Taylor Rule 'Perils' and Constant Gain Learning

J. Huston McCulloch

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Introduction

Benhabib, Schmitt-Grohé and Uribe, in an article entitled "The Perils of Taylor Rules" (BSU 2001), point out that a Taylor Rule (Taylor 1993) with strong (more than 100%) feedback from inflation to interest rates, combined with a Zero Lower Bound (ZLB), is consistent with two steady state expectational equilibria, one at a higher, desired inflation rate, and one at a low or even negative, undesired inflation rate. They argue, assuming Equilibrium Expectations, that the higher inflation equilibrium is dynamically unstable while the undesired lower equilibrium is stable. Bullard (2010), in article entitled "Seven Faces of 'The Peril,'" concedes that this is a fatal problem for Taylor Rule policies, and that they should consequently be replaced with unspecified quantitative easing purchases.

This note considers the stability of a strong Taylor Rule, replacing Equilibrium Expectations with *Constant Gain Learning*. It is found that weak (less than 100%) feedback is destabilizing. Strong feedback can be stabilizing provided the feedback, in conjunction with a structural inflation parameter and the speed of learning, is not too strong.

The ZLB is still an issue, but ordinarily just requires the Central Bank to intervene in longer than normal maturity interest rates. The Taylor Rule raises interesting issues in continuous time, but these are not insurmountable.

The Evans and Honkapohja "E-Stability" learning mechanism used by Bullard and Mehra (2002) is, however, rejected in favor of Constant Gain Learning, at least in discrete time models.

The Taylor Rule with Constant Gain Learning

Let π_{t+1} be inflation from time t to time t+1 and π_t^{a}, "anticipated inflation," be the public's time t expectation of π_{t+1}, based on its experience of relevant data up to and including time t. It is assumed here that the deviation of π_{t+1} from π_t^{a} is driven by the real interest rate gap between the "neutral" or "natural" real rate r_{N} and the actual one period real rate, $i_t - \pi_t^{\mathrm{a}}$, plus microeconomic white noise WN:

$$\pi_{t+1} = \pi_t^{\mathrm{a}} + \mu(r_{\mathrm{N}} - i_t + \pi_t^{\mathrm{a}}) + \mathrm{WN}, \tag{1}$$

for some structural parameter $\mu > 0$.[1] The Central Bank (CB) sets i_t via a Taylor Rule

$$\begin{aligned} i_t &= r^* + \pi_t^{\mathrm{a}} + a(\pi_t^{\mathrm{a}} - \pi^*) \\ &= r^* - a\pi^* + (1+a)\pi_t^{\mathrm{a}}, \end{aligned} \tag{2}$$

where π^* is its inflation target, r^* is its best guesstimate of r_{N}, and $a > 0$ for "strong" ($1+a > 100\%$) inflation feedback or $a < 0$ for "weak" ($1+a < 100\%$) feedback. It is assumed that the CB uses an emulation of the public's experience-driven inflationary expectations in its Taylor Rule.[2] This may differ from what it itself expects the outcome of its own current and intended future policy will be. For simplicity, the zero-mean Unemployment Gap is set aside.

Combining the structural equations (1) and (2) gives the consolidated structural equation (3):

$$\pi_{t+1} = \mu(r_{\mathrm{N}} - r^* + a\pi^*) + (1 - \mu a)\pi_t^{\mathrm{a}} + \mathrm{WN} \tag{3}$$

Under Equilibrium (i.e. Muth-"Rational") Expectations, π_t^{a} will equal the expectation of π_{t+1} in (3), and their common value will be the Expectational Equilibrium steady state inflation rate π^{EE}:

$$\pi^{\mathrm{EE}} = \pi^* + \frac{r_{\mathrm{N}} - r^*}{a}.$$

The Central Bank (CB) can therefore at best expect to miss its inflation target, on average, by an amount that is directly proportional to the amount by which it has mis-estimated the natural real rate, and inversely proportional to its feedback coefficient a.

Following Evans and Honkapohja (EH 2001), it is assumed instead that the public's Perceived Law of Motion (PLM) at time t is empirically based on the potentially relevant observable variables and takes the form

$$\pi_t^{\mathrm{a}} = \pi_t^{\mathrm{PLM}} = \alpha + \beta\pi_{t-1} + \gamma i_{t-1}, \tag{4}$$

summarized by the coefficient vector $(\alpha, \beta, \gamma)'$.[3] Combining (3) and (4), the corresponding Actual Law of Motion (ALM) will be

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[1] A low (or high) one-period real interest rate may also have an indirect stimulative (or restrictive) effect to the extent that agents expect the disequilibrium rates to continue into the future. The present note abstracts from this effect.

[2] E.g. McCulloch (2025).

