Lecture 4
Interest Rates

- Interest Rate Mechanics
  - M&B 6
- Real vs. Nominal Rates
  - M&B 8, M&I 4.1
- Loanable Funds Model
  - M&B 19, pp. 1-3
- Debtor-Creditor Redistribution
  - M&I 7.1

Present Value (PV)

\$PV now invested for m years at interest rate i has Future Value (FV):

$$FV = PV\,(1 + i)^m$$

$\Rightarrow$ Future payment of \$FV to be paid in m yrs has PV:

$$\mathrm{PV} = \frac{\mathrm{FV}}{(1+i)^m}$$

E.g., FV = \$100, i = 5% (.05), m = 1 yr:

PV = \$100 / (1.05) = \$95.24.

or, if m = 20 yr,

PV = \$100 / $(1.05)^{20}$ = \$100 / 2.6533 = \$37.69.

Need x^y or ^ key on calculator to compute!

Note: nominal interest rate "*i*" is "*R*" in M&B, M&I.

- $PV = \frac{FV}{(1+i)^m}$ implies that holding FV constant,

  $i\uparrow \rightarrow PV\downarrow$, $i\downarrow \rightarrow PV\uparrow$, and, $m\uparrow \rightarrow PV\downarrow$, $m\downarrow \rightarrow PV\uparrow$.

- Also, effect of Δi on PV grows proportionately stronger with *m*:

  $\Delta PV/PV \approx - \Delta i \cdot m$

  – *Examples:*

  m = 1 yr, i rises from 5% to 6%, Δ i = +1%, FV = 100:

  $- \Delta i \cdot m = - (+1\%)(1yr) = -1\%$

  (Actual ΔPV/PV = (94.34 – 95.24)/95.24 = - .0094 = - 0.94%)

  m = 20 yrs, i rises from 5% to 6%:

  $- \Delta i \cdot m = - (+1\%)(20YR) = -20\%$

  (Actual ΔPV/PV = (31.18 – 37.69)/37.69 = - .173 = - 17.3%)

  – Leads to *Interest Rate Risk* when banks or thrifts lend long, borrow short. (More later)

i from FV / PV:

$$PV = FV / (1+i)^m \Rightarrow$$

$$(1+i)^m = FV / PV,$$

$$1+i = (FV / PV)^{1/m}, \text{ so}$$

$$i = (FV / PV)^{1/m} - 1$$

E.g., FV = \$100, PV = \$50, m = 10 yrs.,

$$i = (100 / 50)^{1/10} - 1$$

$$= 2^{0.1} - 1 = 1.0718 - 1 = 0.0718 = \underline{7.18\%.}$$

Note: 0.01% = one "Basis Point".

Bonds

Face Value \$F to be paid at maturity *m*

Coupons \$C paid each year for *m* years.

(Assume annual for simplicity – most semiannual)

Bond Present Value (PV_B)

$$PV_B = \frac{C}{(1+i)} + \frac{C}{(1+i)^2} + \ldots + \frac{C+F}{(1+i)^m}$$

$$\Rightarrow \boxed{i \uparrow \rightarrow PV_B \downarrow, \quad i \downarrow \rightarrow PV_B \uparrow}$$

(but $m \uparrow$ could have either effect because payments are being added)

Yield to Maturity (YTM)

= the value of *i* that gives back market price of bond,

holding C, F, m constant.

If

$PV_B = F$, bond is "at par", $YTM = C / F$

$PV_B > F$, bond is "above par", $YTM < C / F$

$PV_B < F$, bond is "below par", $YTM > C / F$

E.g.

$F = \$100$, $C = \$4$, $PV_B = \$100 \Rightarrow YTM = 4\%$

$F = \$100$, $C = \$3$, $PV_B = \$110 \Rightarrow YTM < 3\%$

$F = \$100$, $C = \$6$, $PV_B = \$90 \Rightarrow YTM > 6\%$

Bond Duration*

Effect of Δi on PV_B again proportionally stronger, the longer its m. However, now,

$$\Delta PV_B/PV_B \approx - \Delta i \cdot D,$$

where the bond's ***Duration D*** equals the **present-value-weighted average maturity** of its payments:

$$D = \left(\frac{(1)C}{(1+i)} + \frac{(2)C}{(1+i)^2} + \ldots + \frac{(m)(C+F)}{(1+i)^m} \right) / PV_B$$

Generally,

D = m if C = 0,

D < m if C > 0,

D increases with m

** aka Macaulay Duration*

Consols (Perpetuities)

Pay $C / yr. forever

Exist in UK, conceptually important

$$\mathrm{PV_C} = \frac{C}{1+i} + \frac{C}{(1+i)^2} + \frac{C}{(1+i)^3} + \quad \ldots \quad \infty$$

$$= \frac{C}{1+i} + \frac{1}{(1+i)}\left[\frac{C}{1+i} + \frac{C}{(1+i)^2} + \quad \ldots \quad \infty\right]$$

$$= \frac{C}{1+i} + \frac{1}{(1+i)}\left[\mathrm{PV_C}\right]$$

$\Rightarrow$ $(1+i)\ \mathrm{PV_C} = \mathrm{C} + \mathrm{PV_C}$,

$$\mathrm{PV_C} = \mathrm{C} / \mathrm{i}$$

E.g., C = $100, i = 4% $\Rightarrow$ $\mathrm{PV_C}$ = 100/.04 = $2500

Consol Duration

$D_C = 1 / i$

E.g., $i = 4\% / yr \Rightarrow D_C = 1 / .04 =$ 25 yrs.

