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INTEREST-RISK SENSITIVE DEPOSIT INSURANCE PREMIA

Stable ACH Estimates

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The parameters of interest rate uncertainty are estimated by maximum likelihood for the period 1952–1982, and used to evaluate bank or thrift deposit insurance as a function of duration mismatching, capital/asset ratio, and the recent history of interest rate forecasting errors. Homoskedasticity is overwhelmingly rejected in favor of adaptive conditional heteroskedasticity (ACH). Even after removing this heteroskedasticity, normality gives much lower likelihood than Paretian stable distributions with characteristic exponent in the range 1.614 to 1.714. The conditional deposit insurance values fluctuate by factors in excess of 300 for some duration gaps over the past three decades.

1. Introduction

The Garn–St Germain Depository Institutions Act of 1982 required the Federal Deposit Insurance Corporation (FDIC) and the Federal Savings and Loan Insurance Corporation (FSLIC) to study 'the feasibility of basing deposit insurance premiums on the risk posed by . . . the insured institution . . . rather than the present flat rate system'. The Federal Home Loan Bank Board's report on behalf of the FSLIC recommends risk-sensitive deposit insurance premia but does not provide any hard numbers for setting these premia [FHLBB (1983, pp. 10, 171–210)]. The FDIC (1983, ch. 2) despairs altogether at the prospect of premia which accurately reflect exposure to risks, even interest rate risk which they grant should be relatively easy to measure. Such a system would, it is claimed, entail 'more advanced risk quantification techniques than are currently imaginable'.

It is well established that contingent claims such as deposit insurance can be viewed as put options that entitle the banking firm (or thrift institution) to 'sell' the firm's assets to the insuring agency for a pre-arranged price, determined by the face value of the insured liabilities. The value of the insurance is equal to the value of the corresponding put option [Black and Scholes (1973), Merton (1977), McCulloch (1981a), Pennacchi (1983), Marcus and Shaked (1984)].

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In this paper we exploit this equivalence to estimate the value of insurance that protects deposits against interest rate risk. We evaluate this as a function of the degree of maturity mismatching and of the institution's ratio of capital to assets.

We find that the volatility of interest rates, which greatly affects the value of the insurance, has varied a great deal over the past three decades. Furthermore, large movements in interest rates in either direction are more likely to occur when there have been similarly large movements in recent months, than following periods of interest rate quietude. An efficient insurance pricing scheme must take this predictability of volatility into account.

We show how this volatility has varied over time, and provide tables giving the value of insurance for the most recent available month, as well for months with unusually high, unusually low, and typical volatility. We compare these to 'unconditional' values that do not take into account the variation in the volatility.

As in our earlier paper [McCulloch (1981a)], we use an option pricing model based on the symmetric Paretian stable probability distributions. However, in the present paper we introduce the following major improvements: (1) Stable distribution parameters are estimated using new maximum likelihood software [McCulloch (1979)] rather than the early Fama–Roll quantile method (1971). (2) This maximum likelihood method provides us with estimates of the precision of the estimated parameter values, which in turn enable us to construct confidence intervals for our insurance value estimates, rather than just point estimates as formerly. (3) Variable volatility is captured by introducing an adaptive conditional heteroskedastic (ACH) procedure akin to the autoregressive conditional heteroskedastic (ARCH) procedure recently developed by Engle (1982,1983). (4) The data set is updated to December 1982 and furthermore, the ACH procedure enables us to utilize interest rate experience clear back to the Accord, rather than having to use an arbitrarily selected recent period to obtain a relatively constant volatility. This increases our sample size from 120 to 369.

Section 2 describes the data set we employ, and how we use it to infer net worth variability. Section 3 reviews the Paretian stable option pricing formula and its relation to deposit insurance. Section 4 shows that unconditional parameter values for unexpected returns are misspecified. Section 5 introduces the ACH procedure, and demonstrates that even after adjusting for heteroskedasticity, the disturbances are still substantially non-Gaussian. Fair insurance values for illustrative months are calculated, and weights from which the insurance values can be updated into the future are provided. Section 6 calculates asymptotic standard errors for the logarithms of these estimates, from which confidence intervals may be constructed. Section 7 discusses the practical application of these estimates.

2. Data

As in our earlier paper, we consider the value of insurance for a 'traditional' bank or thrift institution that has very short-term liabilities and longer-term assets. The volatility of such a firm's net worth (based on market value of assets) will depend on the average Macaulay duration of those assets.

We have available the continuous term structure of interest rates for the end of every month from December 1946 to December 1982. These are computed from bid/asked mean quotations on U.S. Government securities by means of a modification of the tax-adjusted cubic spline method described in McCulloch (1975a).[1]

This data enables us to model the bank's short-term liabilities as one-month discount obligations, and its longer-term assets as 'zero-coupon' securities of any desired maturity. The duration of a zero-coupon security exactly equals its maturity, so it serves as a proxy for more complicated assets structures of equal average duration. The duration gap of the banks we are modelling is thus one month less than the asset duration.

Let $\delta(t,m)$ be the market value at time t of a zero-coupon security paying one dollar in m years, i.e., at time $t+m$, and let $\Delta t = 1/12$ year (one month). One dollar invested safely for one month in one-month securities will be worth $1/\delta(t,\Delta t)$ dollars at the end of the month. One dollar invested for one month at risk in securities of maturity m will be worth $\delta(t+\Delta t, m-\Delta t)/\delta(t,m)$ at the end of the month. Let R_i be the excess return during month i (i.e., during the month which begins at time t_{i-1} and which ends at time t_i) on the portfolio of a bank with assets of (initial) duration m, and with liabilities of (initial) duration Δt. This may be calculated as

$$R_i = \log\left[\frac{\delta(t_i, m-\Delta t)}{\delta(t_{i-1}, m)} \Big/ \frac{1}{\delta(t_{i-1}, \Delta t)}\right]. \tag{1}$$

Were it not for the existence of a term premium, this excess return would have mean zero. However, it appears that there is, for reasons which do not concern us here, a systematically positive term premium, i.e., a 'liquidity' premium [see McCulloch (1975b)]. Let p be the liquidity premium (for maturity m) in R_i. Then

$$R_i = p + \varepsilon_i, \tag{2}$$

where ε_i is a random forecasting error with mean zero.