[3] The full PLM VAR would include a similar equation governing the interest rate and would be required to model expectations more than one period into the future. However, since the CB is ordinarily only targeting the one-period rate, this second equation is not required. Including the interest rate equation would make the Jacobian matrix of the

$$\pi_t^{\mathrm{ALM}} = \mu(r_{\mathrm{N}} - r^* + a\pi^*) + (1-\mu a)\alpha + (1-\mu a)\beta\pi_{t-1} + (1-\mu a)\gamma i_{t-1} + \mathrm{WN}$$
$$= \alpha' + \beta'\pi_{t-1} + \gamma' i_{t-1} + \mathrm{WN} \qquad (5)$$

for

$$\alpha' = \mu(r_{\mathrm{N}} - r^* + a\pi^*) + (1-\mu a)\alpha,$$
$$\beta' = (1-\mu a)\beta,$$
$$\gamma' = (1-\mu a)\gamma.$$

The EH "T-Map," ALM= T(PLM), is therefore a simple affine transformation:

$$\begin{pmatrix}\alpha'\\ \beta'\\ \gamma'\end{pmatrix} = \mathrm{T}\begin{pmatrix}\alpha\\ \beta\\ \gamma\end{pmatrix}$$
$$= \begin{pmatrix}\mu(r_{\mathrm{N}} - r^* + \mathrm{a}\pi^*)\\ 0\\ 0\end{pmatrix} + \begin{pmatrix}1-\mu a & 0 & 0\\ 0 & 1-\mu a & 0\\ 0 & 0 & 1-\mu a\end{pmatrix}\begin{pmatrix}\alpha\\ \beta\\ \gamma\end{pmatrix} \qquad (6)$$

The three eigenvalues of this T-Map all happen to all be real and equal to

$$\lambda^{\mathrm{TM}} = 1 - \mu a.$$

Under what EH call "Iterative Learning" (IL), each period's PLM is simply the preceding period's ALM: [4]

$\mathrm{PLM}_t = \mathrm{ALM}_{t-1}$.

The general condition for IL-Stability is that each of the potentially complex eigenvalues of the T-Map must lie inside the unit circle on the complex plane. If they happen to be real, as they are here, this simply means that they must each lie in the interval (−1, 1). In our case, this requires

$$0 < \mu a < 2. \qquad (7)$$

The lower bound on μa is automatically met if there is strong inflation feedback, i.e. if $a > 0$, since μ is assumed to be positive. Weak inflation feedback, i.e. $a < 0$, is clearly IL-unstable. However, if $a > 1/\mu$, there will be overshooting with inflation alternately above and below its equilibrium level. With $a > 2/\mu$, these alternations will be explosive.

T-Map 6x6. When this is done, however, three of the eigenvalues remain as in (6) below, and the other three are all zero, so that learnability is not affected.

[4] An alternative interpretation of IL is that the PLM remains constant during an *era* of several periods, during which the ALM is simply observed and the PLM is constant. Then the PLM in era *e* is given by $\mathrm{PLM}_e = \mathrm{ALM}_{e-1}$.

Iterative Learning is useful for intuition and instruction, but is clearly unrealistic. It is more realistic to assume, in the spirit of Recursive Least Squares (RLS), that agents learn a little about the current ALM each period, rather than all or nothing. This can be modelled by *Constant Gain Learning* (CGL) as in Equation (10.10) of EH (2001: 232):

$$\mathrm{PLM}_t = \omega \mathrm{ALM}_t + (1 - \omega)\mathrm{PLM}_{t-1}, \quad (8)$$

for some constant gain ω in (0, 1). This model generalizes the much-criticized Adaptive Expectations (AE) model of Cagan (1956) to allow for multiple variables and/or transient dynamics, all with time-varying parameters that can accommodate changes in the structural and policy parameters. Constant gain RLS itself is the limiting behavior of generalized RLS when the noise/signal ratio is held constant (McCulloch 2024).

It can easily be shown that if λ^{TM} is an eigenvalue of the T-Map, then

$$\lambda^{\mathrm{CGL}} = \omega\lambda^{\mathrm{TM}} + 1 - \omega$$

is an eigenvalue of the CGL transformation (8). For our Taylor Rule (3), Equation (6) implies that all three of the CGL eigenvalues of (8) again happen to be real and equal, with

$$\lambda^{\mathrm{CGL}} = 1 - \omega\mu a.$$

CGL-Stability therefore requires

$$0 < \omega\mu a < 2. \quad (9)$$

Clearly, weak inflation feedback ($a < 0$) is still destabilizing, and CGL does not help. Too strong inflation feedback can be destabilizing, but this will be offset under CGL so long as $\omega < 2/(\mu a)$. In fact, the alternations that would otherwise occur if $\mu a > 1$ would be efficiently neutralized if $\omega = 1/(\mu a)$.