D_C finite despite infinite final maturity!

$\Delta PV_C / PV_C \approx - \Delta i \cdot D_C$ as for bonds

Real vs. Nominal Interest Rates

i = *nominal* interest rate

on $-denominated loans, not indexed for inflation.

r = *real* interest rate

on purchasing-power-denominated loans, with payments indexed for inflation

Note: nominal interest rate *i* is "*R*" in M&B, M&I. "R" will also be used for bank reserves, so "i" less ambiguous.

US Treasury Inflation-Protection Securities (TIPS)

- All payments indexed to CPI-U (2.5 mo. lag)
- Provide direct observation of real rate *r*
- First issued Jan. 1997
- 5, 10, 20, 30-year initial maturities
- Now $472 B (10.4% of marketable Treasury debt) (2009)

❖ Monthly TIPS yield curves on my webpage:

www.econ.ohio-state.edu/jhm/ts/ts.html

Present Values with P-indexed loans

Same formulas, with *r* in place of *i*

$PV = FV / (1+r)^m$, etc.

e.g. Indexed Consol:

C = *Real* coupon payment (today's \$)

$PV_C = C / r$ (today's \$)

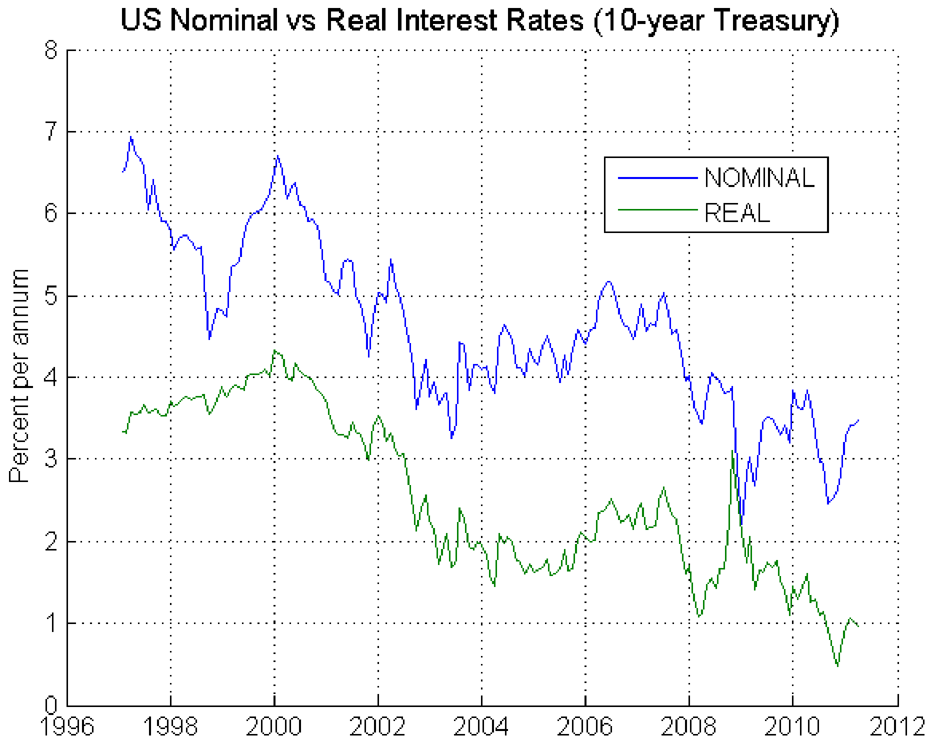
application:

Parcel now pays \$100,000 rent per year, future rent assumed to grow in proportion to P. *r* = 2%.

$\Rightarrow$ PV = \$100,000 / .02 = <u>\$5,000,000</u>

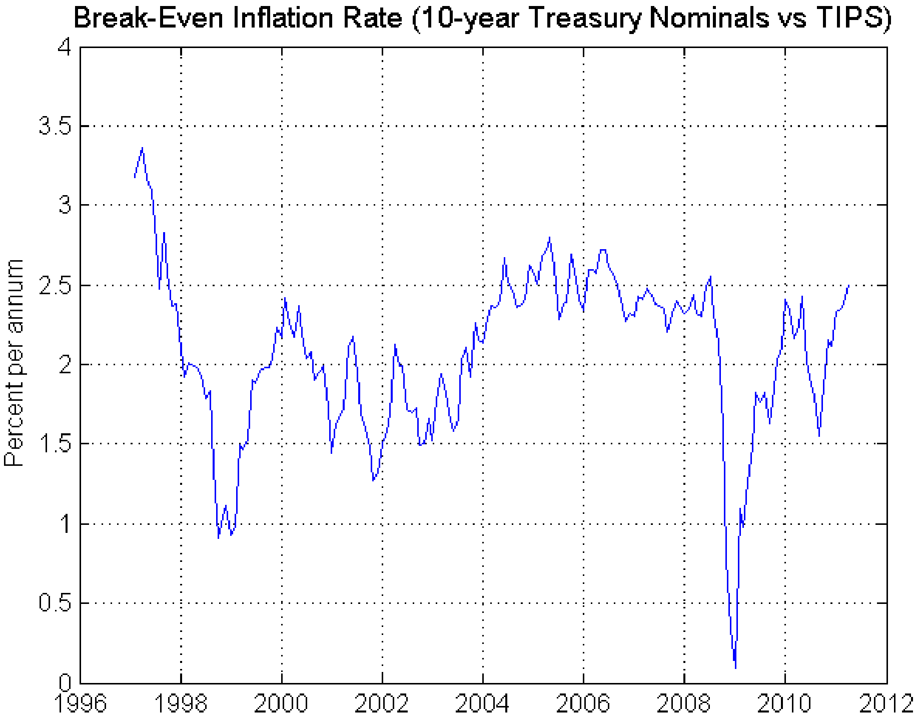
- Real YTM *r* on 10-yr TIPS has been between 0.5% and 4.5% since their introduction in 1997. Recent values under 0.5% very unusual.

