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A Reexamination of Traditional Hypotheses about the Term Structure: A Comment

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ABSTRACT

An example of a continuous time economy is given whose general equilibrium term structure of interest rates obeys the Expectations Hypothesis for continuously compounded interest rates and returns, contradicting the 1981 claim by Cox, Ingersoll, and Ross that such an economy is mathematically impossible. This example does not generate exploitable arbitrage opportunities of the type Cox, Ingersoll, and Ross claim must arise. The "Logarithmic Expectations Hypothesis," as we call it, it therefore an acceptable benchmark from which to measure term premia in continuous time term structure modeling.

THE EXPECTATIONS HYPOTHESIS (EH), broadly construed, postulates that expected returns on risk-free bonds of different maturities are equal or, equivalently, that appropriately calculated forward interest rates equal expected future spot interest rates.

In comparing the expected returns on two bonds of different maturities, however, the returns may be compounded in any of four natural ways: continuously, to the shorter bond's maturity, to the longer bond's maturity, or to the nearest available future date. It has long been understood that if the EH holds in terms of any one of these methods, and if interest rates are stochastic, Jensen's inequality will in general preclude it from holding in terms of any of the other three.

In practice, the convenience and simplicity of continuously compounded interest rates and returns have led many researchers (e.g., Roll (1970), McCulloch (1975), Fama (1984)) to use the first method, and therefore what we call the Logarithmic Expectations Hypothesis or Log EH as the benchmark from which to measure any term premia which may be present. In a widely cited article in this journal, however, Cox, Ingersoll, and Ross (henceforth Cox *et al.*) claim to have proven that the Log EH is "incompatible . . . with any continuous-time rational expectations equilibrium whatsoever" (1981, p.

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777).[1] Although Campbell (1986) has questioned the empirical significance of Cox *et al.*'s allegedly necessary deviations from the Log EH, the mathematical validity of their claim continues to be taken for granted (e.g., Longstaff (1990)).

The Cox *et al.* claim is surprising, since in discrete time models it is easily shown that the Log EH is consistent with a general equilibrium model in which risk-averse investors maximize expected utility subject to an intertemporal budget constraint. One would expect continuous time formulations to provide elegant and concise generalizations of discrete time models, and perhaps even to eliminate bothersome intraperiod simultaneity effects, but not to change the fundamental nature of the economy. In the real world, time in fact passes continuously. The Cox *et al.* claim therefore appears to be a valid statement about the empirical behavior of the real world that cannot be captured by discrete time approximations.

In this paper we demonstrate, contrary to Cox *et al.*, that in fact the Log EH is entirely consistent with a rational expectations equilibrium in continuous time. The Log EH therefore remains an internally consistent benchmark from which to analyze the term structure of interest rates, even in a continuous time framework.

Section I below establishes some preliminary notation. Section II develops a model of a continuous time economy, and derives the rational expectations general equilibrium term premia that obtain in it. Section III gives a specific case of this model in which the Log EH holds for all pairs of maturities, contrary to the assertion of Cox *et al.* Section IV examines the analysis of Cox *et al.* and demonstrates that the Log EH example of Section III does not lead to exploitable arbitrage opportunities. Section V concludes.

I. Notation and Elementary Relationships

Let $P(t, T)$ be the price at time t of a zero-coupon bond that pays one dollar (or unit of output) at future date $T \geq t$. Then

$$R(t,T) \equiv -\frac{1}{T-t} \log P(t,T) \tag{1}$$

is the continuously compounded yield to maturity at time t on this bond. If the partial derivative $D_T P(t,T)$ exists, we define the instantaneous forward interest rate at time t for future date T by

$$\rho(t,T) \equiv -D_T P(t,T)/P(t,T). \tag{2}$$

[1] Our Log EH is equivalent to what Cox *et al.* refer to in continuous time both as the "Yield-to-Maturity EH" and the "Unbiased EH." We find both these terms to be ambiguous, however, since every version of the EH states that *something* is unbiased, and since a "yield" can be compounded in a number of ways besides continuously. Furthermore, in their Appendix (p. 796), Cox *et al.* confusingly assign these same two terms conflicting definitions in discrete time.