[1]For details concerning estate bonds etc., see McCulloch (1981a), 229–230.

3. Paretian stable distributions

In order to evaluate a put option, we must have a parametric representation of the probability distribution involved. Fortunately, the generalized central limit theorem gives us a criterion for singling out one manageable family of distributions from the set of all conceivable distributions.

Each month's forecasting error is the sum of a vast number of contributions that accumulate day by day, hour by hour, and even minute by minute. These forecasting errors must be serially uncorrelated. Furthermore, the interest rate surprise that occurs in each moment is the outcome of the more or less independent decisions of millions of borrowers and lenders, each acting on thousands of bits of information. According to the generalized law of large numbers, if the sum of a large number of independently and identically distributed random variables has a limiting shaped distribution, the limiting distribution must be a member of the *stable* class. These are therefore the natural choice for our purpose. Roll (1970) finds that they adequately model interest rate uncertainty.

The shape of a stable distribution is completely determined by its *characteristic exponent*, α, and *skewness parameter*, β. The characteristic exponent must lie in the interval $(0, 2]$, and determines the rate at which the tails of the distribution taper off. The normal or Gaussian distribution is a special case which occurs when α is at its maximum value of 2.0. When α is less than 2.0, the distribution has 'Paretian' tails that behave asymptotically like a Pareto distribution and which are longer than the tails of a normal distribution. The probability of an event well out in one tail of the distribution — such as a bank failure — depends crucially on the value of α. The Black–Scholes option pricing formula used by Merton and others to evaluate deposit insurance arbitrarily assumes a normal distribution and therefore greatly understates the value of deposit insurance if in fact the distribution is non-Gaussian (i.e., Paretian) stable. Whether or not this is the case is a purely empirical issue.

The skewness parameter β determines the relative size of the two tails of the distribution. As in our earlier study, we assume here that the distribution is symmetrical, which corresponds to the special case $\beta = 0$.

In addition to the two shape parameters, stable distributions have a *scale parameter*, c, and a *location parameter*, which in our case is the term premium p.[2] The location parameter is the mean of the distribution (provided α is greater than 1.0), and simply shifts the distribution to the left or right. The scale parameter expands or contracts the distribution in proportion to c about the location parameter.[3] The scale parameter, like the

[2]The location parameter is ordinarily represented by δ, not to be confused with the discount function defined in the previous section.

[3]The standard deviation cannot in general be used as an index of scale, because if α is less than 2.0, the second moment of the distribution is infinite. If α does equal 2.0, c^2 is one-half the variance.

characteristic exponent, is critical for the value of bank insurance, since higher values of c increase the probability of failure (though not, as it turns out, the expected loss in the event of a failure).

According to Bierwag and Kaufman (1983, p. 4), 'government deposit insurance agencies do not confront a potential loss beyond their control' because they could simply close failing institutions as their net worth reaches zero. This would be true if net worth were a continuous function of time, as it is under the Gaussian diffusion process assumed by the Black–Scholes option pricing model. If, however, net worth undergoes discontinuities such as those which occur under a Paretian stable process in continuous time, the institution's net worth could pass from positive to negative in one instant, giving the insuring agency no time to act. If α is less than 2.0, therefore, the FDIC and FSLIC *do* confront potential losses, even if they continuously examine institutions and immediately close them as they become insolvent.

We assume that the unforeseen monthly return ε_i is symmetric stable with exponent α, scale c, and mean 0. Let q be the firm's ratio of capital to assets (both based on market values). Then it can be shown [see McCulloch (1978a, 1978b, 1981a)] that the annual rate of occurrence of discontinuities in the value of the bank's assets large enough to cause insolvency is given by

$$\lambda = \frac{12}{\pi}\Gamma(\alpha)\sin\left(\frac{\pi\alpha}{2}\right)\left(\frac{c}{-\log r}\right)^{\alpha}, \tag{3}$$

where $r = 1-q$ is the bank or thrift's ratio of liabilities to assets. When α equals 2.0, $\sin(\pi\alpha/2)$ and therefore λ both equal zero and there are no discontinuities in net worth.

Furthermore, the value per year of insurance with continuous surveillance, computed, as is conventional, as a fraction of liabilities, is given by

$$I = \lambda H(r,\alpha)/r, \tag{4}$$

where

$$H(r,\alpha) = r - \alpha\left(\log\frac{1}{r}\right)^{\alpha} \int_{-\log r}^{\infty} e^{-x} x^{-\alpha-1}\,dx. \tag{5}$$

This function $H(r,\alpha)$ represents the expected cost of failure should a failure occur, and has been tabulated in McCulloch (1978b). It increases with $q = 1-r$, roughly in proportion to q for small values of q, and declines with α.

With periodic rather than continuous surveillance, the value of deposit insurance would be somewhat larger because of the possible accumulation of several disturbances, none of which is by itself sufficient to wipe out the last observed net worth. However, this is not an important consideration with

Paretian stable distributions unless we are concerned with the moderately high probability events represented by the shoulders of the distribution. As long as surveillance is sufficiently frequent that the probability of default is very low, we will be concerned only with the Paretian tail of the distribution, which represents the occurrence of single impulses [cf. McCulloch (1978a, p. 604 n.3)], and the continuous monitoring model will be a close approximation to the actual value of insurance. Furthermore, the limiting value of insurance as surveillance becomes continuous is far more tractable analytically than is the discrete surveillance case.