- Nominal YTM *i* on conventional Treasuries higher, to compensate for likely future inflation. Also more volatile, because of fluctuations in inflation premium.



i minus *r* gives “Break-Even” inflation rate, at which returns on real and nominal bonds are equal.

According to the **Fisher Equation** (next slide), this is the market’s **expectation of future inflation** π^e over the life of the loans.



(thru Dec. 2008)

Determination of *r*, *i*

- Loanable Funds Model

  real rate *r* primarily determined by savings, investment decisions, S and D for Credit.

- Fisher Equation

  $$i = r + \pi^e,$$

  where π^e is *expected inflation* over life of loan

  (sometimes written π^a for *anticipated* inflation)

- Adaptive Learning (AL)

  π^e mostly determined by past π,

  with biggest weights on recent past.

  Coefficients may change slowly over time.

- Before 1997, *r* not directly observed.

- But Adaptive Learning allows us to infer r from i, using Fisher Eq'n:

$$r = i - \pi^e$$

plus recent inflation.

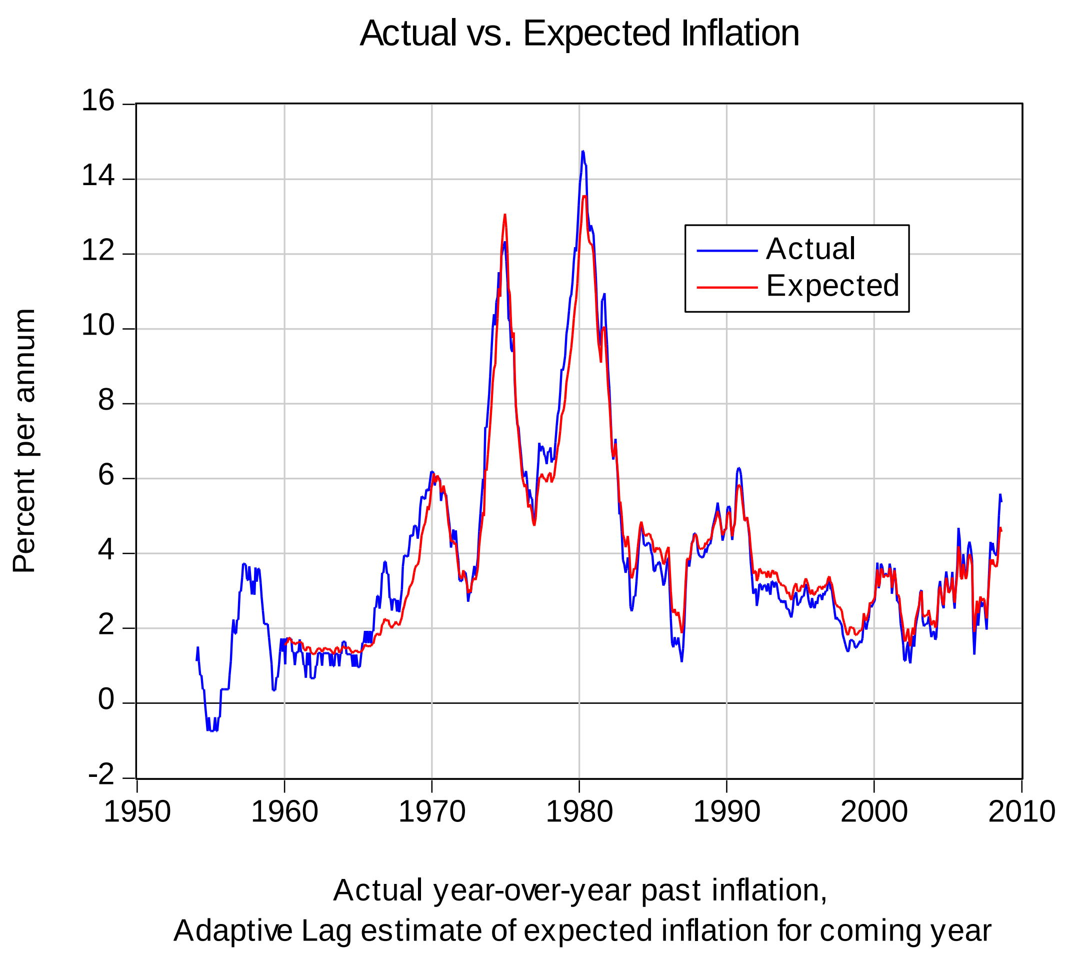
- Simple model for 1-yr horizon using recent experience:

$$\pi^e = 1.24 + .64\,\bar{\pi}$$

where $\bar{\pi}$ is avg. inflation over past 12 mo. (Coefficients change slowly over time.)

