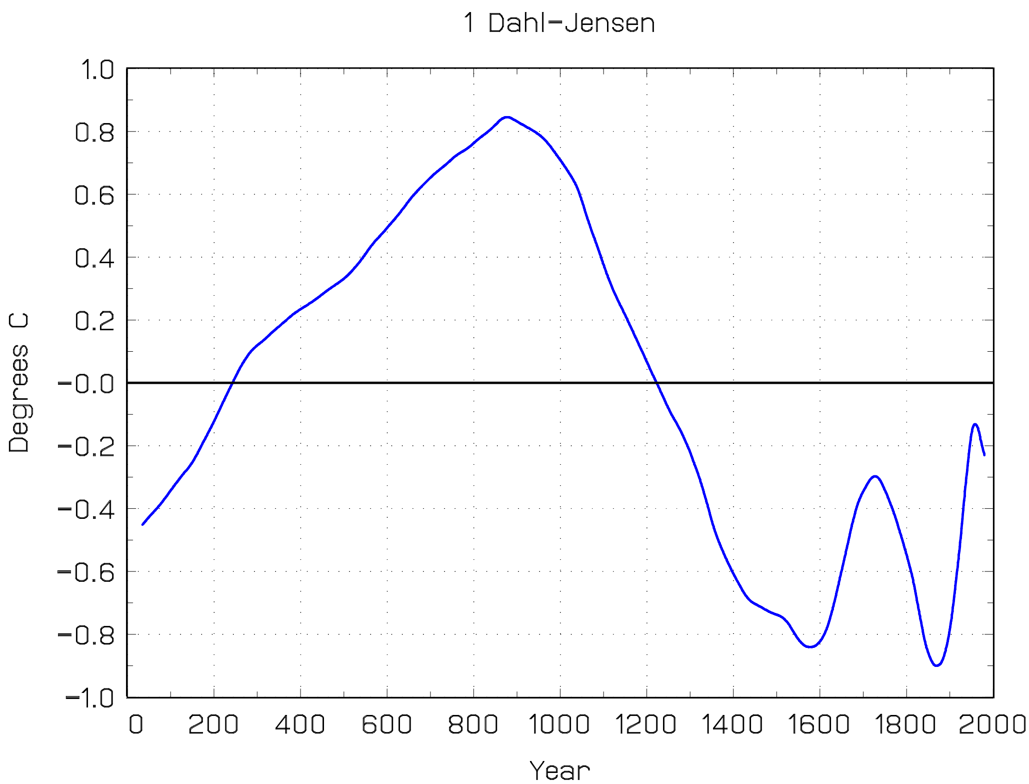
Although the culled and WLS estimates make more efficient use of the data, it was decided to stick with the more readily understood full-sample OLS estimates in the published reconstruction.

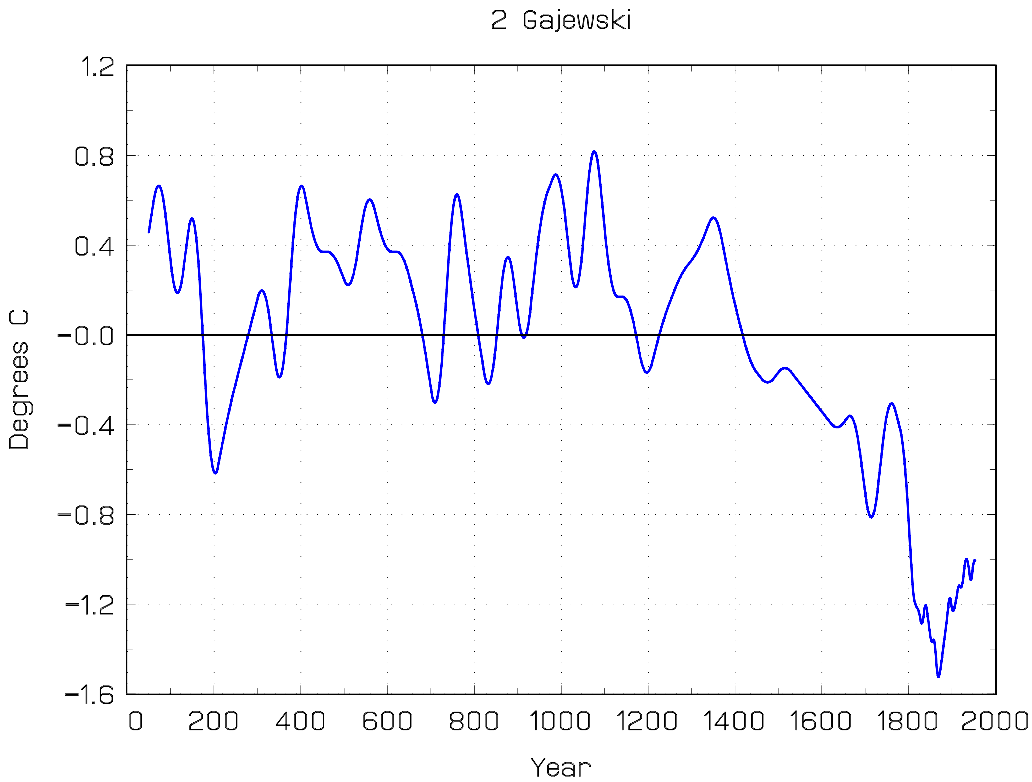
Strictly speaking, the adjustment for degrees of freedom in (1) above is valid only when the errors are homoskedastic – an assumption we are explicitly avoiding. When the errors are heteroskedastic, the correct adjustment requires solving 18 linear equations for the 18 unbiased variance estimates. Although doing this somewhat changes the individual variance estimates, it was found that it had virtually no perceptible effect on the bottom line standard errors. Accordingly, the simpler approximate adjustment in (1) was used.

The maintained assumption of cross-sectional independence of the errors is not unreasonable with the present data set, given the good geographical distribution of the proxies used. In studies with a substantially denser network of proxies, however, spatial autocorrelation would eventually become an important consideration. One way to take this into account would be to model each error as having two components: The first would be an idiosyncratic measurement error with a proxy-specific variance, as in LM. The second component would reflect the difference between the local true temperature anomaly and the global anomaly, and would have a pairwise covariance that was some declining function, to be empirically determined, of the great circle angular distance between the pair of proxies in question. Global temperature could then be efficiently estimated by means of Generalized Least Squares (Aitken's formula), using the complete spatial covariance matrix. Such a strategy would be more efficient than the coarse "gridding" procedure commonly used in climate studies.

REFERENCE

Loehle, C. (2007). A 2000 Year Global Temperature Reconstruction on Non-Treering Proxy Data. *Energy & Environment* 18:1049-1058.

