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THE MONTE CARLO CYCLE IN BUSINESS ACTIVITY

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NBER business "cycle" reference dates and aggregate economic time series are examined for evidence of regular cyclic behavior. A simple contingency table test is used on the reference dates, and aggregate series are fit with a second-order autoregression. The results are negative. Apparently the business "cycle" is an optical illusion or, as Irving Fisher called it, a "Monte Carlo cycle." These are the cycles superstitious gamblers believe govern their luck.

THE MONTE CARLO CYCLE HYPOTHESIS

Some 50 years ago, Irving Fisher (1925, 191) suggested that business cycles are nothing more than "Monte Carlo cycles." These are the cycles superstitious gamblers believe they can discern in their luck at casinos like the one at Monte Carlo. Of course, there are no such cycles in any meaningful sense. Assuming the casino is honestly managed, runs of good or bad luck have no predictive power with respect to future bets. Nevertheless, any rule for dividing a time series (even the size of one's pile of chips at the casino) into periods of preponderant growth and preponderant contraction will result in the expansions relentlessly alternating with the contractions. Most time series thus appear wave-like whether they have a true cyclic character or not. The possible appearance of this optical illusion in business activity was extensively explored by Slutsky (1927), and is the basis of the "random walk hypothesis" of stock market prices (Roberts 1959).[1]

If business fluctuations were just Monte Carlo cycles, if they had no periodicity, rhythm, or pattern except perhaps a trend, then information about past fluctuations would be of no help to us in projecting future activity. Knowledge about detailed economic interrelationships and about the persuasions of policymakers currently in power might be of

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1. See also Nelson (1972, 909).

use, but the course of aggregate activity itself would not. Knowing that business in general has been expanding or contracting would be irrelevant for private and public decision makers.[2] The absolute level of business activity or related indicators might still be a useful piece of information.[3] For instance, it might be very important to know that unemployment is at 25 percent. But still it would be irrelevant whether we got to 25 percent from a value of 20 percent last year or from a value of 30 percent. Furthermore, the age of the current expansion or contraction would tell us nothing about how much longer we could expect it to continue. Thus, if unemployment had stood at 25 percent for the past 20 months, we would be no more justified in saying that the end is in sight than if it had only been at 25 percent for the past 2 months, even if 20 months were the average duration of historical depressions.

With only a few exceptions, little has been done to test the Monte Carlo hypothesis directly.[4] Apparently it is more satisfying intellectually to explain why business exhibits regular cycles, and more rewarding financially to forecast future cycles, than it is to question whether there are cycles at all. In the present paper we address the latter question.

A TEST OF THE MONTE CARLO CYCLE HYPOTHESIS

Is the business cycle nothing more than a Monte Carlo cycle? One of the foremost students of business fluctuations, Arthur Burns, recognizes that any expansion would eventually come to an end due simply to random influences. Still, he goes on to state that "experience strongly suggests that even in the absence of serious extended disturbances, the course of aggregate activity will in time be reversed by restrictive forces that gradually, but insistently, come into play as a result of the expansion process itself."[5] If such restrictive forces do gradually, but insistently, come into play, the probability of an on-going expansion terminating

2. The standard NBER chronology of business "cycles" requires that official contractions and expansions be sustained. Therefore, the foreknowledge that in a year or two today will be declared to have been part of an official expansion or contraction is useful knowledge, since it provides information that the recent state of affairs has a better than usual chance of continuing. We do not deny the value of such foreknowledge. We only question the value of currently available knowledge about the trend of business.

3. I am grateful to Levis Cochin for emphasizing this point.

4. Adelman (1960) is a notable exception. She finds that fluctuations generated by adding random disturbances to naive trend projections of individual series bear a certain resemblance to historical U.S. cycles. She notes that the resulting "specific" turning points (see below) are not as clustered as the specific turning points identified by the NBER, and that randomly shocking the equations of the Klein-Goldberger model as described by Adelman and Adelman (1959) does better in this respect. However, clustering could presumably be increased ad lib. by using sequentially independent random disturbances that are positively correlated iin the cross section.