The Zero Lower Bound

If $\pi_t^a < \frac{a\pi^* - r^*}{1+a}$, the desired one period nominal rate given by (2) will be negative and the CB will be up against the Zero Lower Bound (ZLB). In this case it may still be feasible to achieve the desired stimulus by intervening in a maturity longer than one period.[5]

Assume for simplicity that anticipated inflation and the equilibrium real rates that equate planned consumption and production date by date throughout the

[5] This section extends an argument made already in McCulloch (2018).

future are both constant with respect to maturity. Let m_0 be the time, in years, between the discrete time periods. Then a decrease (or increase) in the continuously compounded nominal and therefore real one period interest rate of Δi that leaves forward rates beyond m_0 unchanged will decrease (or increase) the log value of current output relative to future output at every point in the future, and therefore of aggregated future output, by $\Delta i \times m_0$. The direct stimulative (or restrictive) effect of a decrease (or increase) in the one-period rate is therefore proportional to this product.

When the ZLB is binding, the CB desires a (negative) real interest rate gap of $a(\pi^a - \pi^*)$ at maturity m_0, for a stimulus of $a(\pi^a - \pi^*)m_0$. With a zero nominal interest rate, the real interest rate gap is instead only $-(\pi^a + r^*)$. However, if the CB instead intervenes with a zero nominal interest rate at a somewhat longer maturity m_{ZLB}, it can expect to achieve approximately the same stimulus by setting

$$m_{\text{ZLB}} = \frac{a(\pi^* - \pi^a)}{\pi^a + r^*} m_0, \tag{10}$$

so long as $\pi^a > -r^*$.[6]

In a literally discrete time economy, (10) might not yield an integer multiple of m_0. In that case, the CB could set m_{ZLB} equal to the smallest integer multiple of m_0 that is greater than the value in (10), with an appropriately increased, and somewhat positive, nominal rate.

On paper, an initial $\pi_0^a < -r_N$, if true at all horizons, would still lead to a hyper*de*flation, but this would unrealistically imply that money would already be infinitely valuable in terms of goods so that the hyperdeflation would already have been complete. If instead this were only true for shorter horizons, the CB could still break out of a ZLB situation by targeting an appropriate $m_{ZLB} > m_0$ at which expected inflation did exceed the negative of the real rate, with appropriate adjustments to the formula.

The Evans and Honkapohja "E-Stability" Learning Mechanism

Rather than developing the CGL model (8), EH in their eq. (10.9) (2001: 231) instead propose replacing the unrealistic IL model with the following "E-Stability" differential equation:

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[6] The stimulus would only approximately be equal, because output at m_{ZLB} and beyond is a smaller share of intertemporal wealth than is output at m_0 and beyond. However, the difference is minor so long as m_{ZLB} is small.

$$\frac{\mathrm{d}}{\mathrm{d}t}\mathrm{PLM}_t = (\mathrm{ALM}_t - \mathrm{PLM}_t). \quad (10)$$

Bullard and Mitra (2002) analyze the learnability of Taylor Rule dynamics using this equation.

However, it is inconsistent to model agents as learning continuously in the discrete-time world implied by the type of PLM and ALM that EH consider. CGL (8) is in the same spirit as E-Stability, but consistently assumes discrete time for both the laws of motion and the learning process.

That something is amiss with (10) is evident from its lack of a constant of proportionality on the right-hand side. The E-Stability of a model is determined by the sign of the real part of the eigenvalues of the T-map, and since this sign would not be affected by any positive constant of proportionality in (10), setting this constant equal to unity does not affect the E-Stability condition. This counterintuitive property is contrary to the above results with CGL, in which the rate of learning ω can be consequential for stability.

The Taylor Rule in Continuous Time

To be sure, time is continuous in the real world, and discrete-time models are recognized to be merely a simplifying approximation to reality. However, the continuous-time limit of a discrete-time model in which the CB sets the one-period interest rate would have to contend with the fact that if $m_0 = 0$, an *infinite* change in the m_0 rate would be required to have any stimulative or restrictive effect at all.

On the restrictive side, this would simply mean that the CB would have to engineer a discontinuity at zero maturity in the "discount function" $\delta_t(m)$ that gives the present value of a unit of currency to be paid at maturity m. Let $\delta_t^{\mathrm{N}}(m)$ be the "neutral" nominal discount function that equates planned consumption and production throughout the future, given inflationary expectations. Assuming for simplicity that the natural real rate is constant with respect to both time and maturity and that expected inflation is constant with respect to maturity, the neutral nominal discount function will be

$$\delta_t^{\mathrm{N}}(m) = exp(-(\pi_t^a + r_{\mathrm{N}})m).$$

Then if it desired to increase the cost of present (time t) output relative to all future output by say 1% without directly disturbing forward rates beyond $t = 0$, the CB would have to intervene to alter the discount function to

$$\delta_t(m) = \begin{cases} 1, \ t = 0 \\ exp(-(\pi_t^a + r_{\mathrm{N}})m - .01), \ t > 0. \end{cases}$$

This could be accomplished by offering its entire portfolio for sale at a price of $\exp(-.01)$ times the face value of each issue, including any coupons.