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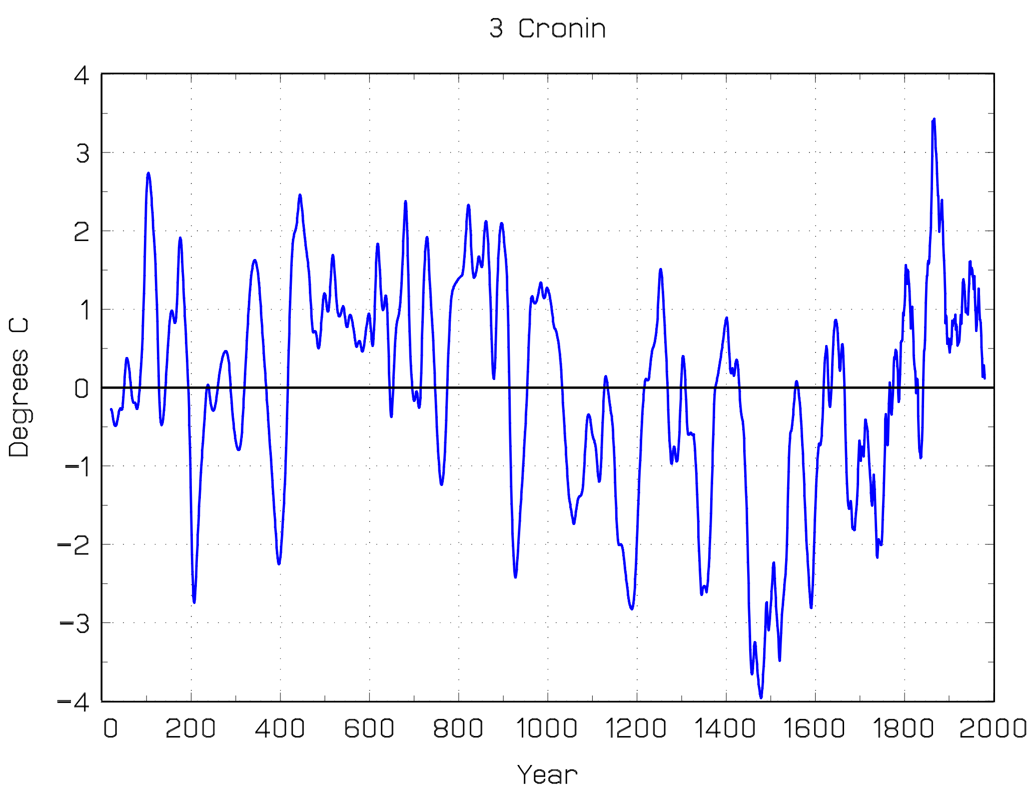
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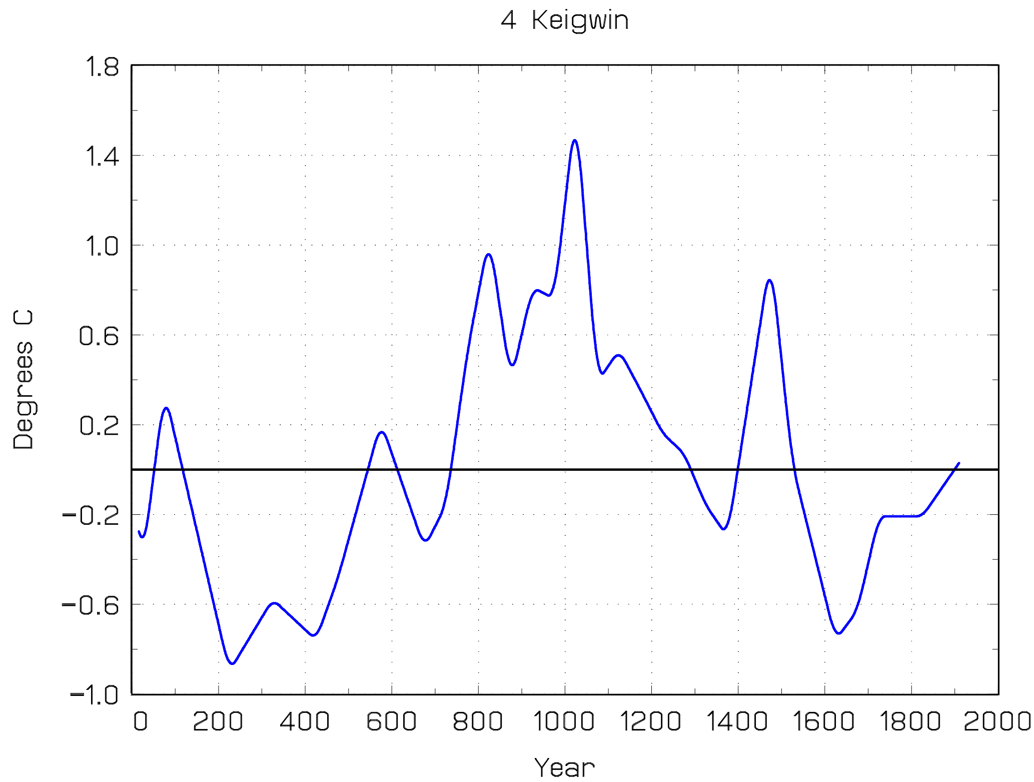
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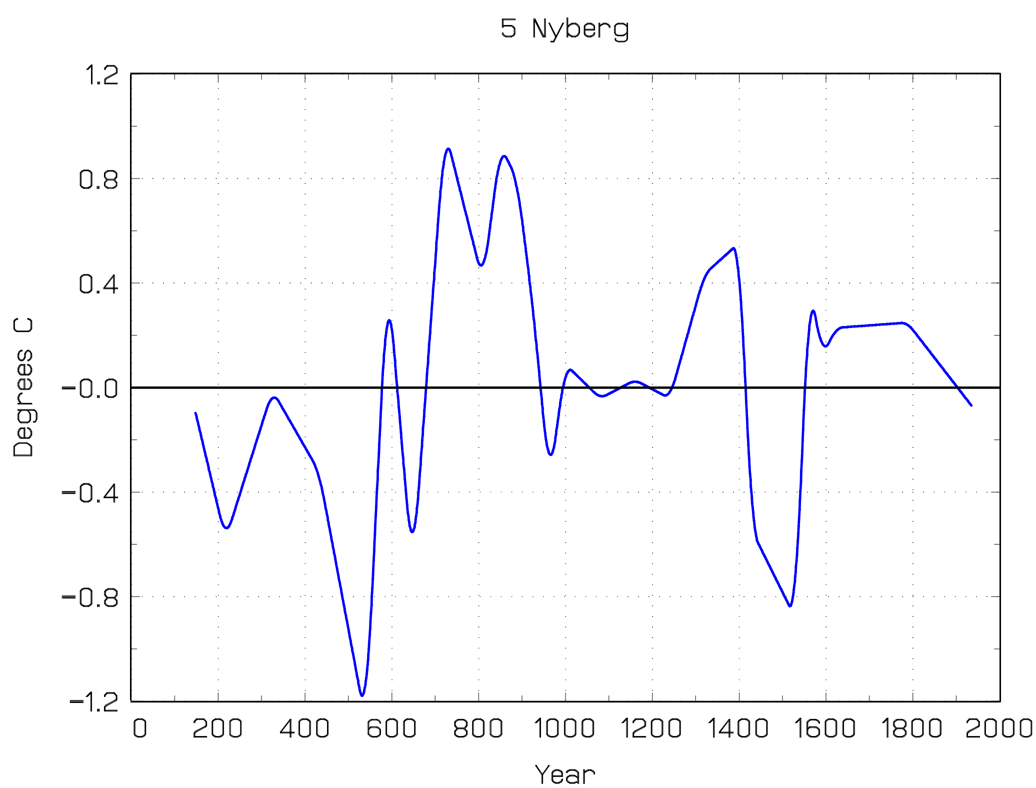
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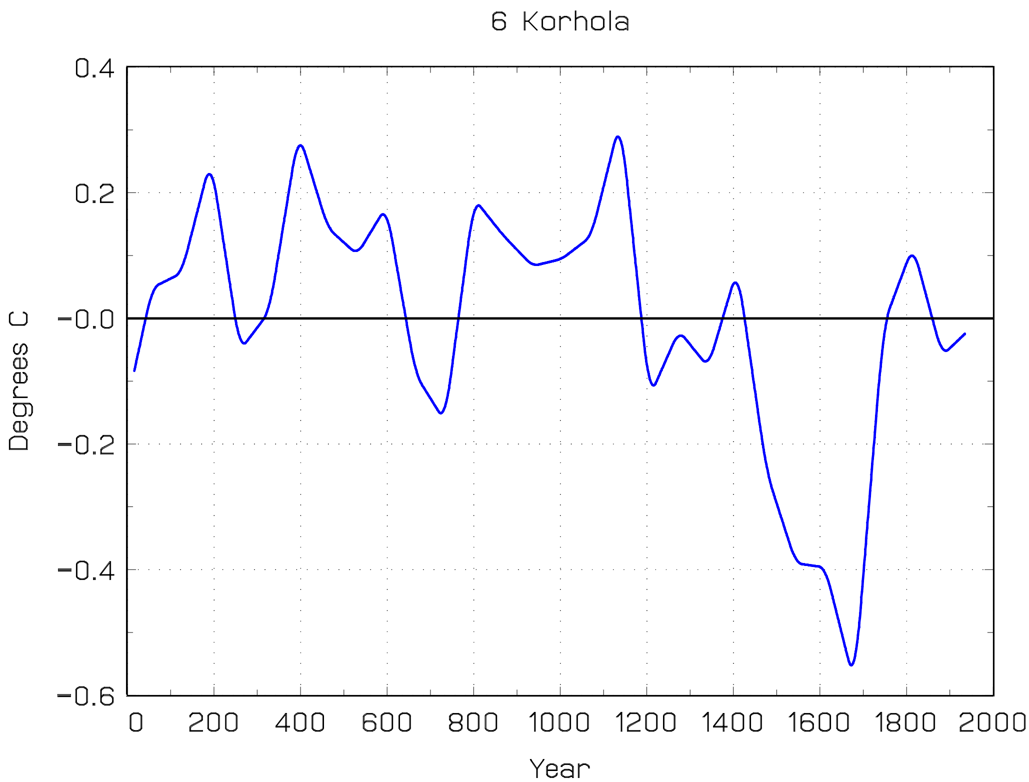
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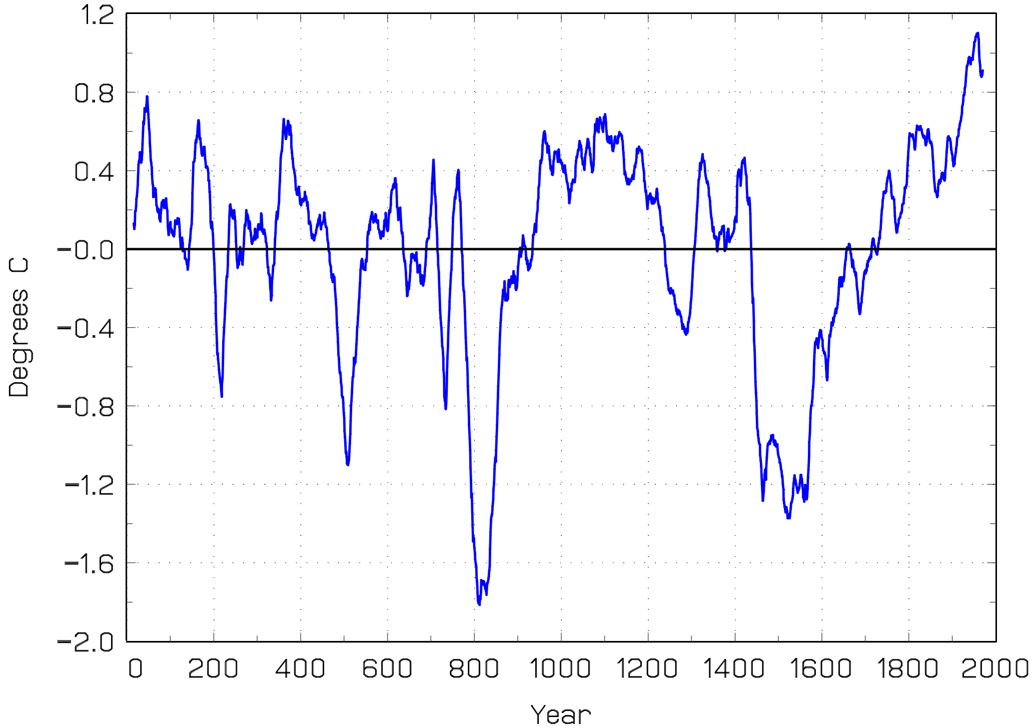

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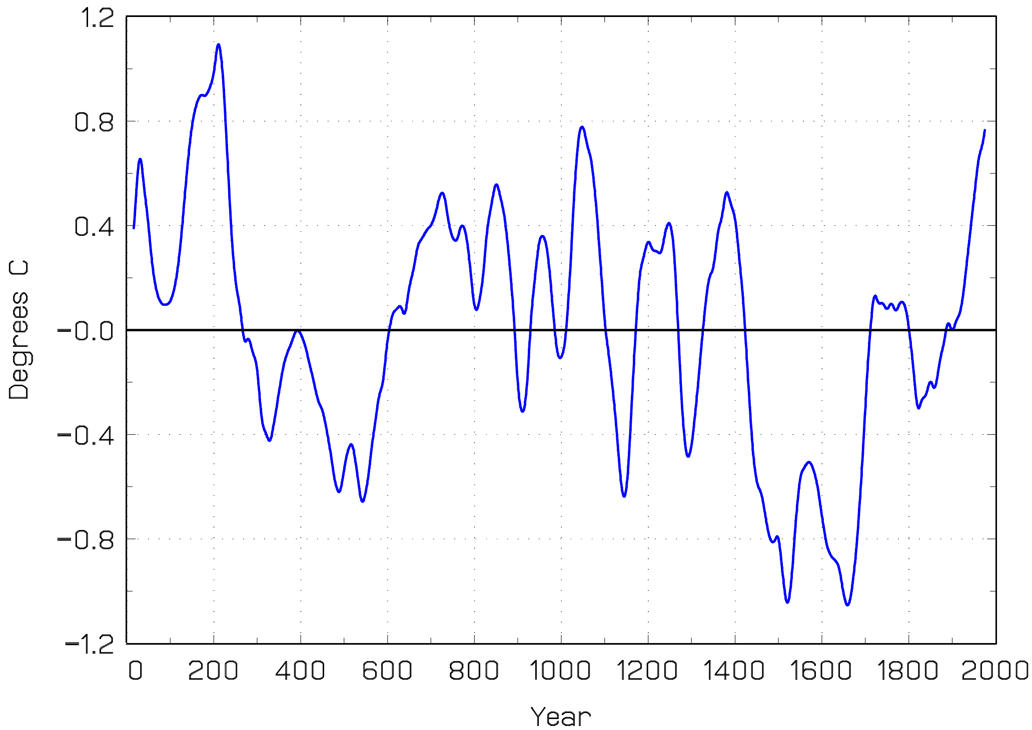
8 Yang