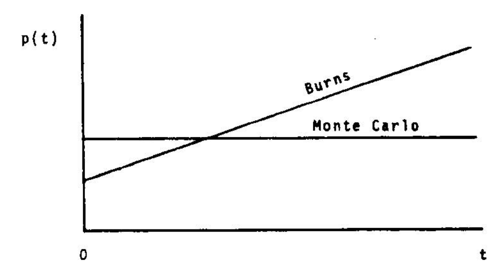
5. (Burns 1969, 29). Burns' collaborator Wesley C. Mitchell does emphasize that the business "cycle" is *not* to be regarded as periodic (Burns and Mitchell 1947, 3, which quotes from Mitchell 1927, 468). See also Schumpeter (1939, I, 168-9).

in any given month would be an *increasing* function of the age t of the expansion. This probability $p(t)$ is illustrated in Figure 1.

This behavior is in contrast to that of a Monte Carlo cycle, in which the probability of a reversal occurring in a given month is a *constant* independent of the length of time elapsed since the last turning point (also shown in Figure 1). Equivalently, the durations of the expansions would be distributed according to the "Pascal" or "geometric" distribution. This implication provides us with a workable test of whether the business "cycle" we observe in the modern world is a mere Monte Carlo cycle: We can approximate the function $p(t)$ with a step function and test whether or not the probability of termination is equal for "young" and "old" expansions. Our test can be performed on contractions as well as expansions. However, it is invalid for full cycles, because the durations of full cycles would be distributed according to a discrete version of the gamma distribution, rather than the Pascal distribution.

FIGURE 1

Probability of an on-going expansion terminating, as a function of its duration to date.



For data, we use the reference cycle turning points computed by the National Bureau of Economic Research.[6] The simplest way to use these dates would be to divide the expansions or contractions into two groups by length and ask whether the probability of a termination is lower when the expansion or contraction is younger than the median age than when it is older than the median age. However, this procedure would cause spurious rejection of the Monte Carlo cycle because of the way in which the dates are computed. Each reference turning point date is a "consensus" of "specific" turning points, which are determined for each of hundreds of time series pertaining to specific aspects of business activity. According to the rules laid down by Burns and Mitchell, "We do not recognize a rise and fall as a specific cycle unless its duration is

6. (Burns 1969, 16-17). A peak at November 1969 and a trough at November 1970, provided by Josephine Su of the NBER, were added to the U.S. data.

at least fifteen months, whether measured from peak to peak or from trough to trough. Fluctuations lasting less than two years are scrutinized with special care . . . " [Burns and Mitchell 1947, 57-58]. This rule implies that the probability of a reversal being recorded will be lower for very young expansions (or contractions) than for medium-aged or old expansions, regardless of how the fluctuations are produced.

In order to perform our test, we therefore must disregard months in which $p(t)$ has thus been reduced. The Burns-Mitchell rule probably does not appreciably alter the distribution of full cycle periods for cycles that are more than 24 months old. If there were no trend in economic time series and if the cyclic component (if any) were symmetric, we would therefore want to disregard the behavior of half-cycles less than $t_0 = 12$ months old. However, because a trend is present, expansions tend to be about 50% longer than contractions. Therefore we will set $t_0 = 14$ months for expansions and $t_0 = 10$ months for contractions.[7]