It follows that

$$R(t,T) = \frac{1}{T-t}\int_t^T \rho(t,\xi)\,d\xi. \tag{3}$$

So long as $D_T P(t,t)$ exists, $R(t,T)$ and $\rho(t,T)$ have a common limit as $T \downarrow t$, defined to be the instantaneous interest rate,

$$r(t) \equiv R(t,t) = \rho(t,t). \tag{4}$$

Now consider two zero-coupon bonds with maturity dates T and $T' > T$. Equating the expected continuously compounded returns on the two bonds from time $t < T$ to either bond's maturity date, we have what we call the Logarithmic Expectations Hypothesis, or Log EH:

$$\log\frac{P(t,T')}{P(t,T)} = E_t \log P(T,T'), \tag{5}$$

where $E_t(\cdot) \equiv E(\cdot|\Omega_t)$, and the representative agent's time t information set Ω_t is assumed to include the history of bond prices and interest rates, as well as any state variables that may be relevant. Dividing (5) by $(T' - T)$ and taking the limit $T' \downarrow T$, the Log EH (5) implies

$$\rho(t,T) = E_t r(T), \tag{6}$$

whence (3) implies

$$R(t,T) = \frac{1}{T-t}\int_t^T E_t r(\xi)\,d\xi. \tag{7}$$

Equations (5), (6), and (7) are equivalent statements of the Log EH.

To avoid potential confusion, in the present paper we introduce the partial differential operator $\partial_{\cdot}$, by analogy to the partial differentiation operator $D_{\cdot}$, to indicate the increment to a function with respect to the argument represented in the function's definition by the dummy argument indicated in the subscript. Thus,

$$\partial_t \rho(t,T) \equiv \rho(t+dt,T) - \rho(t,T), \tag{8}$$

etc. This operator is distinct from the total differential operator d.[2]

II. Equilibrium Term Premia in a Continuous Time Rational Expectations Economy

Consider an economy with an infinite number of identical agents and a single, homogeneous consumption good. Markets are competitive, and there are no transactions costs. Let $X(t)$ and $C(t)$ be the representative agent's real endowment and consumption of the good, respectively. Output $X(t)$ is in

[2] Thus, $dr(t) = d\rho(t,t) = \partial_t \rho(t,t) + \partial_T \rho(t,t) = \partial_t \rho(t,t) + D_t \rho(t,t)dt$, etc. In this paper dt is always taken to be positive. By $\partial_t \rho(t,t)$, we mean $\lim_{T\downarrow t} \partial_t \rho(t,T)$, and similarly for $\partial_t w(t,t)$, introduced below.

fixed, but stochastic supply, so there are no production decisions.[3] Output is perishable, so that storage is infeasible. Each agent is assumed to maximize expected utility

$$E_t U(C(\cdot)) = \int_t^{\infty} E_t u(C(T)) e^{-\theta(T-t)}\, dT, \tag{9}$$

where the utility density function $u(\cdot)$ has the Constant Relative Risk Aversion (CRRA) form

$$u(C) = \begin{cases} \dfrac{1}{1-\eta} C^{1-\eta}, & 0 < \eta < 1 \text{ and } 1 < \eta < \infty, \\ \log C, & \eta = 1, \end{cases} \tag{10}$$

whence $u'(C) = C^{-\eta}$. The parameters θ and η are the pure rate of time preference and the relative rate of risk aversion, respectively.

Define

$$x(t) \equiv \log X(t), \tag{11}$$

$$w(t,T) \equiv E_t x(T). \tag{12}$$

By the Law of Iterated Expectations, $w(t, T)$ is a separate martingale with respect to t for each T, terminating in $w(T, T) \equiv x(T)$.