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-0.4
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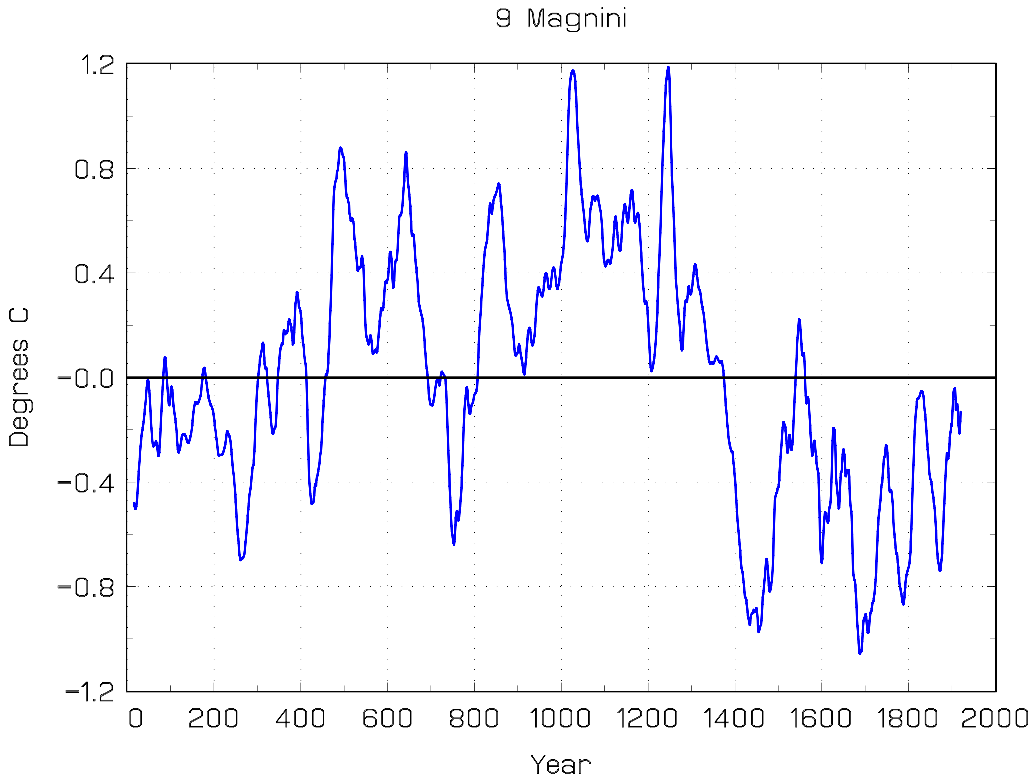
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Degrees C
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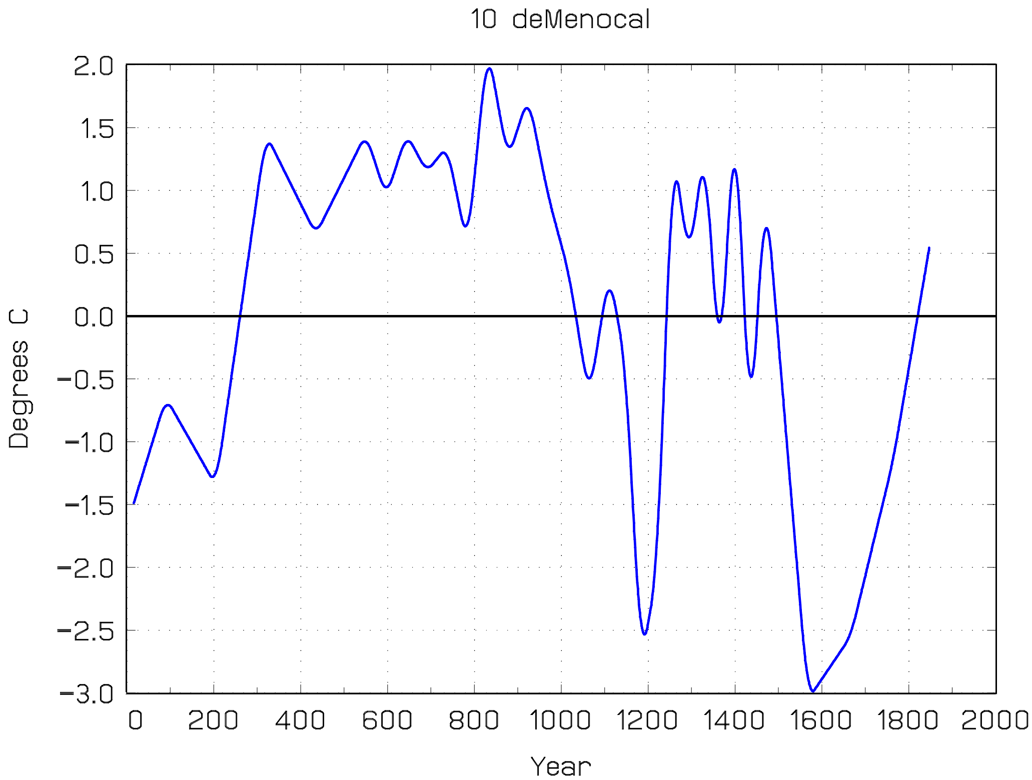
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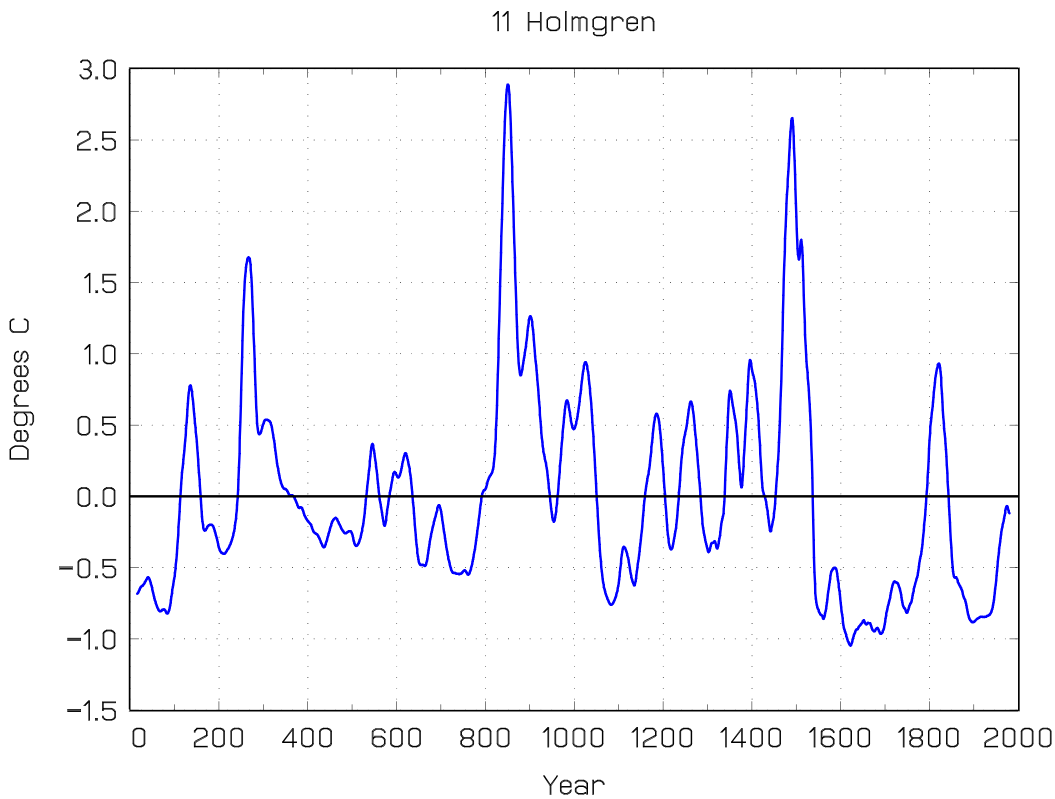
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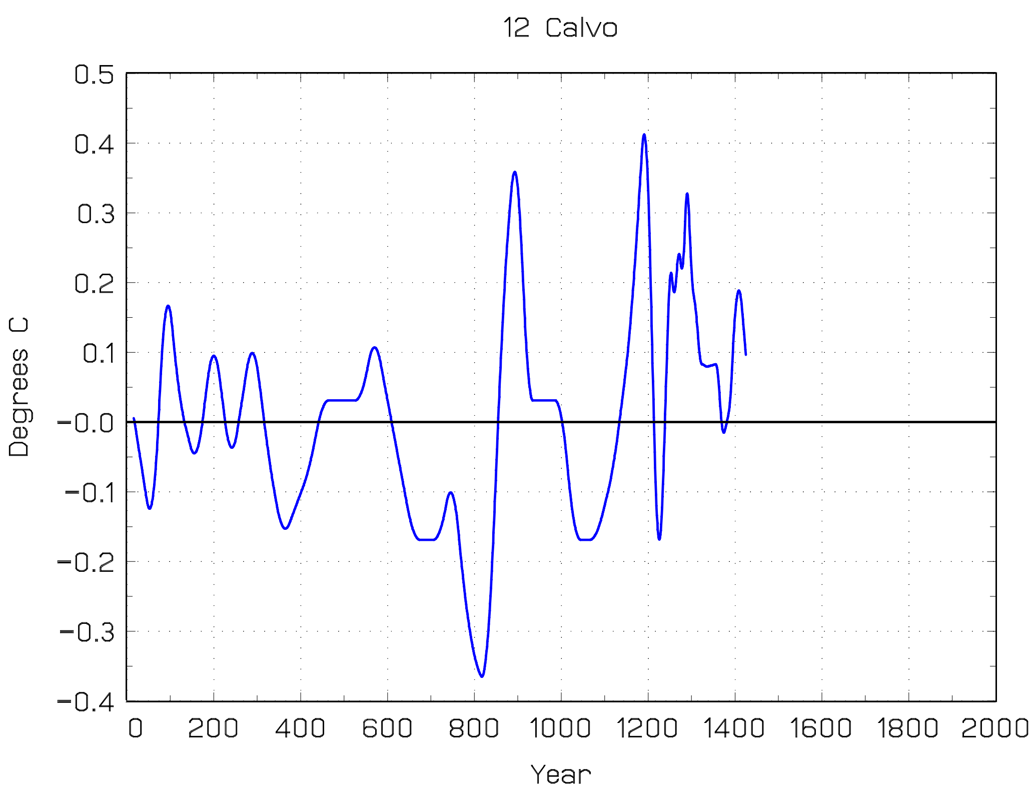
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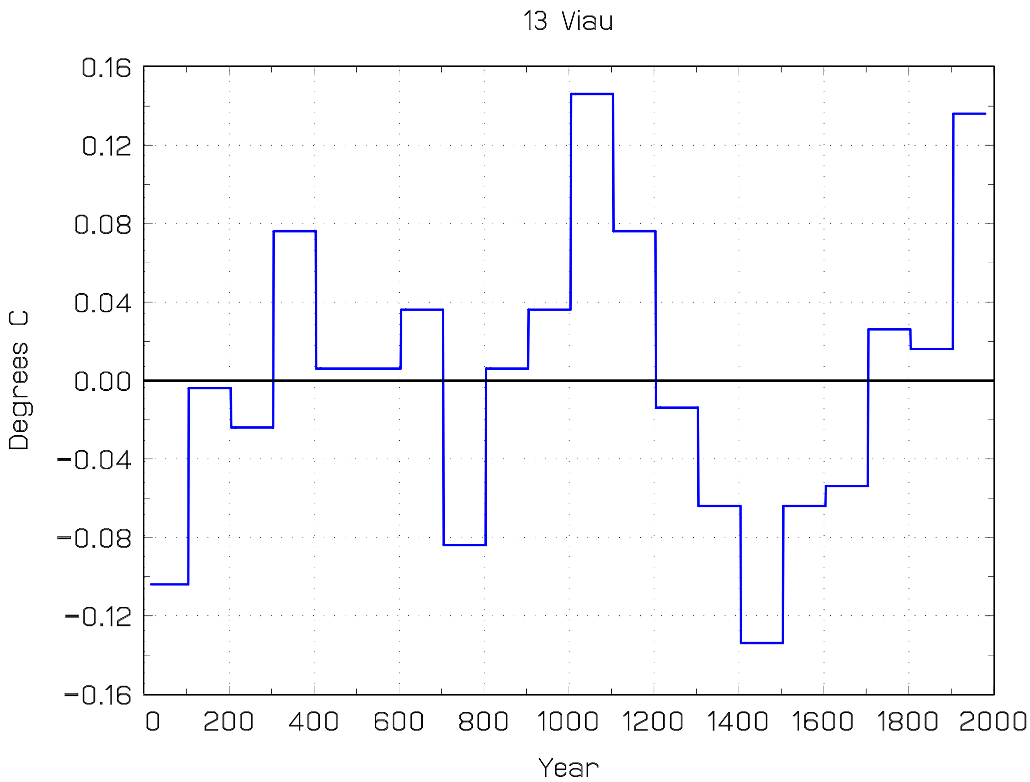
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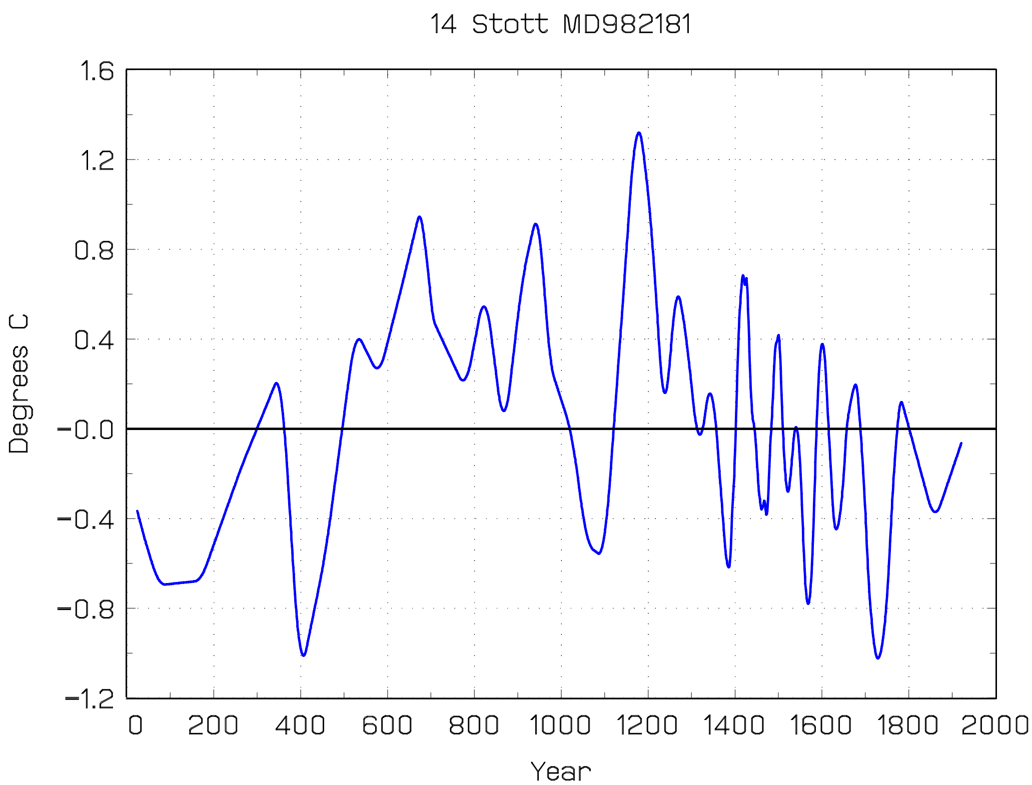
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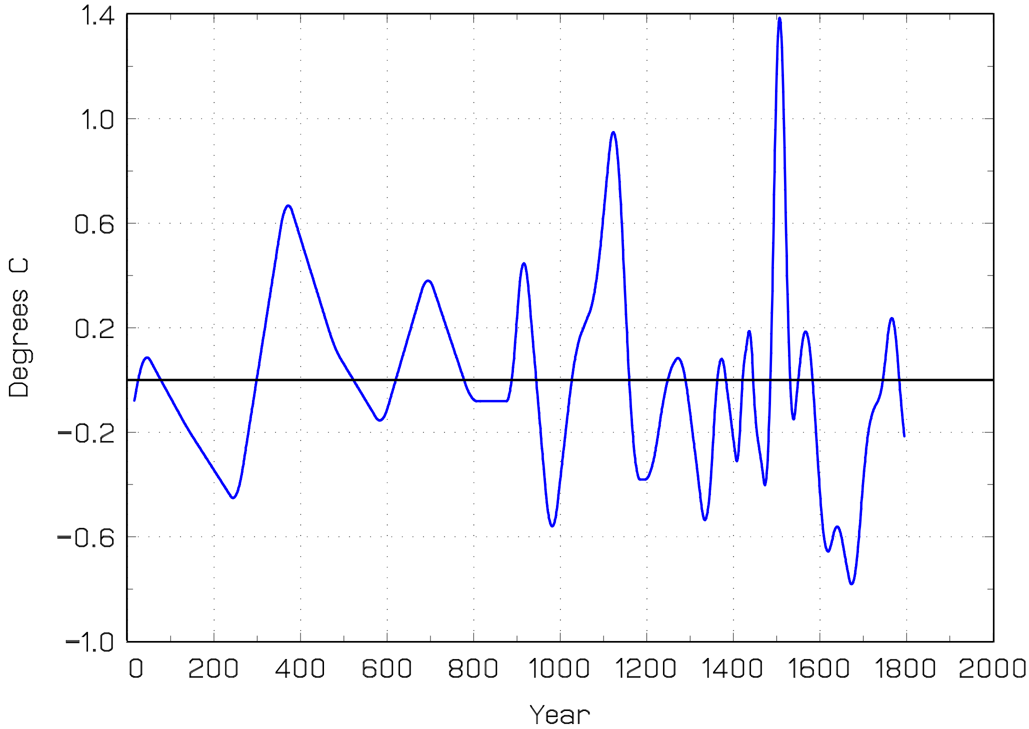