Let t_1 be the median age of the expansions that last more than t_0 months. We wish to approximate $p(t)$ with a step function, as illustrated in Figures 2 and 3. We set

$$p(t) = p_1, \; t_0 < t \leqslant t_1 \tag{1}$$

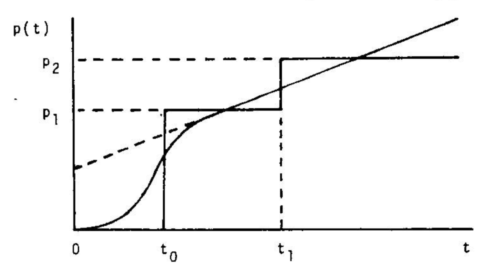
and

$$p(t) = p_2, \; t_1 < t \, . \tag{2}$$

Our null hypothesis H_0 is that $p_1 = p_2$. In this case we will use "p_{12}" to represent the common value of p_1 and p_2. If Burns' statement is correct, we will have $p_1 < p_2$. We do not expect to find $p_1 > p_2$, but that would also be evidence of non-Markov behavior, so our alternative hypothesis H_1 is $p_1 \neq p_2$.

FIGURE 2

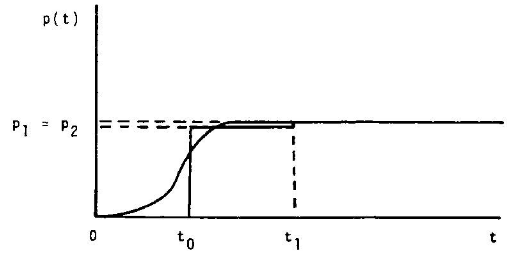
Modification of Burns characterization of p(t) when short cycles are not admitted, and step function approximation.



7. Monte Carlo tests on half-cycle durations derived from a random walk in conjunction with a dating rule that precludes full cycles less than 15 months and reduces the probability of somewhat longer ones suggest that these values of t_0 are if anything higher than necessary.

FIGURE 3

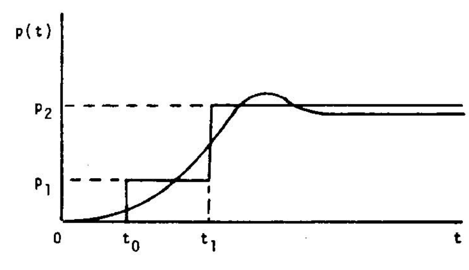
Modification of Monte Carlo characterization of p(t) when short cycles are not admitted, and step function approximation.



If business fluctuations were caused by random fluctuations acting on a second order difference equation with coefficients that imply harmonic oscillations, $p(t)$ would have the form indicated in Figure 4. Provided the theoretical period of the transient oscillations is large compared to the minimal cycle allowed by the dating rule, t_1 tends to fall just short of the theoretical period. A mode in $p(t)$ tends to appear somewhat after the theoretical period. As in the Burns case, we will have p_1 less than p_2 when we approximate with a step function.[8]

FIGURE 4

Probability of an on-going expansion terminating for a cyclic process, with step function approximation.



These hypotheses may be compared by means of a contingency table test based on the likelihood ratio's asymptotic χ^2 distribution. Let m_t be the number of expansions which terminate at age t, for t greater than t_0

8. I am grateful to Milton Friedman for pressing me on this point. The above characterizations were confirmed by Monte Carlo tests with theoretical periods of 40, 80, and 120 months, Q values of 20 and 5 (see below), and with a dating rule that precludes cycles less than 15 months in length. However, these tests indicated that our test is unable to distinguish between a mixture of cycles with different periods and a Markov process.

but less than or equal to t_1. Similarly, define m_2 for t greater than t_1. Let n_1 be the number of months that expansions of age t are observed, including observations on terminations and on expansions that will last more than t_1 months, for t greater than t_0 but less than or equal to t_1. Similarly, define n_2 for t greater than t_1. For convenience, set

(3) $$m_{12} = m_1 + m_2$$

(4) $$n_{12} = n_1 + n_2 .$$

These values may be organized in a simple two-by-two contingency table:

TABLE 1

	Terminations	Nonterminations	
$t_0 < t \leqslant t_1$	m_1	$n_1 - m_1$	n_1
$t_1 < t$	m_2	$n_2 - m_2$	n_2
	m_{12}	$n_{12} - m_{12}$	n_{12}