In the present paper we therefore assume continuous monitoring of the institution's net worth, as well as continuous adjustment of the 'pay-as-you-go' insurance premium to reflect non-fatal changes in net worth and changes in measured interest rate volatility. The derived values will closely approximate those with frequent but discrete monitoring for a bank that pays out continuous dividends equal to the expected return on its position along with a continuous insurance premium based on its most recently evaluated risk. Note that in the case of interest rate risk it is not necessary to actually examine the individual institution in order to evaluate its deposit insurance, so long as its duration gap does not change without warning. The current position of such a bank or thrift can be inferred simply by monitoring the national interest rate market, which can be done virtually continuously.

4. Unconditional estimates

Eq. (2) was estimated for six selected asset durations, using 381 monthly observations from April 1951 to December 1982,[4] under the assumption that the standard scale had a constant, unconditional value c. The estimates of α were all quite low, lying in the range 1.218 to 1.314.

Fig. 1 shows the unanticipated monthly returns for one-year maturity assets, with no weighting. The forecasting errors that are largest in absolute value clearly come in clusters occurring in 1958–1959, 1966–1975, and in particular from 1979 to the present. Similar patterns (not illustrated) appear for the other maturities, which range from three months to ten years. At any point in time, the recent history of surprises obviously has considerable value for predicting the absolute value (though not necessarily the algebraic value) of future surprises. The forecasting errors are therefore not really independent as is tacitly assumed when we apply the stable maximum likelihood program directly to the ε_i's. An unconditional model is therefore misspecified.

Furthermore, insurance values derived from such parameter estimates would not be efficient for risk-sensitive deposit insurance pricing. Most of the

[4]Prior to the Accord of March 4, 1951, the Federal Reserve System rigidly pegged interest rates. The pre-Accord period is therefore not representative of the interest rate volatility of the post-Accord period, so we use only that portion of our data set from the end of March 1951 on.

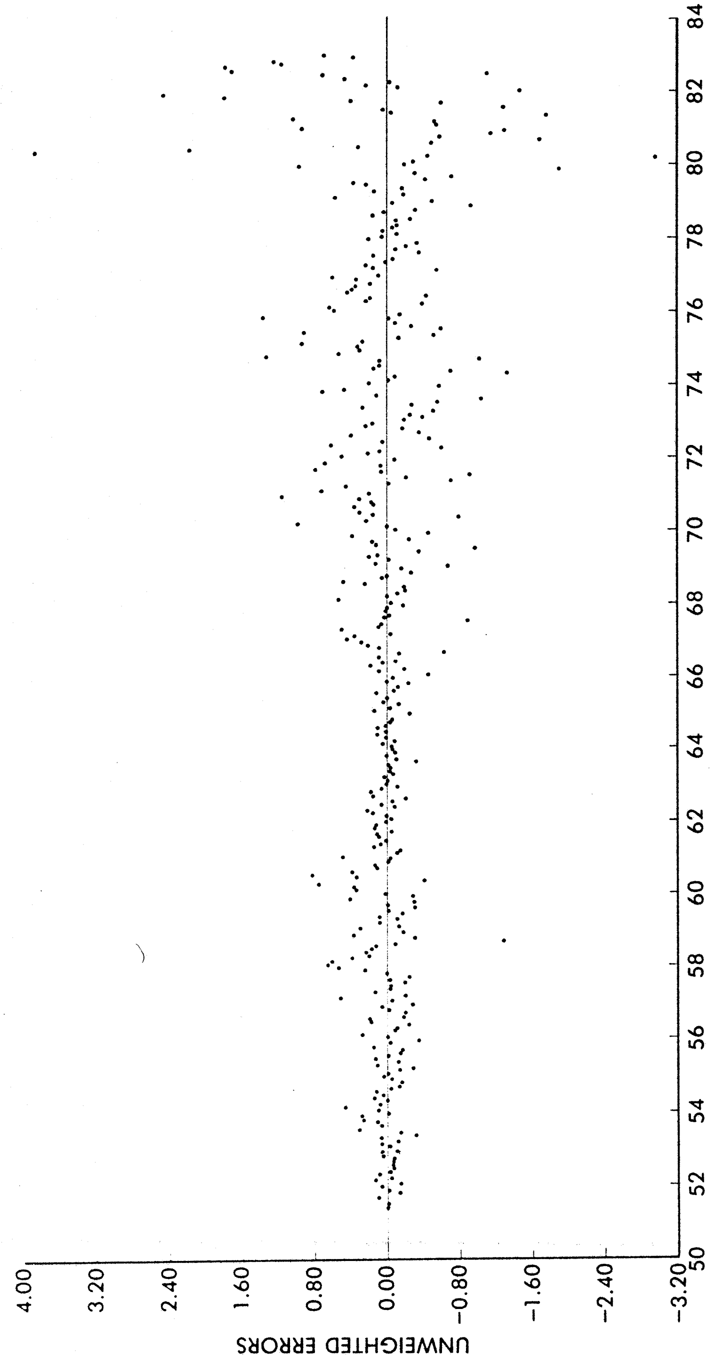

Fig. 1. Unweighted forecasting errors, asset duration 1 yr.

time a much lower value will be warranted if we take into account the recent history of interest rate volatility, and find that we are not in an exceptionally noisy period, but occasionally a much higher value will be required. Efficient deposit insurance pricing must take this heteroskedasticity into account, by making the scale of each month's forecasting error conditional on the recent history of the ε_i's.