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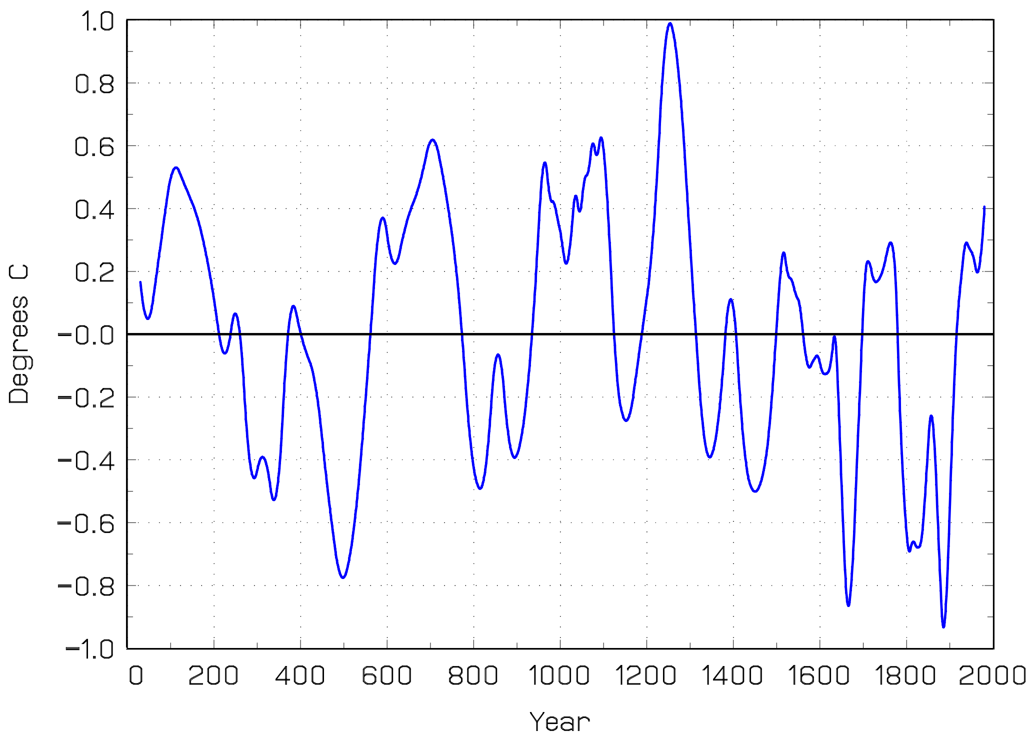