As is well known (Mood et al. 1974, 454), the maximum likelihood estimates of the probabilities are

(5) $$\hat{p}_1 = m_1/n_1$$

(6) $$\hat{p}_2 = m_2/n_2$$

(7) $$\hat{p}_{12} = m_{12}/n_{12}$$

In order to test H_0 against H_1, we look at the statistic

(8) $$\Lambda = -2 \log \lambda ,$$

where

$$\lambda = [n_1^{n1} n_2^{n2} m_{12}^{m12} (n_{12}-m_{12})^{(n12-m12)}]/[n_{12}^{n12} m_1^{m1}\, m_2^{m2} (n_1-m_1)^{(n1-m1)} (n_2-m_2)^{(n2-m2)}]$$

The distribution of Λ is asymptotically χ^2 with 1 degree of freedom. However, since we do not have a very large sample (m_{12} is at most 25), we have tabulated the actual distribution of Λ for small samples. Table 2 shows this distribution for $m_1 = m_2 = 7$ and $p_{12} = 0.10$. It was also tabulated for $m_1 = m_2 = 5$ and $m_1 = m_2 = 9$ and for $p_{12} = 0.05$ and $p_{12} = 0.15$, with very similar results. For comparison, we also show the asymptotic χ^2 distribution. We would not have gone far wrong to have simply used the χ^2 distribution, but since we now have the actual small sample distribution, we should use it.[9]

TABLE 2

Significance Levels for Λ

Confidence Level	Small Sample	Asymptotic χ_1^2
.90	2.78	2.71
.95	3.95	3.84
.975	5.17	5.02
.99	6.8	6.63
.995	8.2	7.88
.999	12.	N.A.

For the sake of illustration, we will go through our calculations for U.S. expansions. From 1854 to 1969 there are 27 expansions whose durations in months, in increasing order, are 10, 12, 18, 18, 19, 20, 21, 21, 22, 22, 22, 24, 25, 27, 30, 33, 34, 35, 36, 37, 44, 45, 46, 50, 80, and 105. Two of these have durations of t_0 = 14 months or less, so we disregard them. The median age of the remaining expansions is t_1 = 27, which gives m_1 = 13, m_2 = 12, and m_{12} = 25. The third through fifteenth expansions each spent their entire lives, less than 14 months each, in the youthful stage and the last 12 expansions each spent t_1-t_0 = 13 months in this stage, so n_1 = 18 + 18 + 19 + 20 + 21 + 21 + 22 + 22 +22 + 24 + 25 + 27 + 27 − (13×14) + (12×13) = 260. The last 12 expansions spent their entire lives save 27 months each in the last stage, so n_2 = 30 + 33 + 34 + 35 + 36 + 37 + 44 + 45 + 46 + 50 + 80 + 105 − (12×27) = 251, and n_{12} = 260 + 251 = 511. These values give us:

$$\hat{p}_1 = 13/260 = 0.050 \tag{9}$$

$$\hat{p}_2 = 12/251 = 0.048 \tag{10}$$

$$\hat{p}_{12} = 25/511 = 0.049 \tag{11}$$

$$\Lambda = 0.01 \tag{12}$$

9. Because n_1 and n_2 may lie anywhere between $m_1 = m_2$ and infinity, we were not able to investigate all contingencies, but had to terminate the calculation after a few hundred thousand combinations of n_1 and n_2. This left events with a total probability of about 0.0002 unaccounted for, somewhat lowering the accuracy for the higher confidence levels. This is indicated in Table 2 by the number of significant digits reported. It is well known that another, closely related statistic, also based on a likelihood ratio derived from a multinomial distribution, has a small-sample distribution that is surprisingly well approximated by the asymptotic χ^2 distribution (Mood et al. 1974, 444).