5. ACH estimates

We now assume that the standard scale of ε_i, rather than being a constant c as in the previous section, is instead of the form

$$c_i = c_0 w_i, \tag{6}$$

where the weight w_i is a function of the magnitude of the forecasting errors up to, but not including, ε_i. If we had an infinite history of forecasting errors, we would like the contribution of each error to decline monotonically toward, but never quite reaching, zero as it fades into the past. This suggests the following formula:

$$w_i = \theta \sum_{j=1}^{\infty} (1-\theta)^{j-1} |\varepsilon_{i-j}|, \qquad 0 < \theta < 1. \tag{7}$$

This formula is equivalent to the adaptive adjustment scheme

$$w_i = \theta |\varepsilon_{i-1}| + (1-\theta) w_{i-1}, \tag{8}$$

which may, after a suitable initialization, be used with a finite data set.[5]

The above adaptive conditional heteroskedastic (ACH) system may be estimated with the existing symmetric stable maximum likelihood linear regression software, simply by dividing each equation

$$R_i = p + \varepsilon_i \tag{9}$$

[5]We base (7) on the magnitudes of the preceding errors rather than on their squares to avoid exaggerating the effect of outliers. In practice, Engle (1982, p. 1002; 1983, section 4) uses a linearly declining weight structure (for the squared errors) rather than a general autoregressive structure, so our adaptive model is not any more restrictive. Eq. (8) could be generalized to allow a reassuringly stationary process with an ergodic long-run distribution. However, examination of fig. 2 suggests that the point estimates of the parameters would indicate an explosive process instead. So (1982) is the first application of conditional heteroskedasticity in a stable context. On the spelling of 'heteroskedasticity', see McCulloch (1985).

through by w_i to obtain

$$\frac{R_i}{w_i}=p\left(\frac{1}{w_i}\right)+u_i, \tag{10}$$

where $u_i=\varepsilon_i/w_i$ is homoskedastic with scale c_0. The adjustment parameter θ may be estimated by scanning over values of θ. For this purpose, the log likelihood of the u_i's must be adjusted by subtracting $\sum \log w_i$ in order to obtain the log likelihood of the ε_i's. Furthermore, since the estimate of p (and therefore the ε_i's from which the w_i's are calculated) varies slightly with θ, (10) must be iteratively estimated for each θ value until the estimate of p stabilizes. However, two iterations are usually sufficient for this to occur, since the location parameter estimate is asymptotically orthogonal to α and c when the stable distribution is symmetric [DuMouchel (1975)]. Note that estimation of α, c_0, p and θ with the entire data set tacitly assumes that these parameters are known to market participants but not to econometricians. Given these four parameters, however, c_i is entirely backward-looking and does not depend on future data. The rough adjustment for heteroskedasticity in the author's earlier study of the liquidity premium (1975b) is flawed by the partially forward-looking nature of the choice of the four homoskedastic sub-periods.

Table 1 shows parameter estimates using the ACH procedure. In order to initialize the adjustment process (8), we computed

$$w_{13}=\tfrac{1}{12}\sum_{i=1}^{12}|\varepsilon_i|, \tag{11}$$

and otherwise discarded the first twelve observations in the maximum adjusted likelihood search for θ. This left 369 observations, from April 1952 to December 1982.

Fig. 2 shows how the weights w_i behave over time for one-year asset durations, and selected values are shown in table 1. For each maturity, the post-Accord low occurs either in 1953 or in 1964–1965. The post-Accord highs almost all occur immediately after the sharp fall in interest rates in late spring of 1980. The 'recent low' values in table 1 are the lowest value since the noisy year 1974. These uniformly occur in 1978. The post-Accord highs are from over 18- to almost 40-fold higher than the post-Accord lows, and range from 7- to 12-fold higher than the recent lows. The most recent values are not far below the post-Accord highs.

Fig. 3 shows the weighted forecasting errors u_i. It is apparent that they are far more homoskedastic than the unweighted errors from the unconditional model. Furthermore, the fact that there are no obvious unusual residual clusters occurring, for instance, at the time of the Franklin National Bank

Table 1
ACH parameter values.[a]

	Asset duration					
	3 mo.	6 mo.	1 yr.	2 yr.	5 yr.	10 yr.
α	1.689	1.625	1.625	1.632	1.614	1.714
	(0.081)	(0.087)	(0.078)	(0.084)	(0.083)	(0.086)
c_0	0.896	0.869	0.859	0.850	0.843	0.891
	(5.4%)	(5.8%)	(5.4%)	(5.5%)	(5.5%)	(5.5%)
$100p$	0.0212	0.0281	0.0155	−0.0061	−0.0708	−0.1327
	(0.0019)	(0.0047)	(0.0100)	(0.0205)	(0.0337)	(0.0600)
θ	0.17	0.14	0.13	0.12	0.20	0.17
min.	0.13	0.10	0.08	0.08	0.14	0.11
max.	0.25	0.20	0.19	0.20	0.29	0.23
$2\Delta \log L$:						
$\alpha=2$	22.09	37.41	39.07	60.76	67.15	32.43
$\theta=0$	128.61	120.38	141.41	121.60	138.35	126.76
selected w_i ($\times$ 100):						
low	0.00767	0.0232	0.0526	0.119	0.134	0.264
	11/64	11/64	11/64	4/53	10/63	8/65
high	0.290	0.644	1.31	2.17	4.91	6.94
	6/80	6/80	6/80	6/80	5/80	12/81
recent	0.0242	0.0600	0.146	0.308	0.528	0.976
low	7/78	6/78	9/78	9/78	10/78	5/78
12/82	0.200	0.488	0.992	1.65	3.18	5.10

[a]Asymptotic standard errors are in parentheses.

failure in 1974 or the Fed's change in operating procedure in 1979, indicates that the ACH procedure has effectively captured the heteroskedasticity in spite of its great parsimony of parameters. The other maturities (not illustrated) exhibit similar behavior.