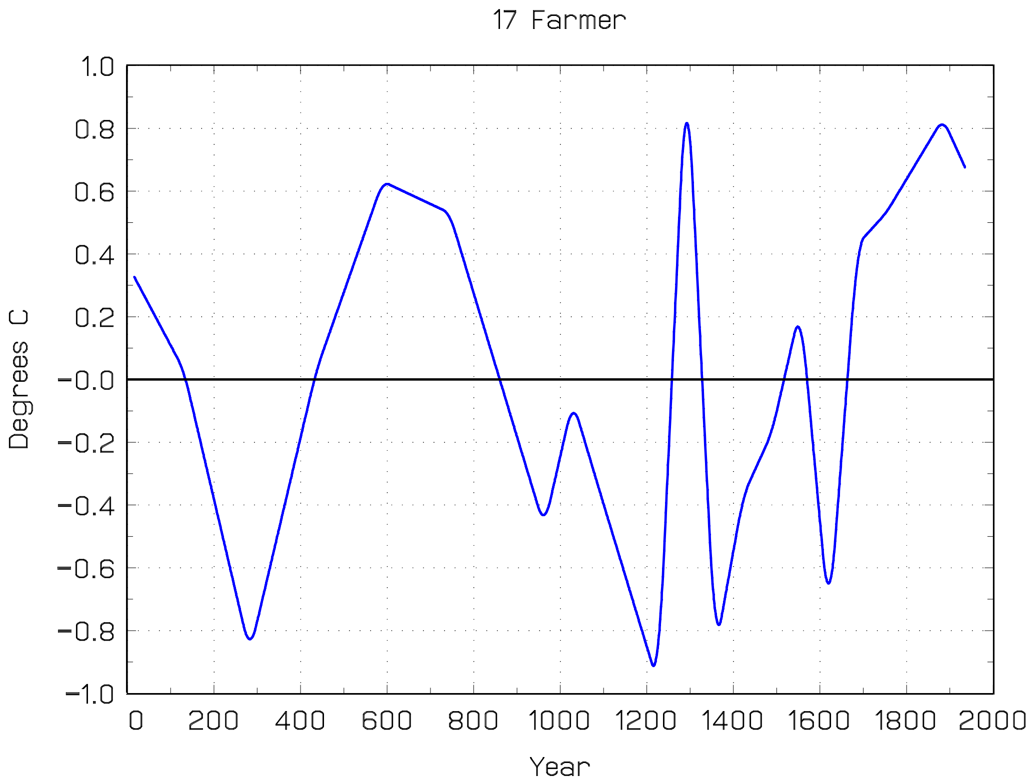
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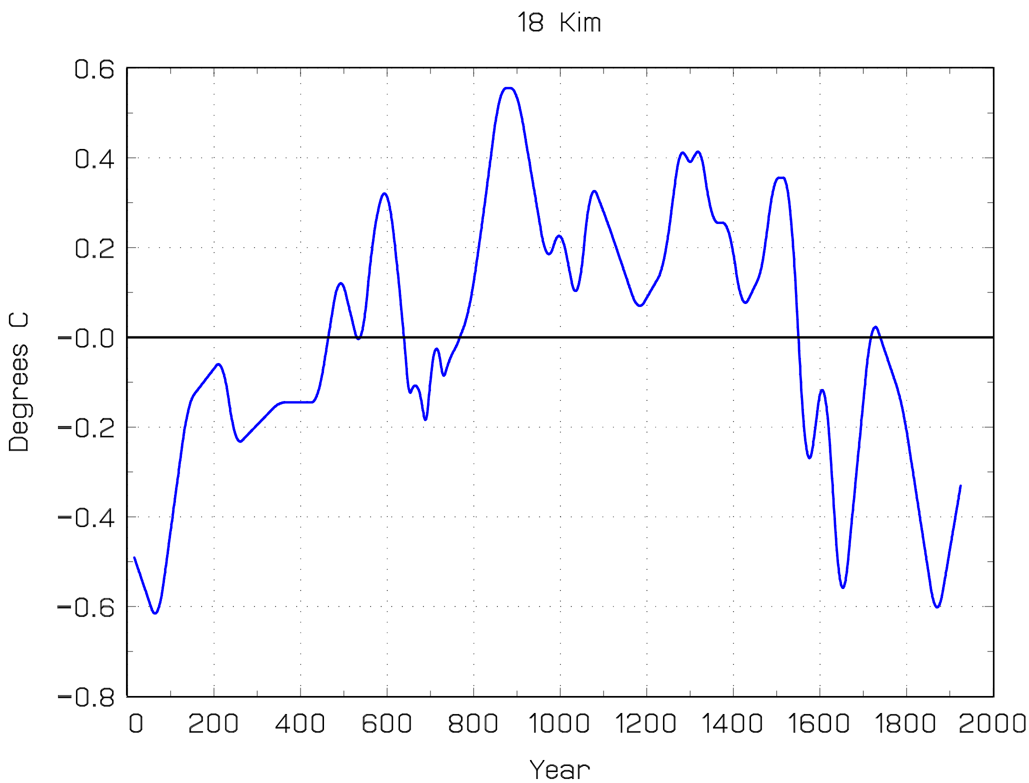
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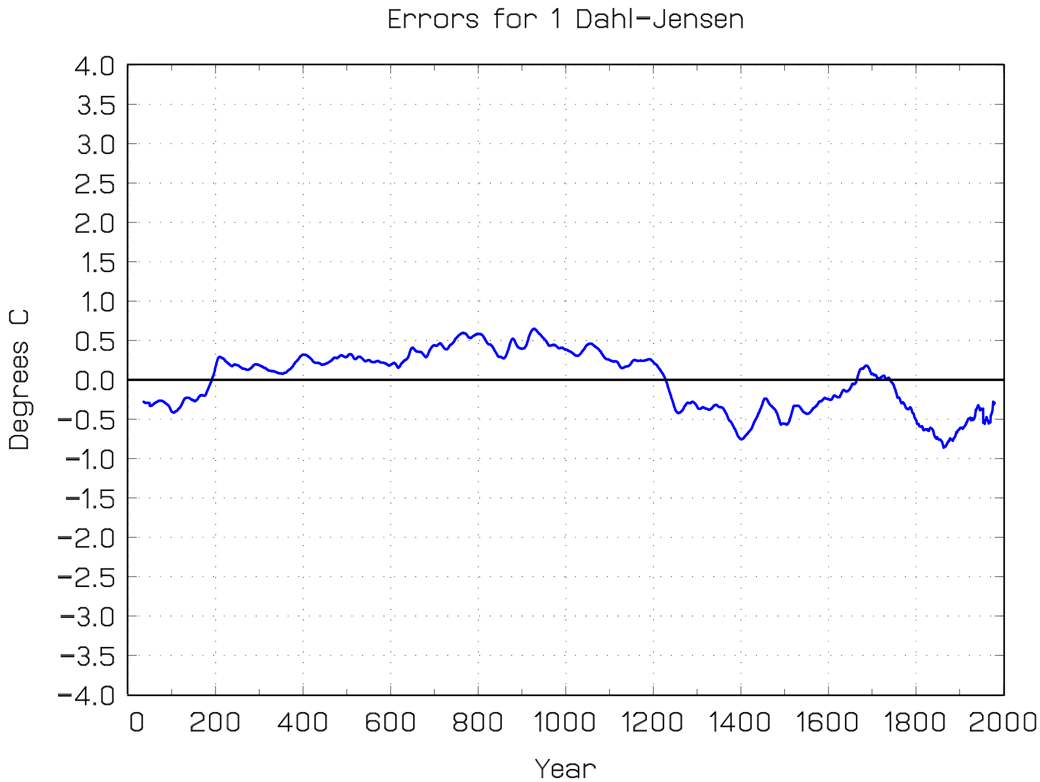
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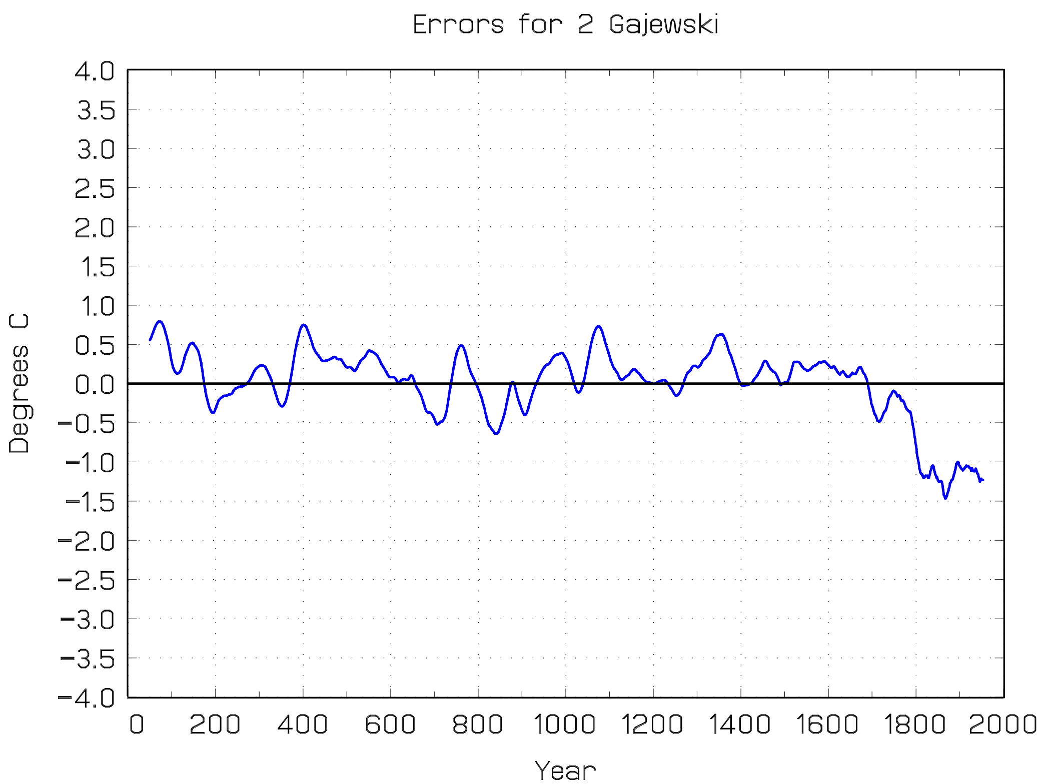
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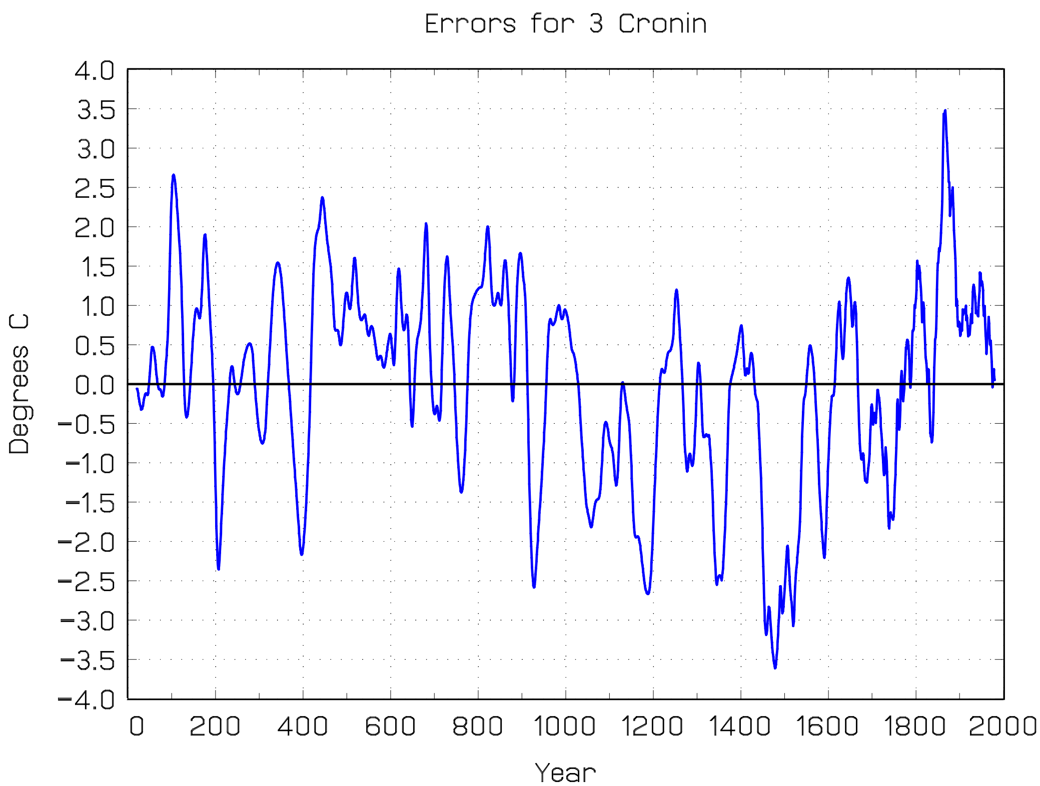
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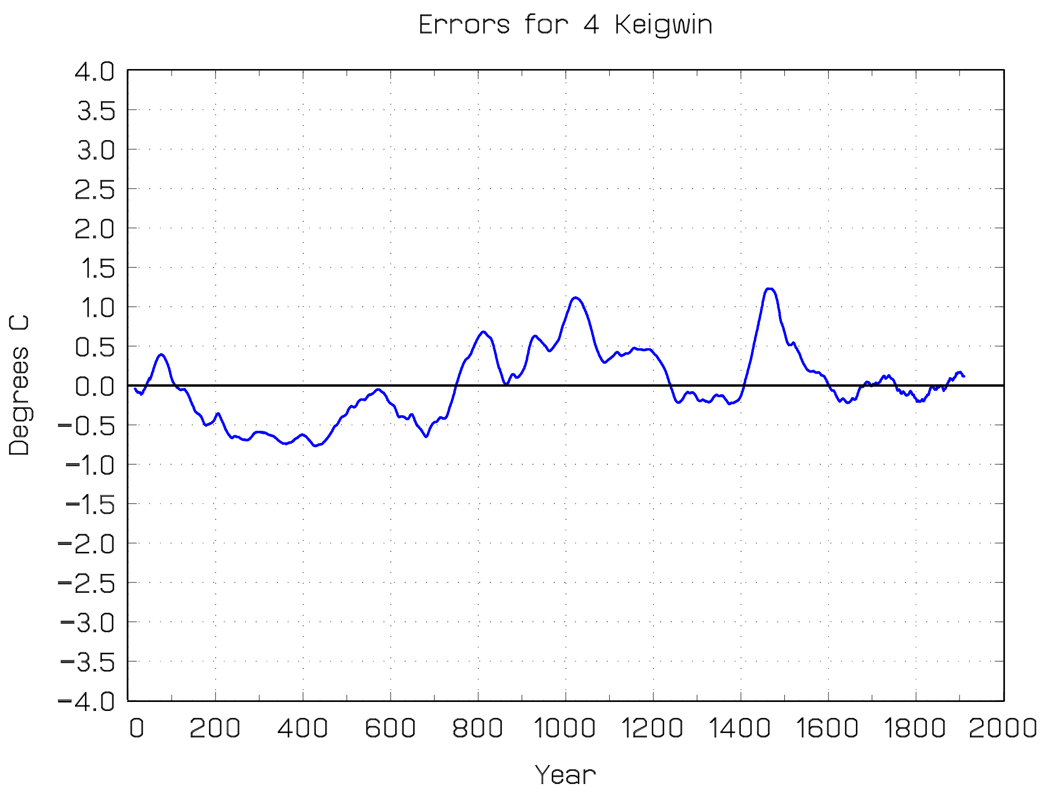
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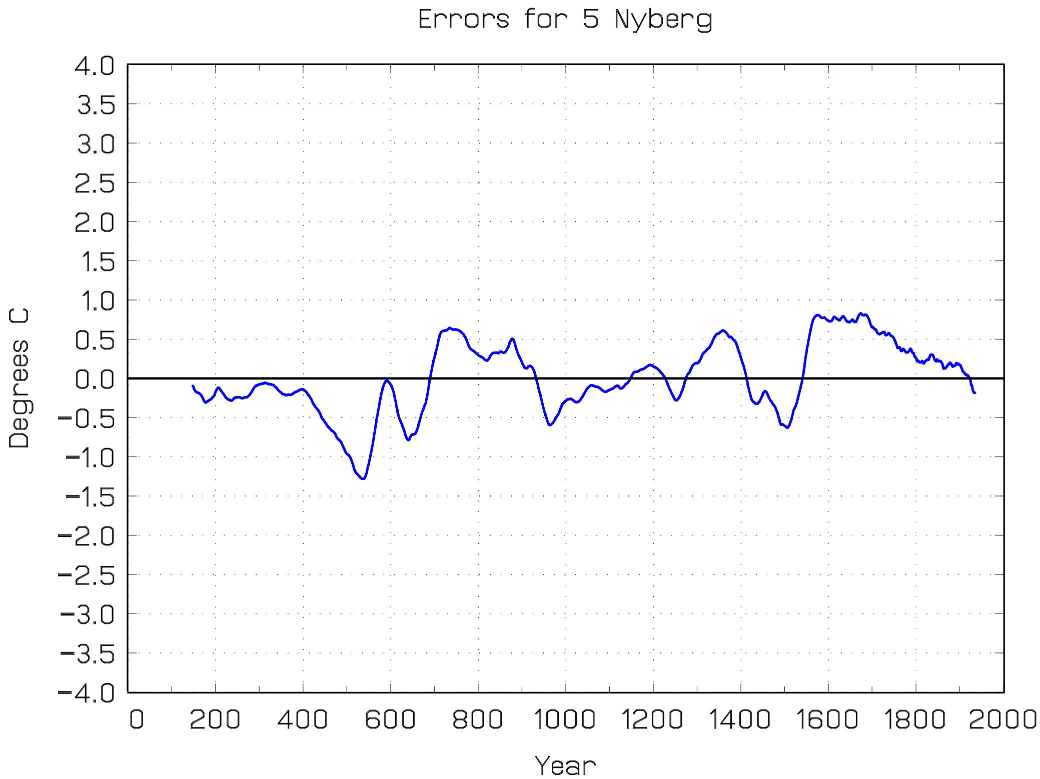
Year