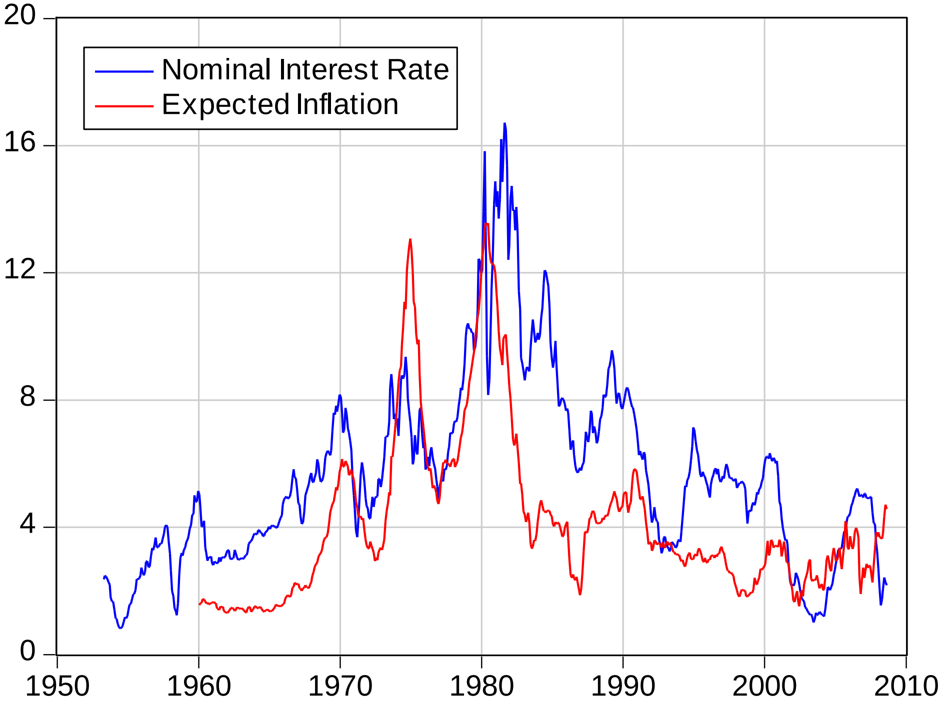
Currently (using 3.8% 8/11 $\bar{\pi}$), this gives

$$\pi^e = 3.6\% \text{ (not plotted)}$$



Changes in π^e account for much of the movement in nominal rates over past 50 years ...

Nominal Interest Rate vs Expected Inflation



One-year const. mat. nominal US Treasury yield, Adaptive Lag estimate of 1-year inflation expectations

... but inferred real rates have not been constant:

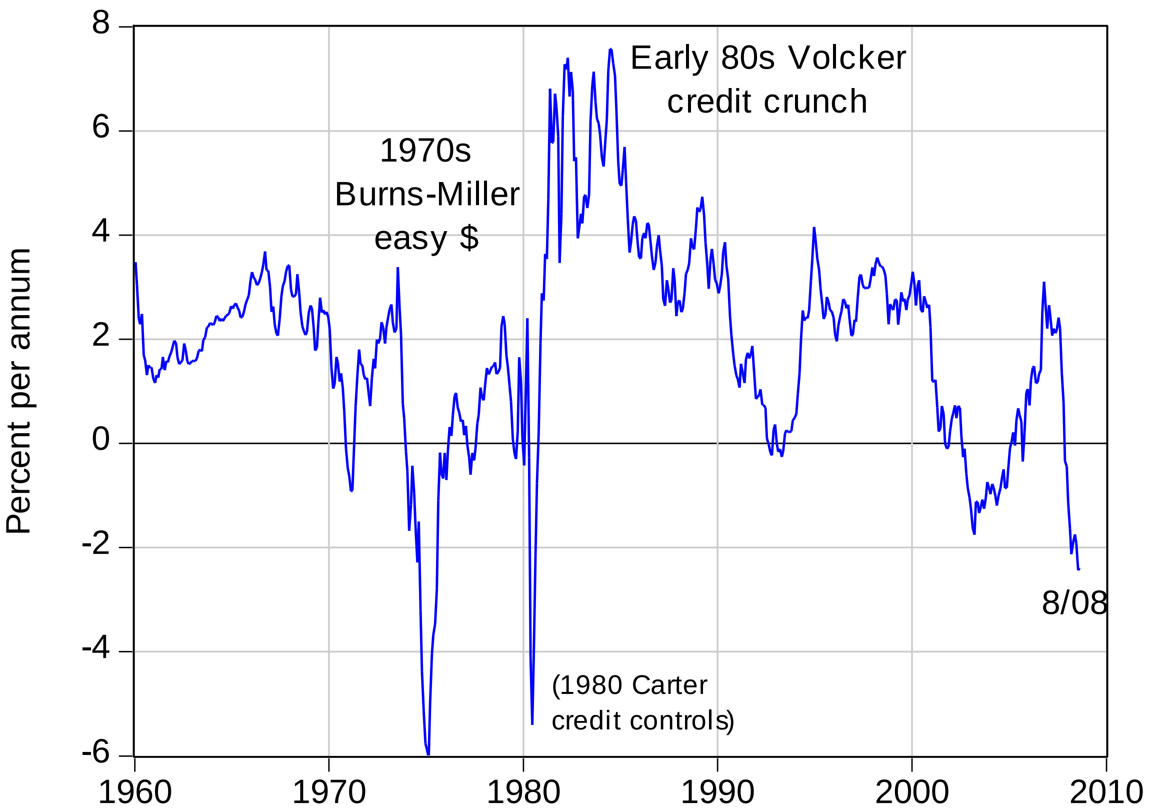
1-yr *r* typically about 2%,

but was 0-1% in 1970s,

5-6% in early 80s,

negative 2003-5, 2008-2011 (not plotted).