The ACH estimates of the characteristic exponent in table 1 are much higher than the unconditional estimates mentioned in the previous section, and therefore warrant much lower deposit insurance premia except for the highest values of w_i. However, they are still substantially less than the Gaussian value of 2.0. Their asymptotic standard errors enable us to reject α values in (1.883, 2.000) at the 0.95 level for all maturities. The actual decline in double the log likelihood that occurs when the constraint $\alpha=2.0$ is imposed is even larger than the asymptotic standard errors would suggest. Unfortunately, since the hypothesis $\alpha=2.0$ is not in the interior of the parameter space, the regularity conditions for the asymptotically χ^2 distribution of this statistic are not met. We therefore do not at present have any

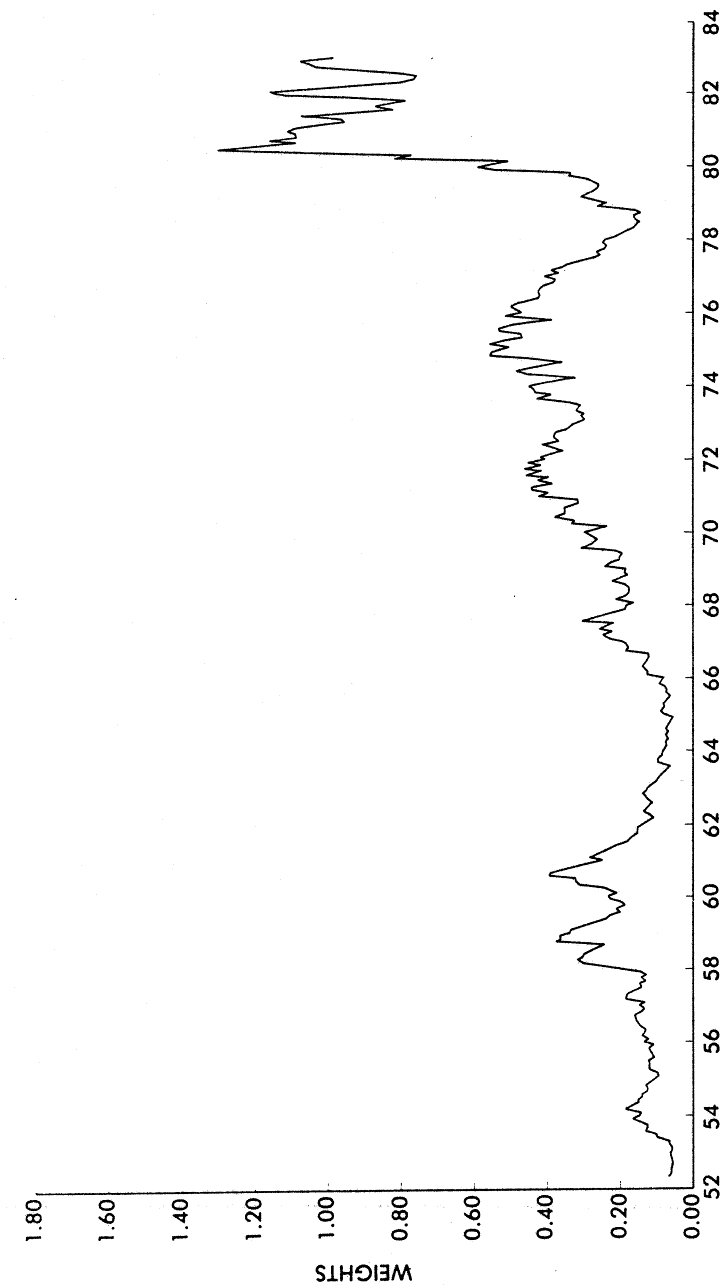

Fig. 2. Weights w_i, asset duration 1 yr.

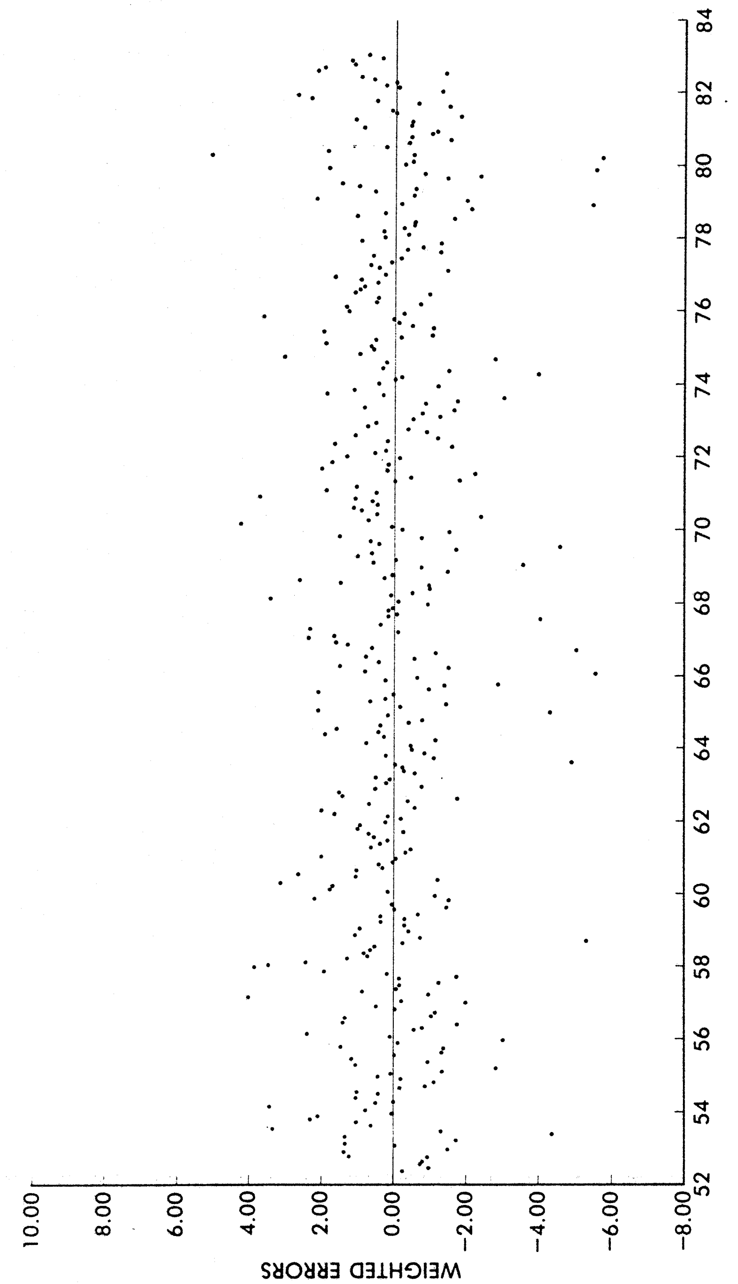

Fig. 3. Weighted forecasting errors, asset duration 1 yr.