The stochastic partial differential

$$\begin{aligned} \partial_t w(t,T) &= w(t+dt,T) - w(t,T) \\ &= E_{t+dt} x(T) - E_t x(T) \end{aligned} \tag{13}$$

gives the revision to the expectation of $x(T)$ that is generated by information that arrives between times t and $t + dt$. Since $w(t, T)$ is a martingale for each T, these revisions are serially uncorrelated innovations. In this paper we make the further assumption that they are in fact normally distributed with instantaneous variance $g(m)$ that depends on maturity $m = T - t$, but not directly on time t itself. Thus,

$$g(m) = \frac{\text{var}}{dt}(\partial_t w(t,t+m)) \equiv \lim_{dt \downarrow 0} \frac{1}{dt} \text{var}(w(t+dt,t+m)|\Omega_t). \tag{14}$$

Future values of $x(T)$ may be built up from the current $w(t, T)$ and subsequent innovations to $w(\cdot, T)$ by integration:[4]

$$x(T) = w(t,T) + \int_{\tau=t}^{T} \partial_t w(\tau,T). \tag{15}$$

[3] Day (1986) compares alternative production scenarios.

[4] The subscript t on ∂ in (15) is, it will be recalled, by definition a reference to the "t" of (12), and not to that of (15).

It follows that

$$h(m) \equiv \operatorname{var}(x(T)|\Omega_t) = \int_t^T g(T-\tau)\,d\tau = \int_0^m g(\mu)\,d\mu. \tag{16}$$

So long as $g(m)$ is positive, the output variance $h(m)$ will be an increasing function of m. Output will therefore be more unpredictable the farther we look into the future, no matter whether the innovation volatility implied by $g(m)$ is an increasing, decreasing, or constant function of m.

Agents have the opportunity to exchange $C(t)$ for $C(T)$ at price $P(t, T)$, where $P(t, T)$ is now interpreted as the time t price of a price level-indexed bond that pays 1 unit of output at future date T. Since agents are identical, in market equilibrium trading in bonds will be zero and hence $C(t) = X(t)$. In equilibrium, the first order condition for expected utility maximization therefore implies

$$P(t,T) = \frac{E_t[u'(X(T))e^{-\theta(T-t)}]}{u'(X(t))}. \tag{17}$$

As of time t, $x(T) = \log X(T)$ is distributed $N(w(t,T),\ h(T-t))$. With our CRRA utility assumption, $\log\ u'(X(T))$ is therefore $N(-\eta w(t,T),\ \eta^2 h(T-t))$. By the familiar formula for the expectation of a log-normal variable, (17) implies

$$\log P(t,T) = \eta[x(t) - w(t,T)] - (T-t)\theta + \tfrac{1}{2}\,\eta^2 h(T-t). \tag{18}$$

Differentiating (18) with respect to T and employing (2) and (16), we obtain

$$\rho(t,T) = \eta D_T w(t,T) + \theta - \tfrac{1}{2}\,\eta^2 g(T-t). \tag{19}$$

Thus, the forward interest rate $\rho(t, T)$ is directly related to the expected rate of growth of output (and therefore of consumption) for future date T.

Assuming (as will be the case throughout this paper) that any deviation from the Log EH is invariant over time and is a function only of maturity $m = T - t$, we define the instantaneous (logarithmic) term premium as in McCulloch (1975) by

$$\pi(m) = \rho(t, t+m) - E_t r(t+m). \tag{20}$$

The Log EH is thus equivalent to $\pi(m) = 0$. Letting $T \downarrow t$ in (19), we have

$$r(t) = \eta D_T w(t,t) + \theta - \tfrac{1}{2}\,\eta^2 g(0), \tag{21}$$

and hence

$$E_t r(T) = \eta D_T w(t,T) + \theta - \tfrac{1}{2}\,\eta^2 g(0). \tag{22}$$

Subtracting (22) from (19), we obtain the equilibrium value of the term premium $\pi(m)$ as defined in (20):

$$\pi(m) = \tfrac{1}{2}\,\eta^2[g(0) - g(m)]. \tag{23}$$

A sufficient condition for the Log EH to hold for all pairs of maturities, infinitesimal or otherwise, in a world with frictionless markets, is therefore simply that the instantaneous variance $g(m)$ of the revision $\partial_t w(t, t + m)$ to the market's expectation of log $X(t + m)$ be a constant

$$g(m) = g_0 \tag{24}$$

for all horizons m. Although the revisions have equal volatility at all horizons under this log EH condition, output variance is still an increasing function, $h(m) = mg_0$, of m.