Inferred 1-Yr. US Real Interest Rate



1-year constant mat. nom. Treasury yield minus Adaptive Lag est. of 1-year expected inflation

Loanable Funds Model of *r* (M&I 19, pp. 1-3)

$(1+r)^m$ is price of present goods in terms of future goods

- $r \uparrow \Rightarrow$ present goods more costly (rel. to future goods)
- $r \downarrow \Rightarrow$ present goods less costly.

"Credit" = command over present goods

= what you get in exchange for your IOU when you borrow

= what you give up in exchange for someone else's IOU when you lend.

Non-monetary equilibrium *r* determined by

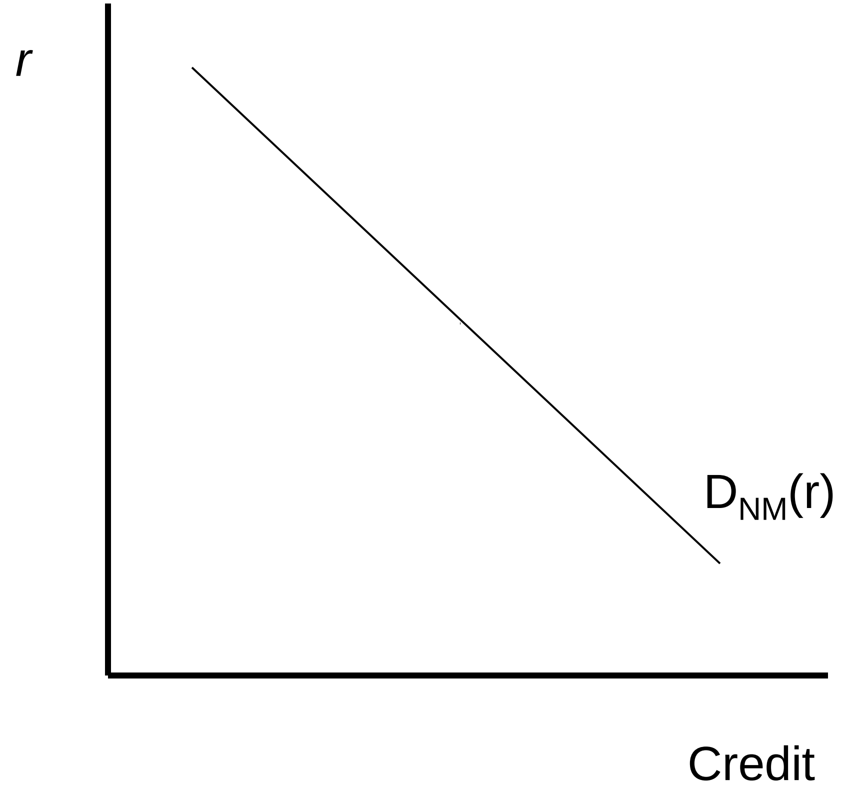
Demand & Supply of Credit.

Non-Monetary Demand for Credit by Borrowers, $D_{NM}(r)$

At low r, borrowers want more credit.

At high r, borrowers want less credit.

$\Rightarrow$ $D_{NM}(r)$ slopes down.

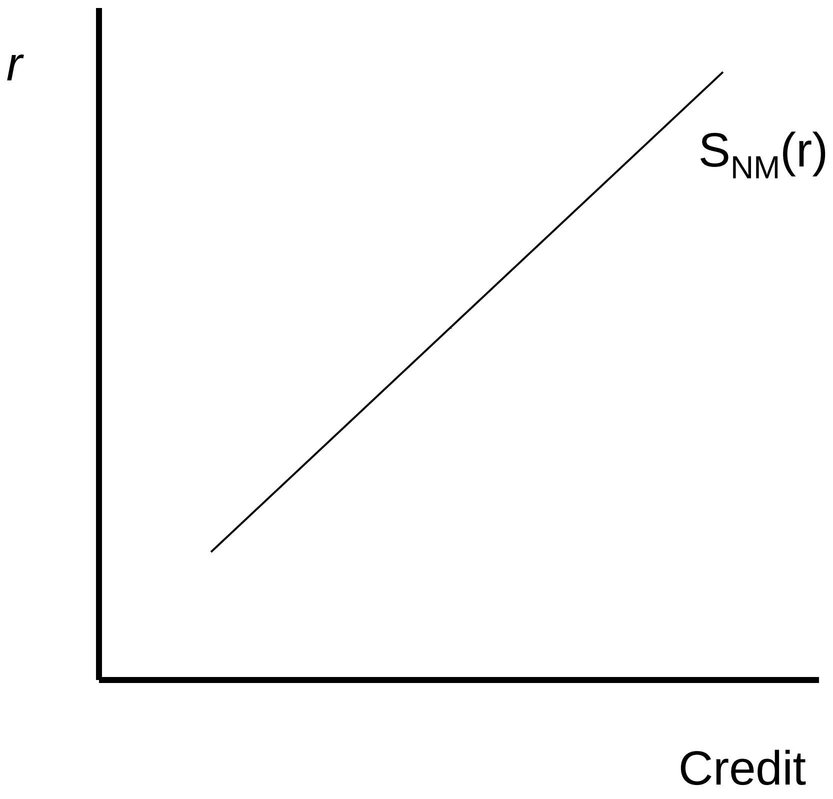


Non-Monetary Supply of Credit by Lenders

At high r, lenders willing to give up more credit

At low r, lenders give up less credit.