way of using this statistic to formally reject normality itself. Note, however, that the decline in double the log likelihood is much greater than 6.63, the 0.99 percentage point of χ^2 with one degree of freedom. Since the log likelihood is a continuous function of α, we *are* formally able to reject values of α *just under* 2.0 at the 0.99 level and above.[6]

Point estimates of the adjustment coefficient θ are shown in table 1, along with the lower and upper bounds of a 95% confidence interval based on the likelihood ratio. The highest lower bound is 0.14, and the lowest upper bound is 0.19, so values of θ within this range may not be rejected for any maturity. A common value of $1/6=0.1666\ldots$ would thus be very representative. This corresponds to an average lag in (7) of six months. An efficient risk-sensitive deposit insurance scheme would therefore have to adjust very quickly to new developments.

The importance of the conditional heteroskedastic model is indicated by the fall in twice the log likelihood under the hypothesis $\theta=0$. This decline is in the range 120.38 to 141.41 for all maturities. Since θ must lie in the range $[0,1]$, we are once again up against the boundary of our parameter space and may not appeal to the standard likelihood ratio test. In this case, however, the standard 'non-standard' conditions [Moran (1971)] are met, and we may apply the usual χ^2 test with one degree of freedom. Homoskedasticity is thus overwhelmingly rejected.

The term premia p in table 1 have been multiplied by 100, so that they show the percent expected excess return per month. A bank with three-month assets and one-month liabilities can expect to make 2.12 basis points per month, or 25.4 basis points (i.e., 0.254%) per annum, more on its assets than it has to pay on its liabilities. This excess, though not very large, is highly significant ($t=11.2$), so there clearly is a payoff to maturity transformation, as long as the duration gap is kept small. With six-month assets, the premium is a little larger (2.81 basis points per month), and is still highly significant. After six months, however, the point estimate falls off and actually becomes negative after two years. There is therefore no significant payoff in terms of increased expected return to increasing the bank's duration gap beyond approximately six months.[7]

[6]It is anticipated that a forthcoming Monte Carlo study will formally confirm the significance of these statistics at a high confidence level. The author is indebted to Jerry Thursby and Randy Olsen for pointing out this problem. Note that Marcus and Shaked (1984), even though they base their insurance value estimates on the Gaussian-based Black–Scholes formula, do find significant evidence of non-Gaussian leptokurtosis in bank stock (and therefore presumably in bank portfolio) behavior, using the Studentized range test.

[7]Our earlier 'free-form' estimate of the term premium in a forward rate for a two-month security to be delivered in one month was 0.12% [McCulloch (1975b, table 6)]. This translates into an excess return over one month on three-month securities of $0.12\times(2/12)=0.020\%$ or 2.0 basis points, virtually the value obtained here. The estimate of the mean of a symmetric stable distribution, unlike the valuation that is placed on a long-odds option, is not highly sensitive to a false assumption of normality.

Table 2
Conditional deposit insurance values as a percentage of liabilities (100 I).

Capital/asset ratio (q)	Asset duration 3 mo.	6 mo.	1 yr.	2 yr.	5 yr.	10 yr.
Typical volatility						
0.01	0.0058	0.0339	0.125	0.347	1.11	1.87
0.02	0.0034	0.0206	0.0762	0.211	0.679	1.08
0.04	0.0020	0.0122	0.0452	0.124	0.405	0.611
0.07	0.0012	0.0078	0.0288	0.0790	0.260	0.372
0.10	0.0009	0.0057	0.0212	0.0579	0.191	0.267
Post-Accord low volatility						
0.01	0.0005	0.0048	0.0177	0.0630	0.0834	0.162
0.02	0.0003	0.0029	0.0108	0.0382	0.0511	0.0937
0.04	0.0002	0.0017	0.0064	0.0226	0.0305	0.0528
0.07	0.0001	0.0011	0.0041	0.0143	0.0196	0.0322
0.10	0.0001	0.0008	0.0030	0.0105	0.0144	0.0230
Recent low volatility						
0.01	0.0033	0.0224	0.0928	0.298	0.766	1.53
0.02	0.0019	0.0136	0.0565	0.181	0.469	0.883
0.04	0.0011	0.0081	0.0335	0.107	0.280	0.498
0.07	0.0007	0.0052	0.0214	0.0678	0.180	0.303
0.10	0.0005	0.0038	0.0157	0.0498	0.132	0.217
Post-Accord high volatility						
0.01	0.220	1.06	3.30	7.19	27.9	44.0
0.02	0.129	0.645	2.01	4.36	17.1	25.5
0.04	0.0740	0.383	1.19	2.58	10.2	14.4
0.07	0.0456	0.244	0.760	1.64	6.55	8.75
0.10	0.0329	0.179	0.559	1.20	4.83	6.26
Most recent volatility (12/82)						
0.01	0.117	0.674	2.10	4.61	13.9	26.0
0.02	0.0689	0.411	1.28	2.80	8.52	15.0
0.04	0.0394	0.244	0.758	1.65	5.09	8.47
0.07	0.0243	0.155	0.482	1.05	3.26	5.16
0.10	0.0175	0.114	0.355	0.770	2.40	3.70

Finally, the estimates of c_0 in table 1 are all close to unity, since the average value of w_i approximately equals the average value of $|\varepsilon_i|$.