Errors for 5 Nyberg
Degrees C
4.0
3.5
3.0
2.5
2.0
1.5
1.0
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−0.5
−1.0
−1.5
−2.0
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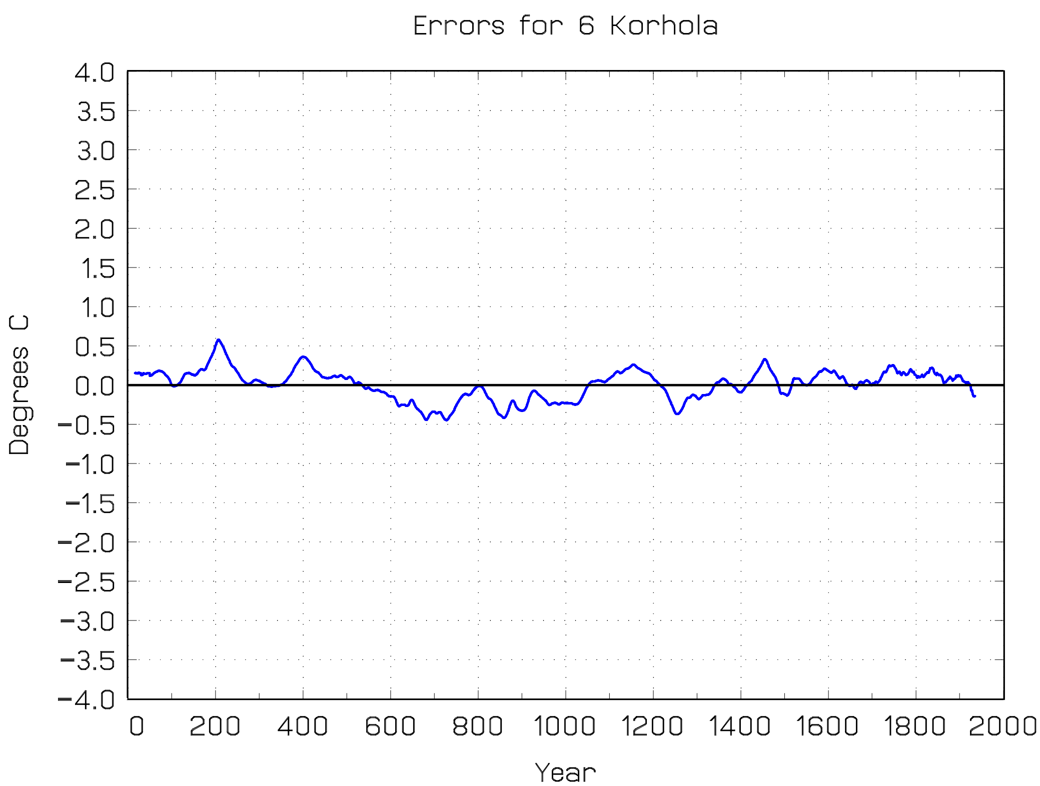
Year



Errors for 6 Korhola
Degrees C
4.0
3.5
3.0
2.5
2.0
1.5
1.0
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−0.5
−1.0
−1.5
−2.0
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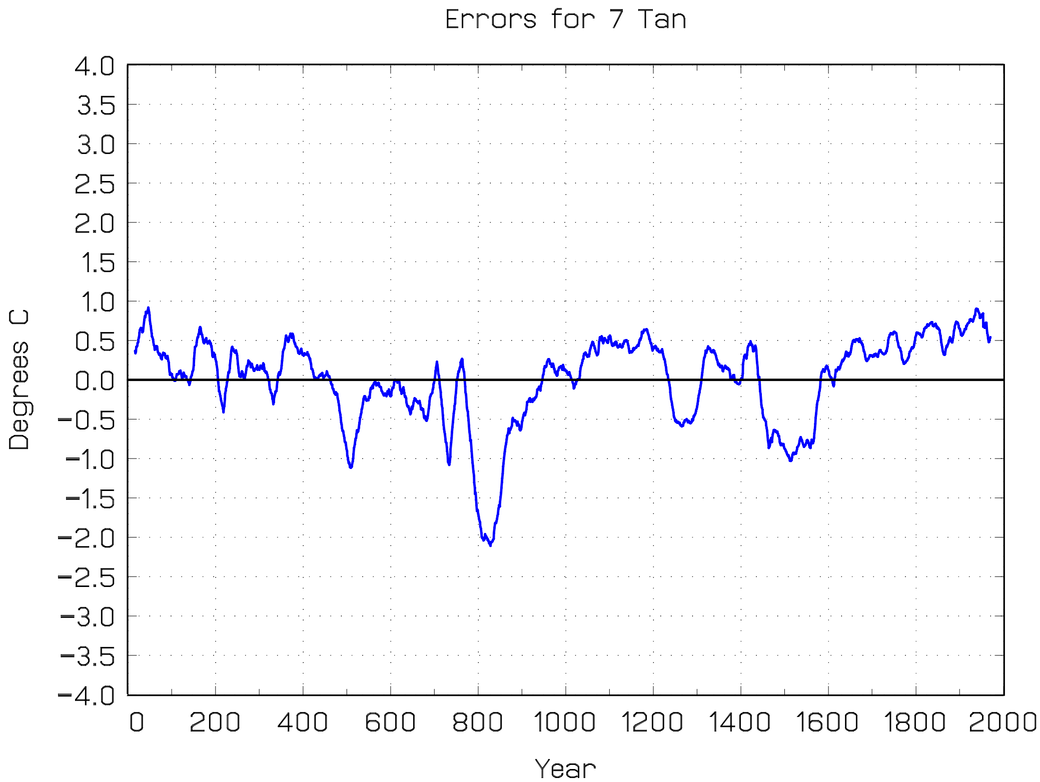
Year



Errors for 7 Tan
Degrees C
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3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
-0.5
-1.0
-1.5
-2.0
-2.5
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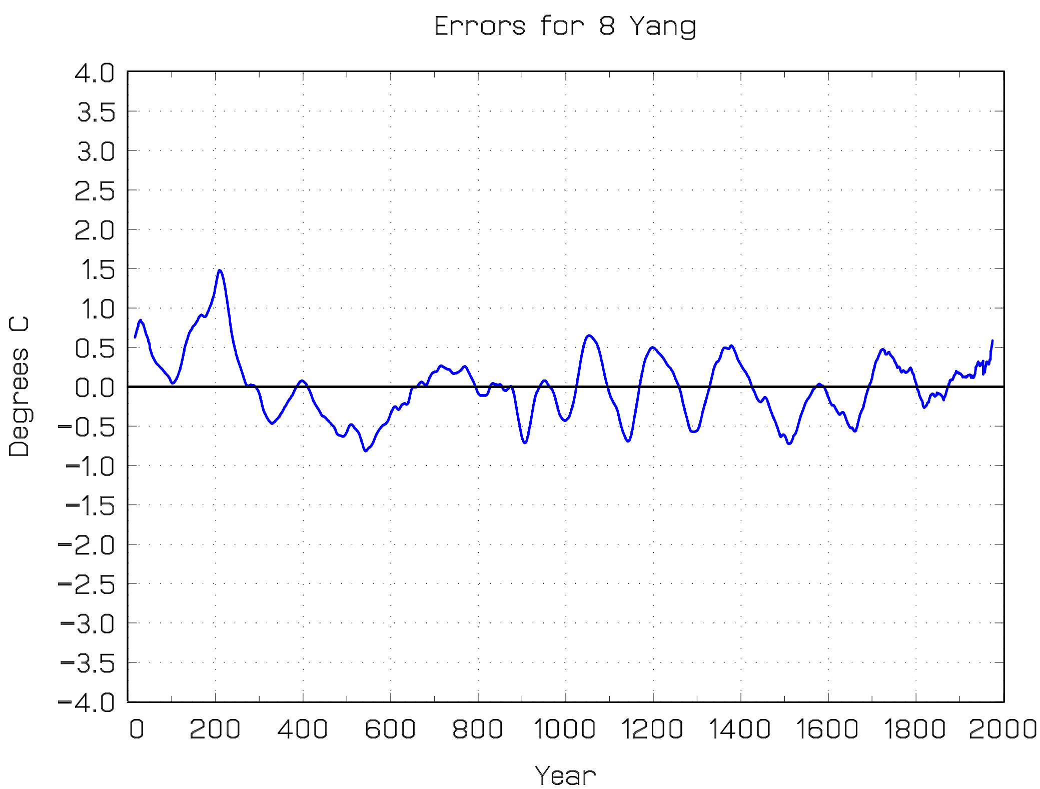
Year



Errors for 8 Yang
Degrees C
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3.0
2.5
2.0
1.5
1.0
0.5
0.0
-0.5
-1.0
-1.5
-2.0
-2.5
-3.0
-3.5
-4.0
0
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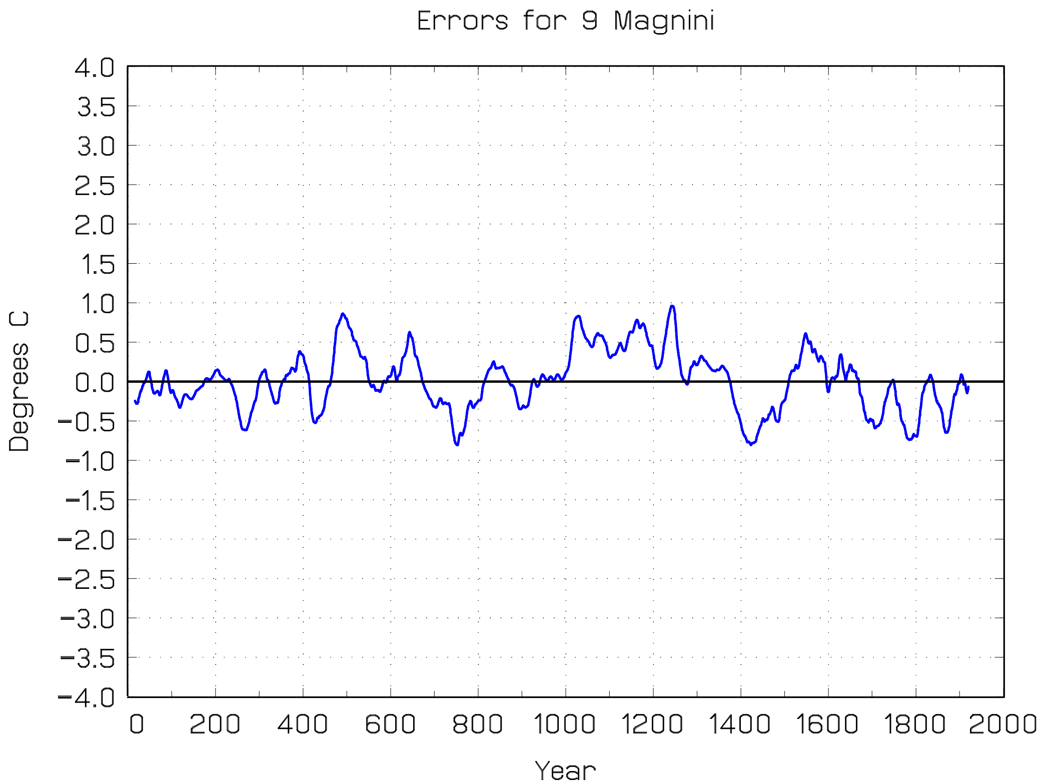
2000
Year