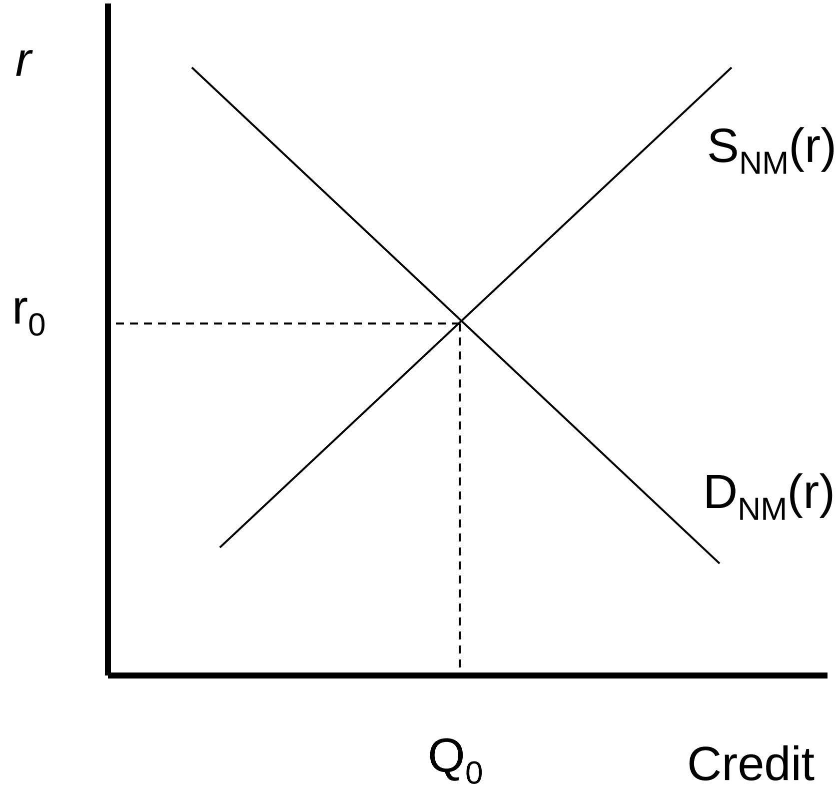
⇒ $S_{NM}(r)$ slopes up



Credit Market Equilibrium

(non-monetary economy)

r_0 = Non-Monetary Equilibrium real interest rate



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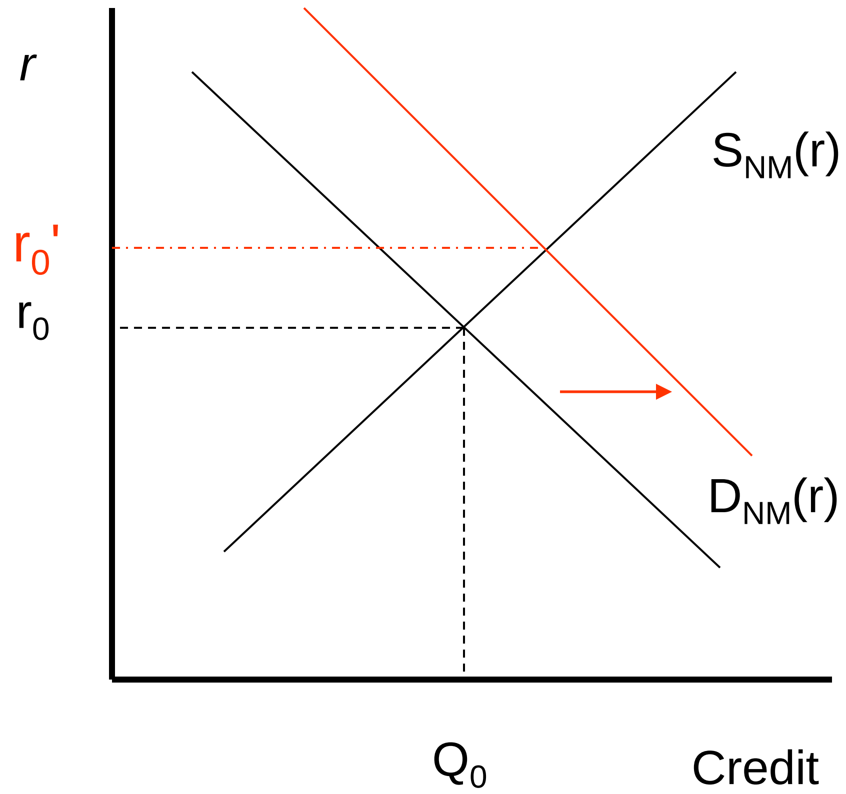
(non-monetary economy)