Table 2 shows the conditional value of deposit insurance for the post-Accord low, post-Accord high, recent low, 'typical', and most recent values of interest rate volatility. The 'typical' values are found by setting 'c' equal to $c_u c_0$, where c_u is the unconditional scale estimate (not tabulated here) that was found in section 4. The latter is representative of the $|\varepsilon_i|$'s and therefore of the w_i's.

The 'typical' deposit insurance values shown in table 2 are far less than unconditional insurance values (not tabulated here) that were derived from the parameter estimates found in section 4. A bank with 4% capital and one-year assets, for example, imposes an annualized burden of only 4.52 basis points on the FDIC in a typical month, far less than the corresponding unconditional estimate of 58.6 basis points or even our former (1981a) estimate of 21.4 basis points. The post-Accord low volatility estimates are even lower, an almost imperceptible 0.64 basis points with 4% capital and one-year duration assets. This is the conditional estimate for November 1984.

Unfortunately, a period of high volatility such as has prevailed since approximately October 6, 1979 can easily produce even higher conditional than unconditional insurance values. In June 1980, for example, the conditional value of insurance on a bank with 4% capital and one-year duration assets was 119 basis points, almost twice the unconditional value. In December 1982, it was still 75.8 basis points. Only the safest bank parameter values (high capital/asset ratios and short duration assets) have yielded lower conditional than unconditional insurance values during this noisy period.

The reason for this extreme variability in insurance value is that the frequency of failure given in eq. (3) is proportional to c^{α} and therefore to w_i^{α}. The greater-than-unity power of α exaggerates the already substantial variation in the w_i's.

6. Accuracy of insurance value estimates

Let C be the asymptotic covariance matrix of the estimates of α and $\log c$ that is returned by the symmetric stable maximum likelihood program. Then

$$\operatorname{var}(\log I) \approx z'Cz, \tag{12}$$

where

$$z_1 = \frac{\partial \log I}{\partial \alpha} = \frac{b'(\alpha)}{b(\alpha)} + \log\left(\frac{c}{-\log r}\right) + \frac{H_\alpha}{H}, \tag{13}$$

and

$$z_2 = \frac{\partial \log I}{\partial \log c} = \alpha. \tag{14}$$

In (13), $b(\alpha) = \Gamma(\alpha)\sin(\pi\alpha/2)$.

The corresponding standard errors are shown in table 3. The positive correlation between the estimates of α and $\log c$[8] works in favor of a lower standard error for $\log I$, since α and c have opposite effects on I. However, the asymptotic variance of $\log I$ contains a term that is approximately equal to $[\log(c/q)]^2 \operatorname{var}(\alpha)$. When c is very small in comparison to q, which is often the case, this term dominates and can make the asymptotic standard error of

[8]Asymptotically about 0.34 at $\alpha = 1.7$ and 0.29 at $\alpha = 1.3$ according to DuMouchel (1975, p. 388).

basis points, or as high as 150.0 basis points, but has only a 0.05 chance of lying outside this range.

The reason for such broad confidence intervals is that we are attempting to project the probability of events that lie far outside our experience. This extrapolation compounds the measurement error that is present in the parameter estimates. The uncertainty in our projections is unavoidable without a larger sample, however, since our maximum likelihood parameter estimates are already asymptotically efficient.[9]

Risk-sensitive deposit insurance premia should be at least as high as the *lower* bound of a 0.95 confidence interval as constructed above. In order to be reasonably certain that government deposit insurance is not inadvertently subsidizing both the banks and a socially undesirable level of misintermediation, however [see McCulloch (1981b)], FDIC and FSLIC premia should actually be set at the *upper* bound of such a confidence interval. In all probability private insurers (or depositors themselves in exchange for higher rates on their deposits) would be willing to accept this risk for a premium somewhere inside this interval, but exactly where would be difficult to predict.

7. Application

In this paper we have found the value of insurance for a bank or financial intermediary with zero-coupon assets of one maturity (3 mo., 6 mo., 1 yr., 2 yr., 5 yr., and 10 yr.) and zero-coupon liabilities of as second, shorter maturity, namely 1 month. The institution therefore has a duration gap exactly one month less than the asset maturity, i.e., 2 mo., 5 mo., 11 mo., etc. Individual bank or thrift assets and liabilities may represent compound payment streams, and furthermore have widely differing final maturities. However, as long as the asset and liability payment streams do not overlap, the Macaulay duration gap between assets as a whole and liabilities as a whole will be a valid index of interest rate exposure. And as long as forward interest rate volatility is independent of maturity (which appears to be at least approximately the case), the value of insuring this exposure will be purely a function of the duration gap and correspond exactly to the values we have given above (adjusting, of course, for the one-month liability maturity we have used). For this purpose, the duration gap could be computed along the lines proposed by the FHLBB (1983, 363–376). It should be kept in mind for this calculation that the effective maturity of short-term liabilities

[9]In table 3, we used $\mathrm{var}(\log c_0)$ as a proxy for $\mathrm{var}(\log c)$ when applying (12). Strictly speaking, we should have taken into account the fact that the uncertainty of θ makes w_i uncertain and therefore contributes to the uncertainty of c_i. However, this is unlikely to cause problems in the long run unless banks shop for insurance on a monthly basis. Note that as in Engle's Theorem 4 (1982), the estimator of θ is asymptotically uncorrelated with those of α and $\log c_0$.

Table 3
Asymptotic standard errors of $\log I$.