FINDINGS

Table 3 shows our test statistic for the U.S., Great Britain, Germany and France, for expansions and contractions. The U.S. figures are given both for the full period available, December 1854 to November 1970, and for a truncated period, December 1854 to August 1929. The truncated period is shown because it might be objected that in recent times, beginning perhaps with the New Deal, the government has learned how to prolong expansions and curtail contractions artificially, so that the later fluctuations might not be representative of the workings of a pure market economy. Indeed, the two longest expansions did occur since 1933. Therefore, calculations were performed omitting the turning point of March 1933 and all subsequent turning points.[10]

TABLE 3

Country	Fluctuations Used	m_{12}	t_1	$\hat{p}_1$	$\hat{p}_2$	$\hat{p}_{12}$	Λ
United States (Dec. 1854-Nov. 1970)	Expansions (27)	25	27	0.050	0.048	0.049	0.01
	Contractions (27)	22	17	0.088	0.090	0.089	0.00
United States (Dec. 1854-Aug. 1929)	Expansions (20)	18	22	0.070	0.087	0.078	0.25
	Contractions (19)	17	18	0.102	0.062	0.083	1.09
France (Dec. 1865-Aug. 1938)	Expansions (17)	15	27	0.054	0.060	0.056	0.05
	Contractions (17)	15	18	0.092	0.057	0.071	0.93
Germany (Feb. 1879-Aug. 1932)	Expansions (10)	10	35	0.030	0.081	0.044	2.48
	Contractions (10)	10	18	0.092	0.037	0.058	2.19
Great Britain (Dec. 1854-Sept. 1938)	Expansions (16)	13	42	0.023	0.079	0.034	4.76
	Contractions (16)	14	20	0.061	0.048	0.054	0.21

In every U.S. case the statistic is easily insignificant, even for the truncated pre-New Deal period. For all the countries together, there is one statistic which by itself is significant at the .95 level. Under H_0, however, the probability of at least one being significant at this level is roughly 1 in 3. (Eight of the ten statistics are nearly independent. We have $1 - .95^8 = .337$.) In order to reject H_0 at the .95 level, we must have at least one statistic by itself significant at the .9936 level

10. It should be noted that in order to attribute the 80 and 105 month post-Hoover expansions to deliberate government intervention in the economy, it would be necessary to reinterpret American involvement in World War II and Viet Nam in terms of fiscal policy rather than foreign policy.

($.95^{1/8} = .9936$). None is significant at even the .975 level, so we may not reject H_0.

Contrary to Burns' statement quoted above, experience, at least that of the NBER reference dates, does not strongly suggest anything other than that reversals come about through random disturbances. There is no evidence here that "the course of aggregate activity will, in time, be reversed by restrictive forces that gradually but insistently come into play as a result of the expansion process itself." The business "cycle" is apparently a superstition, a Monte Carlo cycle.

This finding is consistent with the hypothesis that fluctuations do permeate business activity, making otherwise unrelated series move more or less together. It says nothing about the usefulness of leading indicators for predicting forthcoming turning points (though it should be kept in mind that we may not know until after the date of the reference turning point whether or not the specific leading indicator has officially turned). Nor does it detract in any way from the value of the NBER reference dates as benchmarks of the broad ups and downs of business activity.

THE SECOND ORDER AUTOREGRESSION APPROACH

By way of qualification to the preceding analysis, it should be noted that a lot of information is lost in going from raw economic time series to the NBER reference dates. Tests performed on the actual time series are potentially more powerful than our simple procedure.