However, a symmetrically *expansionary* policy in continuous time that *reduced* the price of current consumption relative to all future consumption by the same 1% would immediately be up against the ZLB. The best the CB would be able to do would be to set nominal rates equal to zero out to maturity

$$m_{\mathrm{ZLB}} = .01/(\pi^a + r_N),$$

resulting in a discount function of

$$\delta_t(m) = \begin{cases} 1, \ t \leq m_{\mathrm{ZLB}} \\ exp(-(\pi_t^a + r_{\mathrm{N}})m + .01), \ t > m_{\mathrm{ZLB}}. \end{cases}$$

This reduces the price of time t consumption relative to time $t + m$ consumption by somewhat less than 1% for m in $(0, m_{\mathrm{ZLB}})$, but that portion of the future accounts only a small fraction of its total value.

It is not clear how Adaptive Learning should be modelled in a continuous time world. In equation (8), we would expect ω to be roughly proportional to m_0, so that $\omega \approx \rho m_0$ for some positive ρ. Therefore (10), with this ρ added to the right hand side, would seem to be the natural limit of CLG, but as noted above, it is suspect in my mind because its stability would not depend on ρ.

In the US, financial markets operate in continuous time, but the FOMC only meets 8 times a year and almost never changes its overnight target between meetings.[7] It is therefore virtually an arbitrage relationship that the approximately 1/8-year interest rate equals its overnight target, so that the Fed behaves much like a discrete time CB with an effective time interval of 1/8 year.

Benhabib, Evans and Honkapohja

Benhabib, Evans and Honkapohja (2014: 227-8) investigate a discrete time 4-variable Taylor Rule model with learning, but model the expectation of each variable with constant gain univariate AE as in Cagan (1956) rather than multivariate CGL. Although they call their model "Constant Gain Learning," it is only the very narrow AE special case of (8).

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[7] Recent exceptions that prove the rule are 3/3/20 and 3/15/20 during the covid crisis.

Concluding Comments

The Taylor Rule admittedly still has a number of uncertainties and potential "perils": The neutral real rate r_N is not directly observable and surely fluctuates over time with intertemporal demand and supply shocks. The public's π_t^a can only be roughly estimated. The Central Bank can force equilibrium inflation π^{EE} close to its target with a high value of its feedback coefficient *a*, but as has been shown, that also has a destabilizing effect. Any instability could in theory be offset with sufficiently slow learning, but it is an empirical question whether the noise/signal ratio in the historical data (McCulloch 2024) would imply a sufficiently low asymptotic gain.

REFERENCES

Benhabib, Jess, George W. Evans, and Seppo Honkapohja (2014). "Liquidity Traps and Expectation Dynamics: Fiscal Stimulus or Fiscal Austerity?" *Journal of Economic Dynamics and Control* **49**: 220-38.

Benhabib, Jess, Stephanie Schmitt-Grohé, and Martin Uribe (2001). "The Perils of Taylor Rules." *Journal of Economic Theory* **96** (1-2): 40-69.

Bullard, James (2010). "Seven Faces of 'The Peril.'" Federal Reserve Bank of St. Louis *Review* **95** (2): 339-52.

Bullard, James, and Kaushik Mitra. (2002). "Learning about monetary policy rules." *Journal of Monetary Economics* **49**: 1105-29

Cagan, Phillip (1956). "The Monetary Dynamics of Hyperinflation," Ch. 2 in M. Friedman, ed., *Studies in the Quantity Theory of Money*. Chicago: Univ. of Chicago Press.

Evans, George W., and Seppo Honkapohja (2001). *Learning and Expectations in Macroeconomics.* Princeton, NJ: Princeton Univ. Press.

McCulloch, J. Huston (2018). "The Optimum Quantity of Money and the Zero Lower Bound." *Cato Journal* **38** (Spg./Sum.): 428-45.

__________(2024). "Adaptive Least Squares: Recursive Least Squares with Constant Noise-to-Signal Ratio." <https://www.asc.ohio-state.edu/mcculloch.2/papers/ALS/ALS.pdf>

_________ (2025). "Entrenched Inflation Update for 5/25." June 26, 2025. <https://www.asc.ohio-state.edu/mcculloch.2/papers/ALS/Entrenched Inflation 5-25.pdf>

Taylor, John (1993). "Discretion vs. Rules in Practice." *Carnegie-Rochester Conference Series on Public Policy* **39** :195-214.

The author is Adjunct Professor, NYU and Emeritus Professor, OSU.
Comments welcome via mcculloch.2@osu.edu.