Capital/asset ratio (q)	Asset duration					
	3 mo.	6 mo.	1 yr.	2 yr.	5 yr.	10 yr.
Typical conditional estimates						
0.01	0.580	0.494	0.392	0.368	0.301	0.324
0.02	0.626	0.545	0.437	0.416	0.348	0.374
0.04	0.672	0.595	0.482	0.464	0.396	0.427
0.07	0.712	0.638	0.521	0.505	0.437	0.467
0.10	0.735	0.664	0.544	0.530	0.461	0.495
Post-Accord low conditional estimates						
0.01	0.700	0.598	0.485	0.454	0.431	0.444
0.02	0.746	0.649	0.530	0.503	0.479	0.495
0.04	0.792	0.699	0.575	0.551	0.527	0.549
0.07	0.832	0.742	0.614	0.592	0.568	0.589
0.10	0.855	0.768	0.637	0.617	0.593	0.617
Recent low conditional estimates						
0.01	0.607	0.516	0.406	0.375	0.319	0.334
0.02	0.653	0.567	0.452	0.424	0.367	0.384
0.04	0.699	0.617	0.497	0.472	0.415	0.437
0.07	0.738	0.660	0.535	0.513	0.456	0.478
0.10	0.762	0.686	0.558	0.538	0.480	0.505
Post-Accord high conditional estimates						
0.01	0.407	0.314	0.240	0.218	0.148	0.174
0.02	0.453	0.364	0.284	0.265	0.191	0.221
0.04	0.498	0.413	0.328	0.312	0.236	0.272
0.07	0.538	0.456	0.366	0.352	0.275	0.312
0.10	0.561	0.481	0.389	0.377	0.299	0.339
Most recent conditional estimates						
0.01	0.437	0.337	0.261	0.240	0.179	0.198
0.02	0.483	0.387	0.305	0.287	0.224	0.246
0.04	0.528	0.437	0.349	0.334	0.270	0.298
0.07	0.568	0.480	0.387	0.375	0.310	0.338
0.10	0.591	0.505	0.410	0.399	0.334	0.364

$\log I$ quite large. Furthermore, b'/b in (13) tends toward minus infinity as α approaches 2.0, which reinforces the large standard error, particularly for our conditional estimates.

Recall from table 2 that the most recent conditional point estimate of the value of deposit insurance for a bank with 4% capital and one-year assets was 75.8 basis points per year. According to table 3, the logarithm of this estimate has an asymptotic standard error of 0.349, or 34.9%. A 95% confidence interval extends 1.96 standard errors above and below the log of the point estimate. A 95% confidence interval for the estimate itself therefore extends above and below the point estimate by a factor of $\exp(1.96 \times 0.349) = 1.98$. The value of deposit insurance on this date could therefore be as low as 38.2

may be somewhat longer than their stated maturities [Flannery and James (1984)].

If asset and liability payment streams do have some overlap, however, the duration gap does not completely summarize the situation. Consider, for example, a bank with two-year zero coupon liabilities and assets which are 50% each (in terms of present values) one-year and three-year zero coupon bonds. This bank's assets and liabilities have equal durations so there is no duration gap, yet it is still not immunized against interest rate risk. Suppose, for instance, that at the beginning of the year the one-, two-, and three-year interest rates were all 10%, and that at the end of the year the one-year rate was 9% and the two-year rate 10%. In spite of its zero duration gap, this bank would have taken a 1% loss on both the short-funded and long-funded portions of its balance sheet.

In a case like this, the value of deposit insurance depends on the exact payment structure and on the covariability of forward interest rates at different maturities, a problem that would go far beyond the scope of the present paper. Nevertheless, the duration gap still indicates a valid lower bound to the value of deposit insurance, and asset duration by itself (or liability duration if that is greater) still gives a valid upper bound.

In order to update the ACH interest rate volatility estimates we have presented, it is not necessary to redo the entire maximum likelihood estimation, since the next several years' experience will have very little effect on the estimators of α, c_0, and θ. During this time it will be sufficient merely to update the w_i for 12/82 as given in the last line of table 1 using routine term structure estimates, eq. (8), and the adjustment parameter (θ) values given in table 1. Updated deposit insurance values may then be obtained simply by multiplying the conditional estimates for 12/82 from the end of table 2 by $(w_i/w_T)^\alpha$, where w_i is the updated w value, w_T is the weight for 12/82 from table 1, and α is the maturity-specific characteristic exponent from table 1.[9]

In a system of joint private and public insurance in which the private insurer covers the first x percent of the losses and the government insuring agency covers only the remainder [as suggested by the FHLBB (1983, pp. 40–43 and 211–262)], or of partial self-insurance in which depositors themselves take the first x percent of the losses as a 'deductible', the total capital buffer protecting the government insurer from losses is, to a close approximation, essentially increased by x percent, and the risk-sensitive premium may be reduced accordingly. For example, the point estimate of the value of insurance for a bank with one-year assets and 4% capital is, from table 2, 4.52 basis points under conditions of 'typical' volatility. If private

[9]For example, by April 1984, w_i for one-year asset durations had fallen to 0.345, reducing the insurance value with 4% capital from 75.8 basis points to 13.5 basis points. By January 1985 these were back up to 0.508 and 25.3, respectively.

insurers or the depositors shouldered the first 3% of the losses, capital protecting the FDIC would effectively be increased to 7%, so that the FDIC's premium could be reduced to 2.88 basis points. Presumably the private insurer (or the depositors) would demand the difference (1.64 basis points) to compensate themselves for their share of the risk. In the case of joint private/public insurance, however, the public insurer would have to ascertain that the private insurer actually had the resources to cover 100% of its share of the losses in order to warrant such a reduction in the governmental deposit insurance premium.

For other application issues such as dividend payments, variable rate loans, price level indexed loans, and the 'going business' value of the bank or thrift, the reader is referred to McCulloch (1981a).

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