r_0 = Non-Monetary Equilibrium real interest rate

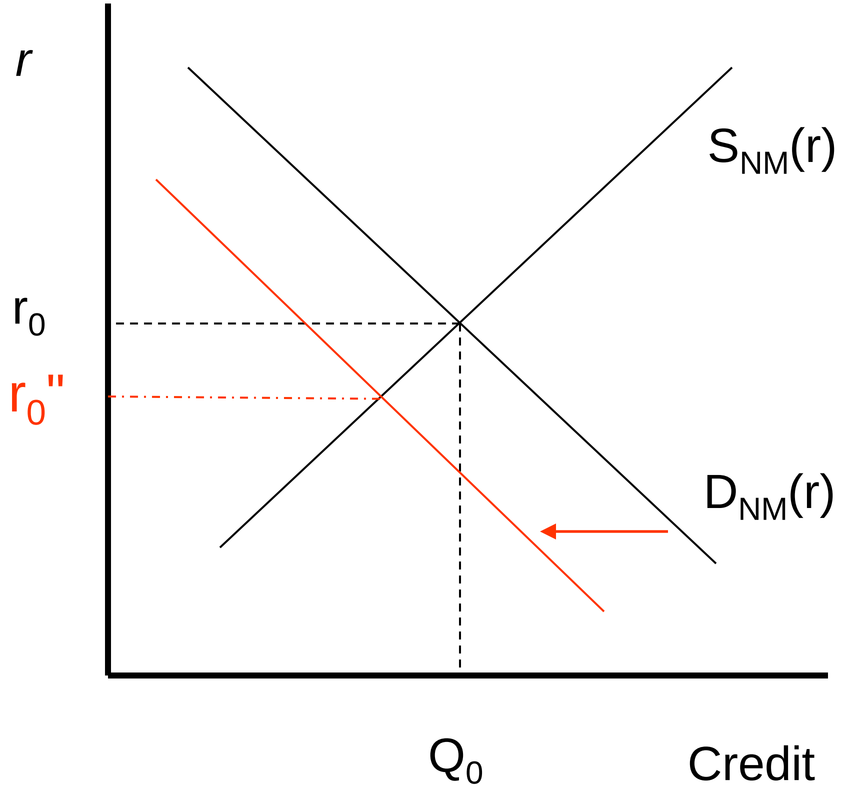
Increase in **D** for Credit (**right**ward shift in $\mathbf{D}_{NM}(r)$) **in**creases $\mathbf{r_0}$ to r_0'

Credit Market Equilibrium

(non-monetary economy)

r_0 = Non-Monetary Equilibrium real interest rate

Decrease in **D** for Credit (**left**ward shift in $\mathbf{D}_{NM}$(r)) **de**creases $\mathbf{r_0}$ to r_0''



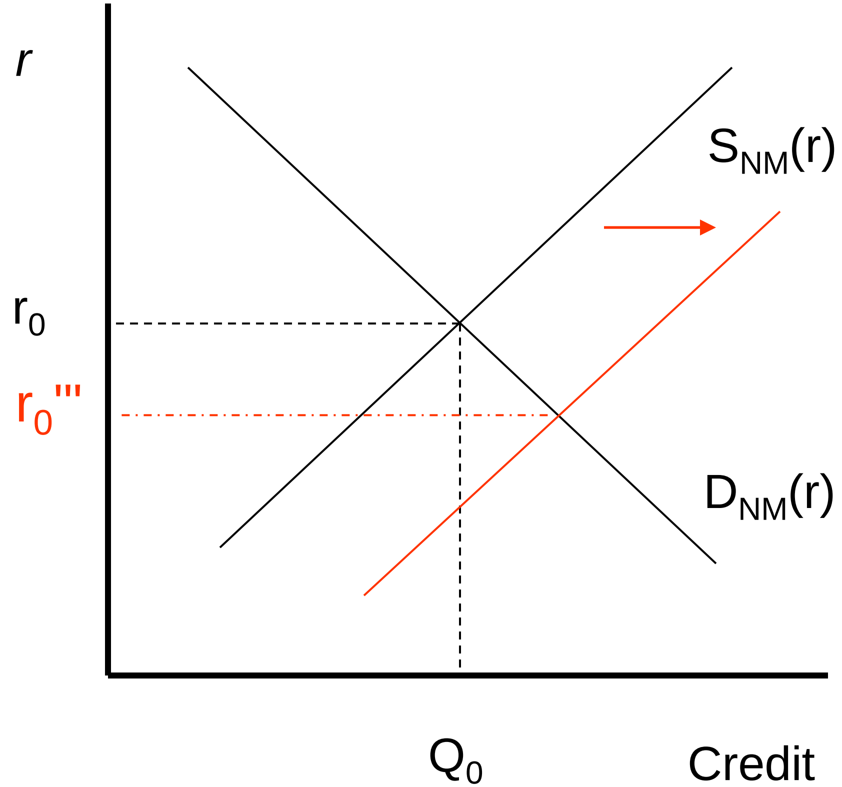
Credit Market Equilibrium

(non-monetary economy)

r_0 = Non-Monetary Equilibrium real interest rate

Increase in **S** of Credit (**right**ward shift in $\mathbf{S}_{NM}$(r)) **DE**creases $\mathbf{r_0}$ to r_0'''