One approach, first proposed by Udny Yule, is to assume that the time series in question follows a second order autoregressive scheme:[11]

$$x_t = c\,x_{t-1} + d\,x_{t-2} + u_t \, . \tag{13}$$

It is well known [e.g., Goldberg 1958, 169-72] that this system produces cycles of period

$$T = 2\pi/\arccos[c/2(-d)^{1/2}] \tag{14}$$

if and only if

$$d < -c^2/4 \, . \tag{15}$$

The system is stable if and only if

$$1 + c - d > 0 \tag{16}$$

11. A similar suggestion was made by Frisch (1933). Simulations by Adelman and Adelman (1959) using the Klein-Goldberger model suggest that that model is stable, non-cyclic, and converges very quickly.

$$(17) \qquad 1 - c - d > 0$$

and

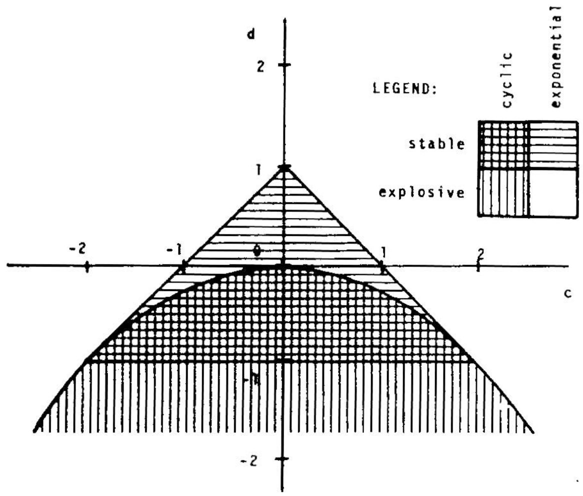
$$(18) \qquad 1 + d > 0 .$$

These conditions are illustrated in Figure 5.

If the system is not cyclic, it has solutions that are exponential in t. If it is stable, the coefficients on t in the exponentials will be negative so that x_t will converge on zero if the random disturbance u_t should go away. If it is unstable, at least one of the coefficients will be positive so that x_t will eventually increase or decrease without limit. If the system is cyclic and stable, it will have a sinusoidal solution that decreases in amplitude with time toward zero if the random disturbances cease. If it is cyclic and unstable, it will oscillate about zero with ever larger amplitude.

FIGURE 5

Values of c and d in Equations (20) and (23) for which the Yule system is cyclic or exponential, stable or explosive.



If an economic series is cyclic, it is most likely cyclic about an exponential trend, with cycles that grow in size in proportion to its own secular growth, even if the cycles are stable in the sense that the series would converge on its exponential trend if the random shocks ceased. If g is the long-run equilibrium trend growth rate of such a series, its natural logarithm z_t would have the form

(19) $$z_t = e + gt + x_t ,$$

where x_t is the series' cyclic component, obeying (13). Equations (13) and (19) imply

(20) $$z_t = a + bt + cz_{t-1} + dz_{t-2} + u_t ,$$

where

(21) $$a = e(1 - c - d) + g(c + 2d)$$

and

(22) $$b = g(1 - c - d) .$$

Equation (20) was fit to annual data on the logarithm of real income for 1929-1973. If income is the sum of several series each of which obeys an equation like (20) with different values of the parameters, income would not necessarily obey such an equation itself. Therefore it is desirable to disaggregate income somewhat to see if its components are cyclic. To this end, we also fit equation (20) to the logarithm of real investment and the logarithm of real consumption over the same period. The results of these regressions are shown in Table 4. This table also shows estimates using quarterly data on real income, since annual data cannot reveal cycles with periods less than two years.[12] This is because the arccosine in equation (14) must lie between 0 and π.