Risk-averse investors are concerned about both present and future consumption uncertainty, and are willing to pay a premium to avoid either. If the instantaneous variance of current consumption is greater ($g(0) > g(m)$), they will pay a greater premium to avoid it, and this will show up in the term structure as a lower expected return on short term bonds than long-term bonds ($\pi(m) > 0$). If the instantaneous variance of distant consumption is greater ($g(0) < g(m)$), they will instead pay a greater premium to avoid distant future uncertainty, and this will show up as a higher expected return on short-term bonds than on long-term bonds ($\pi(m) < 0$). And if these instantaneous variances are equal, there will be no observed term premium one way or the other. The (logarithmic) term premium is thus best conceived of as a risk *gradient* rather than a risk *premium*. Note, however, that the uncertainty gradient that is relevant is that of the *revisions* to expected consumption, $g(m)$, not that of consumption itself, $h(m)$.[5]

Because of our assumption that output volatility depends only on m and not on t, either directly or indirectly, through any state variables observed at time t, our term premium is also time invariant, as assumed in (20) above.

Although (24) is a sufficient condition for the Log EH, it is not necessary, since if investors are risk neutral ($\eta = 0$), by (23) the term premium will vanish and the Log EH will hold, regardless of $g(m)$. However, this case is trivial, since by (19) interest rates will be nonstochastic, and will all simply equal the pure rate of time preference θ. This is true because if $\eta = 0$, indifference curves are flat, so that intertemporal prices and interest rates will be entirely demand determined, barring corner solutions (which we have precluded with our assumption that output is log normal).

Equation (19) implies that forward rate increments may be represented as

$$\partial_t \rho(t, T) = {}^1\!/_2\, \eta^2 g'(T - t)\, dt + \eta \partial_t D_T w(t, T), \tag{25}$$

so that the forward rate $\rho(t, T)$ is stochastic if and only if the expected rate of growth of future output $D_T w(t, T)$ is stochastic and $\eta \neq 0$.

[5] This point was first made clear, in a three-period model, by Woodward (1983).

III. A Continuous Time Log EH Economy

Processes $w(t, T)$ that satisfy the Log EH condition (24) and lead to nontrivial, stochastic interest rates can easily be generated from two uncorrelated Wiener processes $z_1(t)$ and $z_2(t)$ with zero means and unit volatilities by specifying

$$\partial_t w(t,T) = \phi_1(m)\,dz_1(t) + \phi_2(m)\,dz_2(t), \tag{26}$$

where $\phi_1(\cdot)$ and $\phi_2(\cdot)$ are nonnegative differentiable functions of $m = T - t$, ϕ_1 is monotonically increasing, ϕ_2 is monotonically decreasing, and

$$\phi_1(m)^2 + \phi_2(m)^2 = g_0, \tag{27}$$

so that $g(m)$ indeed is a constant, g_0, for all m.[6] A specific example of such functions is

$$\phi_1(m) = g_0^{1/2}(1 - e^{-m}), \tag{28a}$$

$$\phi_2(m) = g_0^{1/2}(2e^{-m} - e^{-2m})^{1/2}. \tag{28b}$$

Combining (24), (25), and (26) implies that under (27),

$$\partial_t \rho(t,T) = \eta[\phi_1'(m)\,dz_1(t) + \phi_2'(m)\,dz_2(t)]. \tag{29}$$

Thus under specification (28a–b), forward rates are stochastic at all maturities, but with an instantaneous variance $\eta^2[\phi_1'^2(m) + \phi_2'^2(m)]$ that declines toward 0 as $m \to \infty$, despite the constant volatility of output innovations. $R(t, T)$ and $r(t)$ are likewise all stochastic.

We have therefore shown by counterexample that the conclusion of Cox *et al.* that the Log EH is "incompatible ... with any continuous-time rational expectations equilibrium whatsoever" is erroneous.

IV. The Cox, Ingersoll, and Ross Analysis

In this section, we relate the model of Sections II and III to the apparatus of Cox *et al.* (1981). We show that our counter-example does not violate their no-arbitrage condition (their (25)), and show why the "proof" they present of the impossibility of the Log EH is invalid.

The instantaneous logarithmic return $\partial_t \log P(t, T)$[7] on a bond maturing at date T may be found by taking the partial differential of our general

[6] The even simpler stochastic process $dx(t) = g_0^{1/2}\,dz(t)$ would formally yield the log EH, but only in a trivial sense, since interest rates themselves would then be nonstochastic with our homothetic utility function.