(corrected 10/5/11)



Credit Market Equilibrium

(non-monetary economy)

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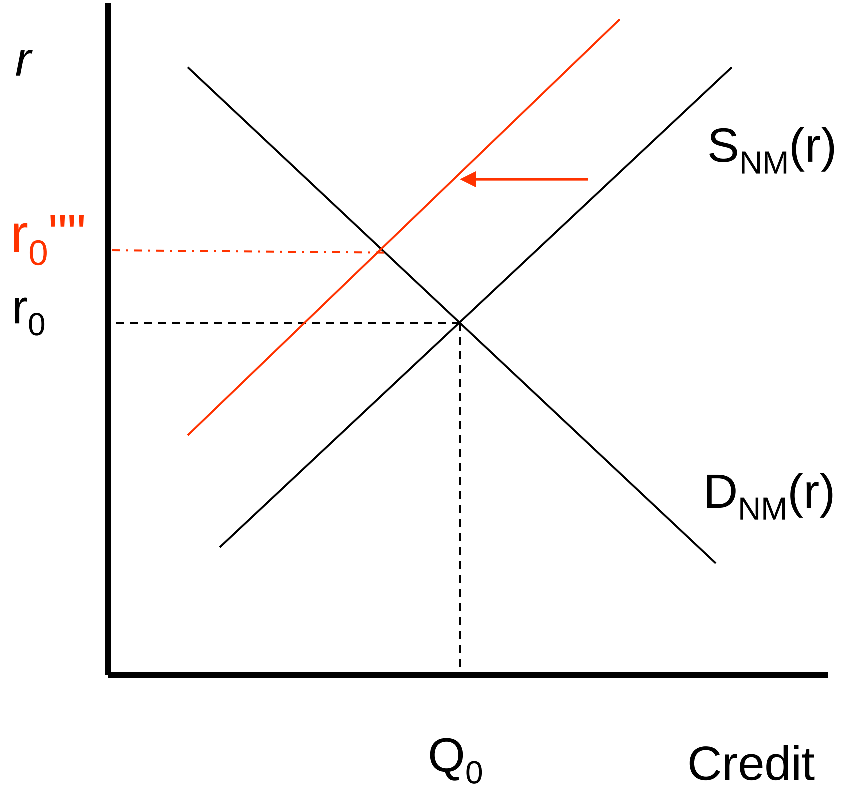
Decrease in **S** of Credit (**left**ward shift in $\mathbf{S}_{NM}$(r)) **in**creases $\mathbf{r_0}$ to r_0''''



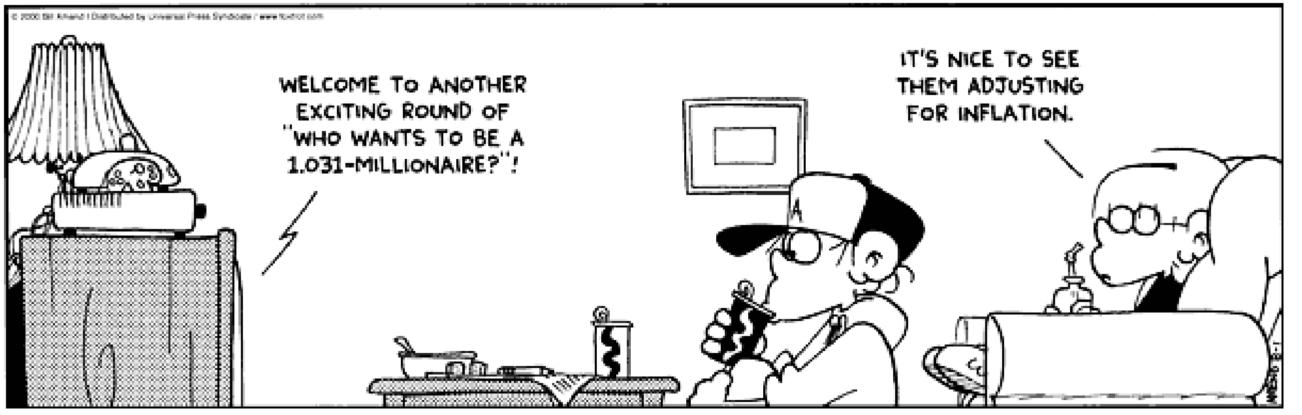
Debtor-Creditor redistribution (M&I 7.1)

- Nominal Debt paying $i = r + \pi^e$
  1. $\pi = \pi^e$
     $\Rightarrow$ $i - \pi = r$, No transfer.
  2. $\pi > \pi^e$ (as in 1970s)
     $\Rightarrow$ $i - \pi < r$. Creditors lose, Debtors gain.
  3. $\pi < \pi^e$ (1930's, 1980's)
     $\Rightarrow$ $i - \pi > r$. Debtors lose, Creditors gain.*

- if they can collect – Bankruptcies & foreclosures rise!

- Transfer may be eliminated with Price-Level Indexed Debt.
  - Payments indexed to CPI-U or other index
  - Real return independent of inflation
  - TIPS since 1997
- Nominal debt = safe indexed debt + lottery ticket on CPI.
  - Serves no function for risk-averse investors, borrowers
  - But still no private indexed securities to speak of!

FOXTROT by Bill Amend



- Next:
  - Velocity and the Quantity Equation
    - M&I 3, 4, 7.4