Errors for 9 Magnini
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3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−2.0
−2.5
−3.0
−3.5
−4.0
Degrees C
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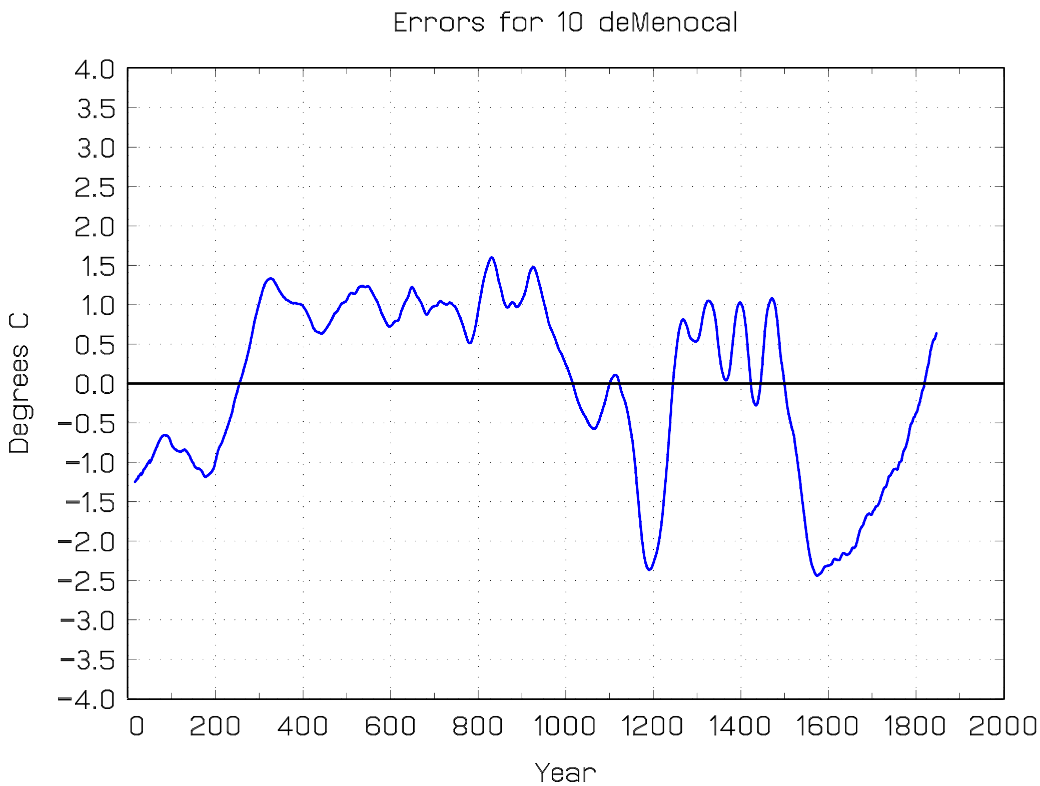
2000
Year


Errors for 10 deMenocal
4.0
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−2.0
−2.5
−3.0
−3.5
−4.0
Degrees C
0
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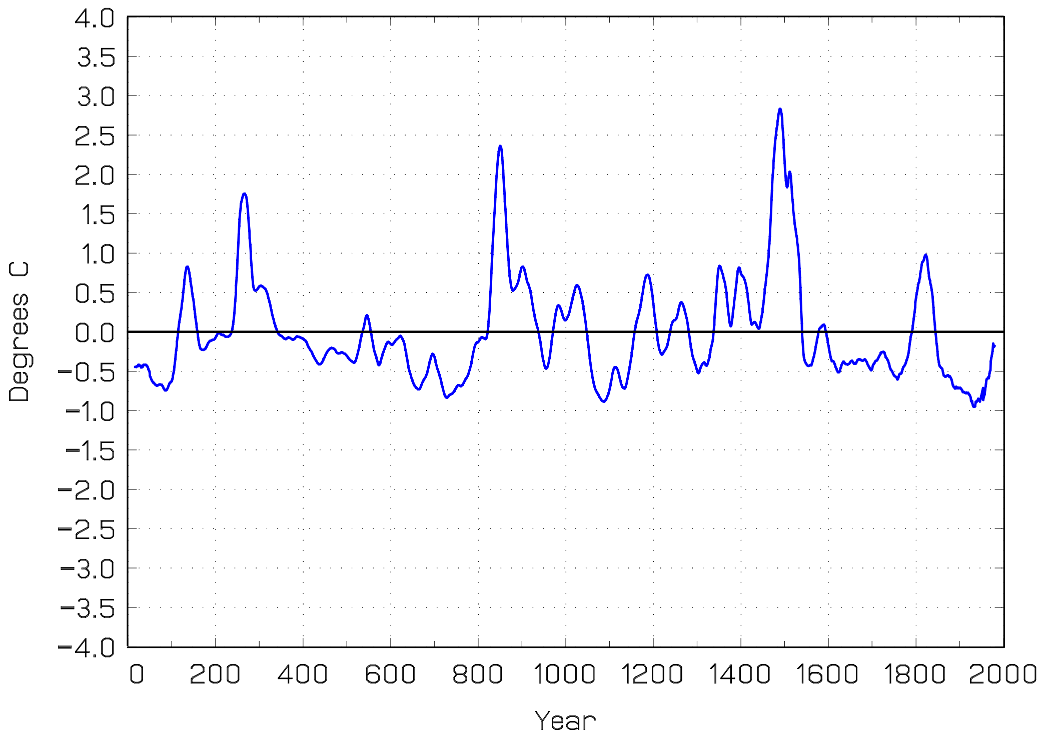
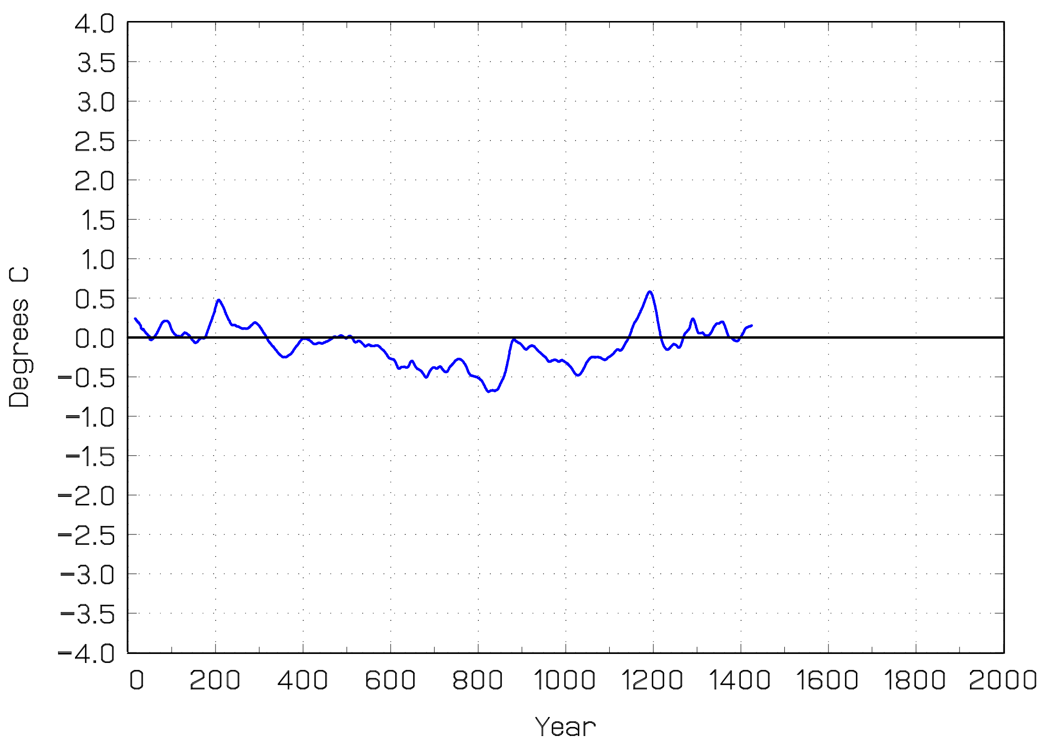
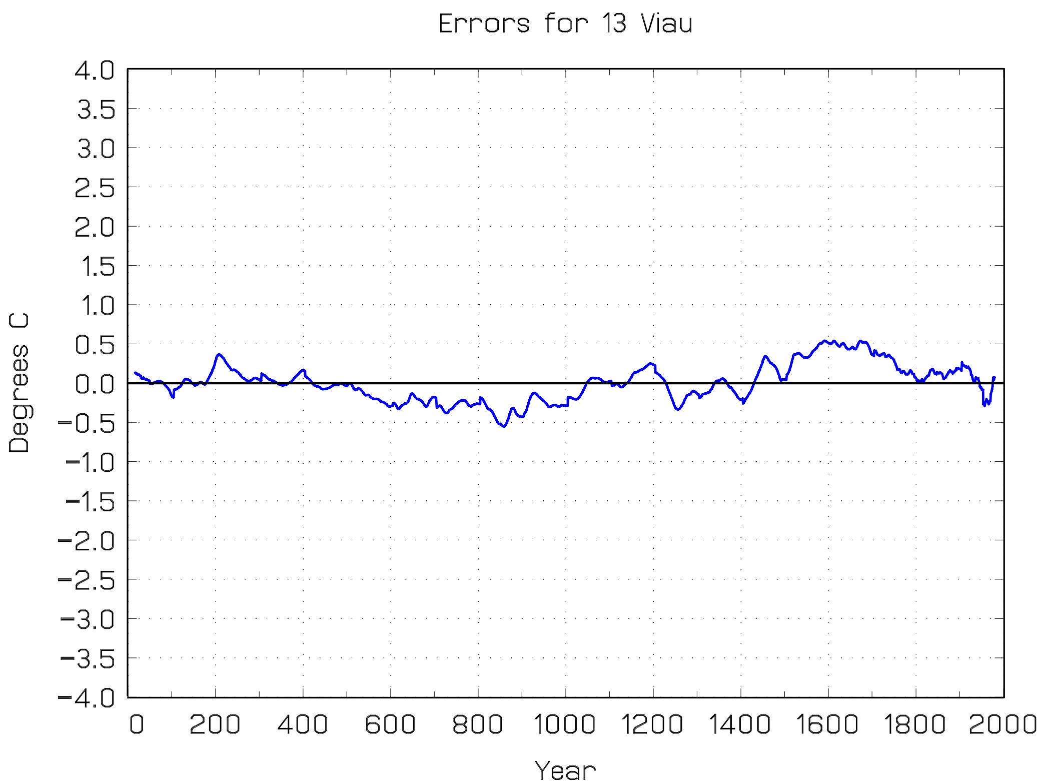
2000
Year

Errors for 11 Holmgren



Errors for 12 Calvo




Errors for 13 Viau
Degrees C
4.0
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−2.0
−2.5
−3.0
−3.5
−4.0
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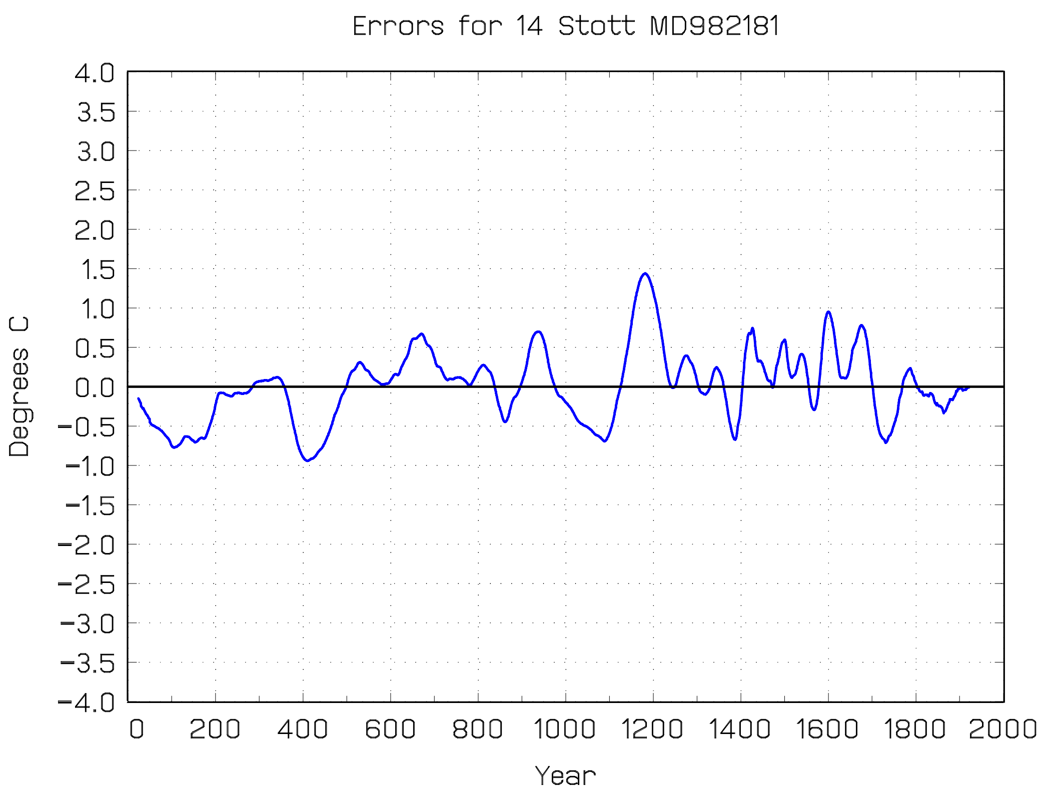
Year