[7] By $\partial_{.} \log P(t, T)$, etc., we mean $\partial_{.}\phi(t, T)$, where $\phi(t, T) \equiv \log P(t, T)$, etc.

bond-pricing formula (18) with respect to t, while keeping in mind that $dx(t) = \partial_t w(t, t) + D_T w(t, t)\, dt$:

$$\partial_t \log P(t,T) = \left[\eta D_T w(t,t) + \theta - \tfrac{1}{2}\, \eta^2 g(T-t)\right] dt + \eta[\partial_t w(t,t) - \partial_t w(t,T)]. \tag{30}$$

Substituting (21) into the above yields

$$\partial_t \log P(t,T) = \left[r(t) + \tfrac{1}{2}\, \eta^2\{g(0) - g(m)\}\right] dt + \eta[\partial_t w(t,t) - \partial_t w(t,T)]. \tag{31}$$

Applying Itô's Lemma,

$$\frac{\partial_t P(t,T)}{P(t,T)} = \left(r(t) + \frac{1}{2}\,\eta^2\left\{g(0) - g(m) + \frac{\text{var}}{dt}\,[\partial_t w(t,t) - \partial_t w(t,T)]\right\}\right) dt + \eta[\partial_t w(t,t) - \partial_t w(t,T)]. \tag{32}$$

Under our unrestricted two-process model (26), (32) becomes

$$\begin{aligned}\frac{\partial_t P(t,T)}{P(t,T)} &= \left(r(t) + \eta^2\left[\phi_1(0)^2 + \phi_2(0)^2 - \phi_1(0)\phi_1(m) - \phi_2(0)\phi_2(m)\right]\right) dt \\ &\quad + \eta[\phi_1(0) - \phi_1(m)]\, dz_1 + \eta[\phi_2(0) - \phi_2(m)]\, dz_2 \\ &= \left[r(t) + \hat{\boldsymbol{\lambda}}'\boldsymbol{\delta}(m)\right] dt + \boldsymbol{\delta}'\, dz,\end{aligned} \tag{33}$$

where

$$\delta_k(m) = \eta(\phi_k(0) - \phi_k(m)), \qquad \hat{\lambda}_k = \eta\phi_k(0). \tag{34}$$

Equation (33) is (2) and (8) of Cox *et al.*, in the special case of our unrestricted two-process model.

As Cox *et al.* (p. 788) correctly note (and as can be seen directly from (31) using (23)), (7) implies that under the Log EH, the expected continuously compounded instantaneous return on a bond with any maturity date T is given by

$$E_t \partial_t \log P(t,T) = r(t)\, dt. \tag{35}$$

Cox *et al.* then correctly demonstrate that implication (35) of the Log EH, together with Itô's Lemma, implies that in (33),

$$\hat{\boldsymbol{\lambda}}'\boldsymbol{\delta}(m) = \tfrac{1}{2}\, \boldsymbol{\delta}'(m)\boldsymbol{\delta}(m). \tag{36}$$

This is equation (25) of Cox *et al.* in the Log EH case, which corresponds to their $a = \frac{1}{2}$. They then go on to argue that if (36) were true for all states of the world at all points in time t, there would have to be a functional relationship among the driving processes $z_k(t)$, an impossibility. They therefore conclude that (36) cannot be universally true, and hence that disequilibrium arbitrage opportunities must exist under the Log EH.

In fact, under the Log EH condition (27),

$$\frac{1}{2}\boldsymbol{\delta}'(m)\boldsymbol{\delta}(m) = \frac{1}{2}\eta^2\big[\phi_1(0)^2 + \phi_1(m)^2 - 2\phi_1(0)\phi_1(m) + \phi_2(0)^2 + \phi_2(m)^2 - 2\phi_2(0)\phi_2(m)\big]$$
$$= \eta^2[g_0 - \phi_1(0)\phi_1(m) - \phi_2(0)\phi_2(m)]$$
$$= \hat{\boldsymbol{\lambda}}'\boldsymbol{\delta}(m). \tag{37}$$

Equation (36) is therefore automatically valid for all m under the Log EH condition (27). Thus, the Log EH does not lead to arbitrage opportunities in any counterexamples of the type (26), (27).