Errors for 14 Stott MD982181
Degrees C
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3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−2.0
−2.5
−3.0
−3.5
−4.0
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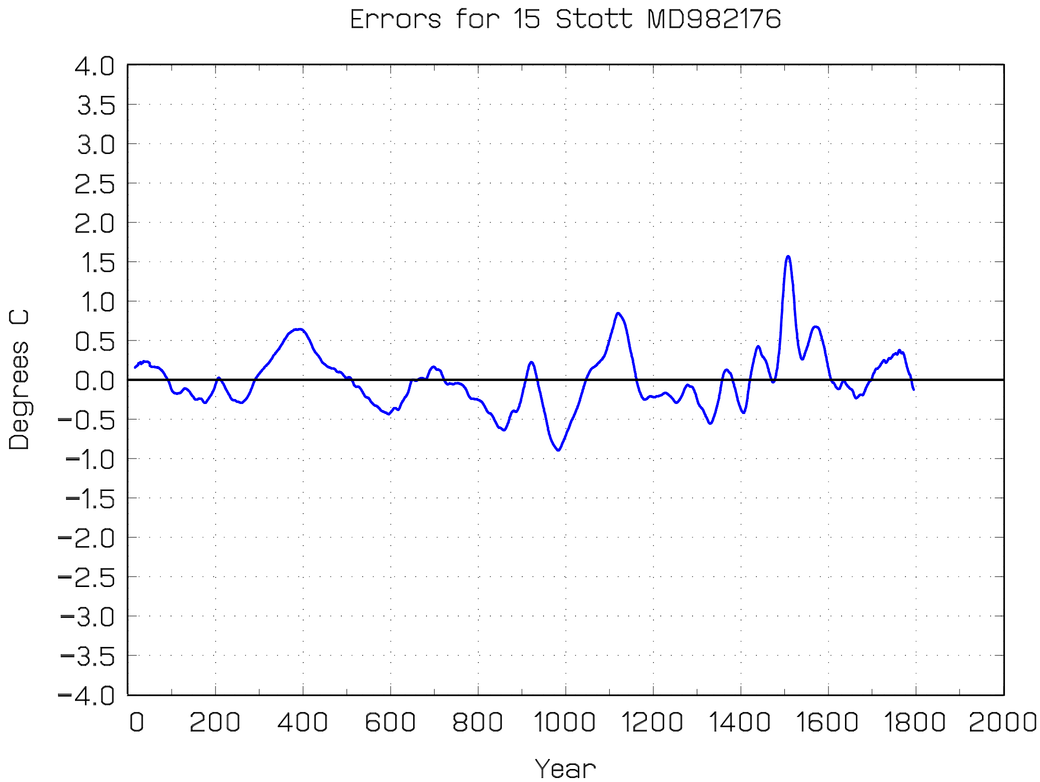
Year


Errors for 15 Stott MD982176
Degrees C
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3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−2.0
−2.5
−3.0
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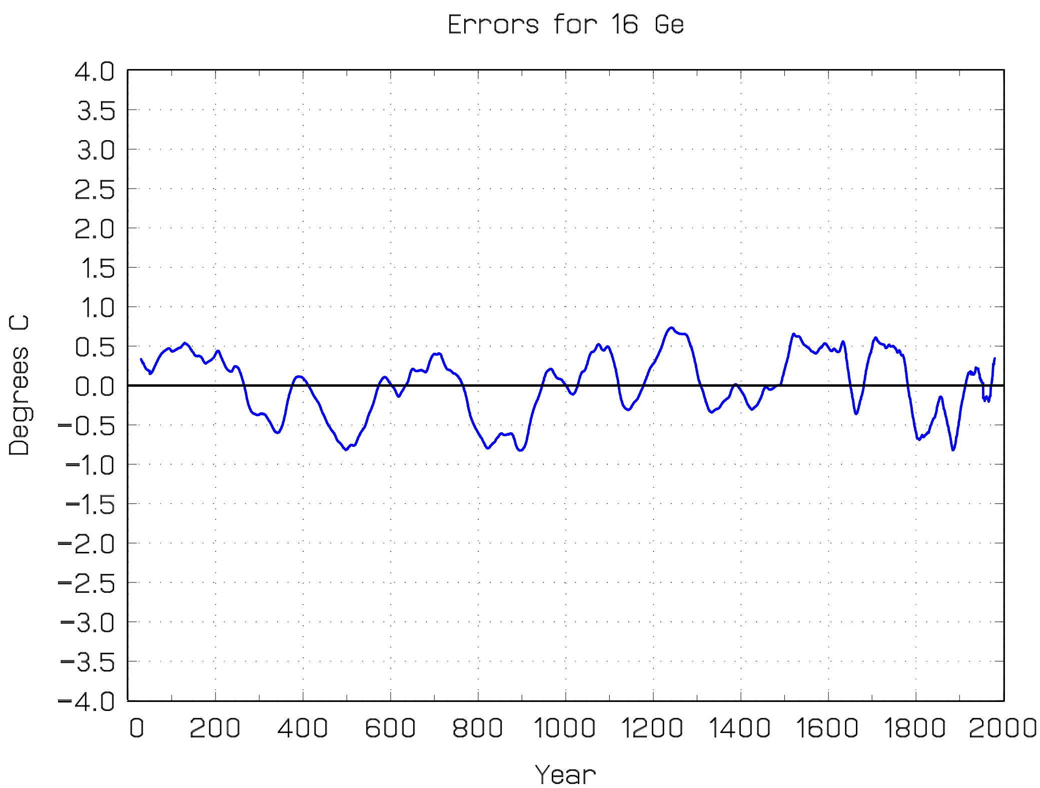
Errors for 16 Ge
Degrees C
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3.0
2.5
2.0
1.5
1.0
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0.0
−0.5
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−1.5
−2.0
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−3.0
−3.5
−4.0
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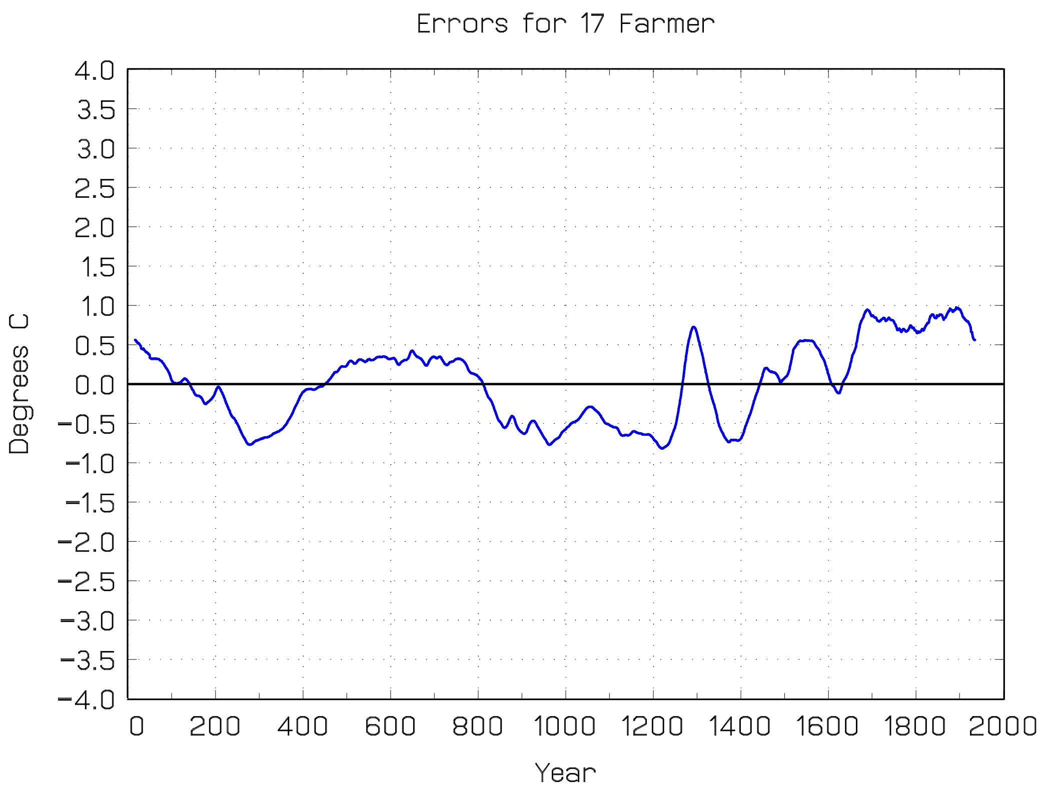
Year


Errors for 17 Farmer
Degrees C
4.0
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−2.0
−2.5
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−4.0
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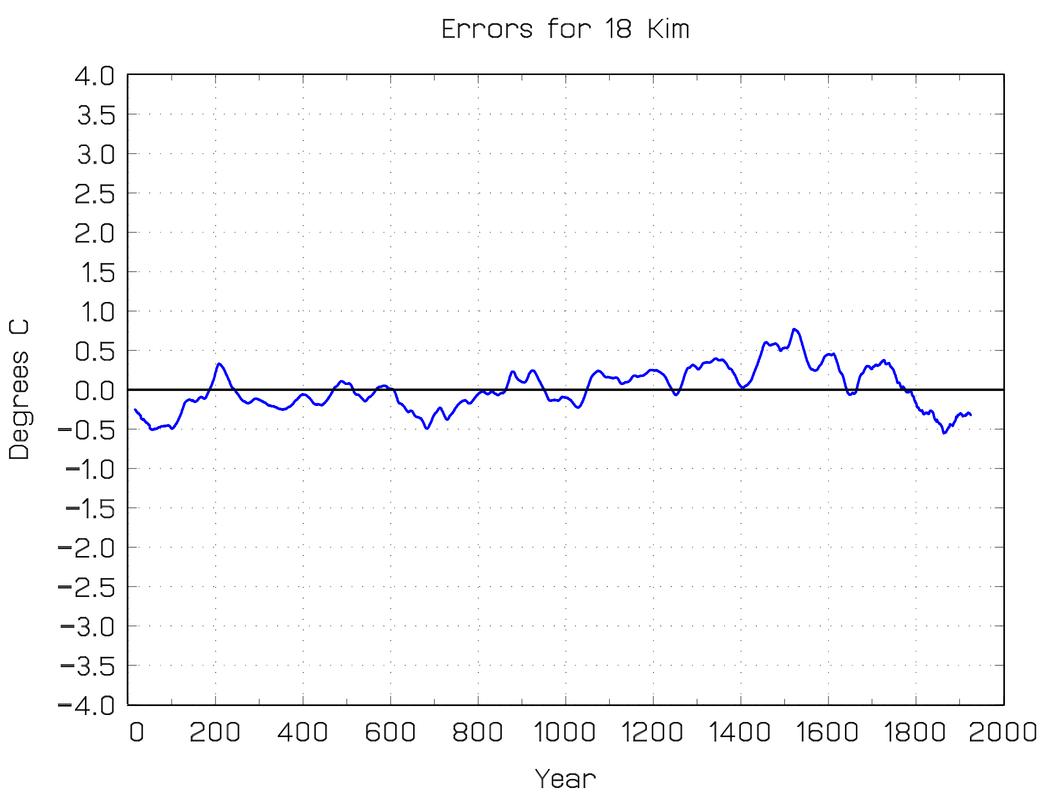
Year


Errors for 18 Kim
Degrees C
4.0
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
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