Although the Cox *et al.* proof of the impossibility of the Log EH is defective, it may well nevertheless be true, in their model, that the Log EH *is* impossible, because of a critical assumption in their model, in addition to rational expectations, general equilibrium, and continuous time, which we have not incorporated here. Whereas in our model, the state of the economy at time t is described by a potentially *infinite* dimensional state function $w(t, T)$, Cox *et al* make the seemingly innocuous assumption that it can always be described by a *finite* set of N state variables (their $Y_n(t)$). Although Cox *et al.* do not mention it, there may well in fact be no way the Log EH can hold nontrivially in continuous time *for more than N maturities* if all bond prices are a function of an N-dimensional state vector.[8] Although our example required only two sources of uncertainty, dz_1 and dz_2, under (26) $w(t, T)$ is a different compound of the infinite-dimensional history of these two shocks for each value of T, and, because of the nonlinearity of (27), these values cannot be summarized exactly by any finite subset.

In order to make the statement of Cox *et al.* quoted above valid, therefore, "whatsoever" must at a minimum be replaced with the phrase, "in which bond prices of all maturities may be expressed as a function of a finite number of state variables." With this modification, their conclusion loses much of its apparent generality. In fact, given that the infinite-dimensional continuous time future is potentially infinitely complex, their assumption is highly restrictive and unrealistic.

The technical error in their proof appears to lie in their tacit assumption that the diffusion coefficients $\partial_k(m)$ (their $\partial_k(Y, t, T)$) must be nontrivial functions of all the state variables (their $Y_n(t)$), and time t, as well as maturity. If they were, (36) (and (27)) could not be valid for all values of Y, and the Log EH could not be generally valid. However, while the diffusion coefficients *might* depend nontrivially on all the state variables, there is nothing in the nature of a continuous time rational expectations equilibrium

[8] This statement is true for $N = 1$, but the author has been unable to prove it in the general case.

that requires this to be so, nor is this required for all interest rates to be nontrivially stochastic.[9]

Note that the Log EH does not depend on risk neutrality ($\eta = 0$), or even on logarithmic utility ($\eta = 1$), but merely on the constancy across m of the volatility of the innovations to expected future log output. The Log EH does require at least two sources of uncertainty to be nontrivially valid.[10]

V. Conclusion

Cox, Ingersoll, and Ross claim to have demonstrated that the logarithmic version of the Expectations Hypothesis is "not sustainable in a continuous-time rational expectations equilibrium." (1981, p. 778) Since in the real world time passes continuously, this statement has been widely accepted as a mathematically inescapable fact, with unavoidable empirical implications for real world financial markets.

We have demonstrated that in fact the Log EH is a mathematically permissible equilibrium characterization of the term structure in continuous time. There are two problems with the Cox, Ingersoll, and Ross analysis: The first, more important problem is that the validity, if any, of their claim rests only on a seemingly innocuous, yet in fact unrealistic assumption that they have made, namely that all information about the infinite-dimensional, continuous time future can be summarized precisely in a finite number of state variables. The second, merely technical problem is that even under this special assumption, their proof is defective.

We do not argue that the constant volatility condition required for the Log EH is particularly compelling on theoretical grounds, nor that the Log EH is empirically valid.[11] We merely claim that it is an internally consistent and mathematically feasible benchmark, from which we may measure any empirical term premium that may in fact be present.

[9] The $\phi_k(m)$ and therefore the $\overset{\delta}{\cancel{\phi}}_k(m)$ could be made to depend proportionately on any *single* function of the state variables, such as $D_T w(t, t)$, or equivalently $r(t)$, without disturbing (27) or (36), except to the extent that g_0 would become a different constant for each point in time. It would appear, however, that they could not be made to depend nontrivially on more than one such function of the state variables.

[10] Cox *et al.* (1981, p. 778) illustrate their defective proof of the internal inconsistency of the Log EH using a model with only one source of uncertainty. Naturally, the Log EH fails to hold unless the term structure is nonstochastic.

[11] In fact, Roll (1970), McCulloch (1975), Fama (1984), and others have long since shown that it is *not* empirically valid, particularly in the first six months of maturities.

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