The Durbin-Watson statistics in Table 4 for equation (20) are not indicative of serial correlation. Nevertheless, we should be cautious in drawing conclusions from the regression coefficients shown. As has been pointed out by Griliches (1967, 36), if serial correlation is present in the residuals of a regression equation containing lagged dependent variables, the coefficients will be inconsistent. Furthermore, the Durbin-Watson statistic will be biased, so that we have no reliable warning of the inconsistency. If our regression suffers from this problem, the conclusions we draw from it may be invalid.[13]

In order to explore the robustness of our equation (20), we also estimate this equation in first difference form:

(23) $$\Delta z_t = b + c\,\Delta z_{t-1} + d\,\Delta z_{t-2} + e_t$$

where

12. For quarterly data, the period used was 1947 I to 1972 IV.

13. I am grateful to Charles Brown for calling my attention to this problem.

(24) $$\Delta z_t = z_t - z_{t-1} \text{ etc.}$$

and

(25) $$e_t = u_t - u_{t-1} .$$

Results from equation (23) are also reported in Table 4. The residuals u_t in equation (20) and the residuals e_t in equation (23) cannot both be serially independent. Yet neither of the two sets of Durbin-Watson statistics indicates serial correlation.

TABLE 4
Regression Coefficients for Equations (20) and (24)
(Standard Errors in Parentheses)

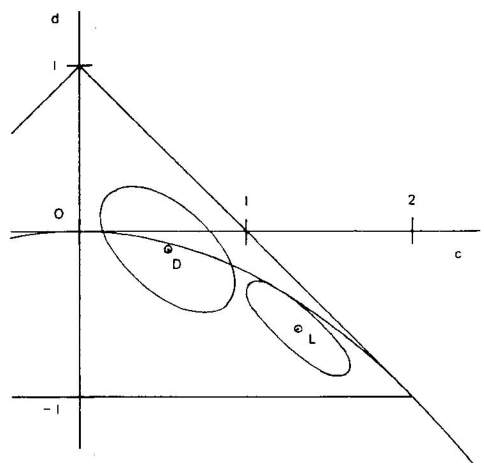
Variable	a	b	c	d	D-W	h
Log Real Income	1.40 (0.36)	0.0107 (0.0028)	1.31 (0.12)	−0.59 (0.11)	2.21	−1.12
Δ Log Real Income		0.0231 (0.0097)	0.53 (0.16)	−0.11 (0.15)	1.98	undefined
Log Real Investment	1.45 (0.29)	0.0337 (0.0071)	0.93 (0.13)	−0.50 (0.12)	2.32	−2.18
Δ Log Real Investment		0.0416 (0.0540)	0.30 (0.16)	−0.17 (0.15)	2.05	undefined
Log Real Consumption	2.13 (0.40)	0.0172 (0.0032)	0.80 (0.15)	−0.25 (0.11)	2.15	−1.97
Δ Log Real Consumption		0.0248 (0.0074)	0.31 (0.16)	0.01 (0.15)	1.90	undefined
Log Real Income (quarterly)	0.72 (0.26)	0.0008 (0.0003)	1.36 (0.09)	−0.45 (0.09)	2.12	−1.56
Δ Log Real Income (quarterly)		0.0054 (0.0015)	0.38 (0.10)	−0.07 (0.10)	1.95	undefined

Durbin's h statistic is meant to avoid the bias of the Durbin-Watson statistic when lagged dependent variables are present. If the disturbances are serially independent, this h statistic is asymptotically distributed normally with zero mean and unit standard deviation. It may be biased for small samples, but does not have the asymptotic inconsistency of the Durbin-Watson statistic. This h statistic is shown in Table 4. It is significant at the .95 level for equation (20) using investment data, but not for any of the other regressions. In fact, for all the equation (24) estimates, it is actually undefined, taking on imaginary values. This leads us to suspect that the sample sizes used are perhaps not yet large enough for the asymptotic distribution of the statistic to have set in.

Figures 6, 7, 8, and 9 show .95 confidence regions for c and d for the regressions in Table 4. In many cases the point estimates indicate stable cycles. However, the discrepancies between the confidence regions for equation (20) and (23) estimates using the same data leads us to doubt the validity of any of the estimates. If we believed these regression results, for annual income data we would be able to reject *all* values of c and d at the .975 level, while for quarterly income data we could reject all values at the .999 level. The investment and consumption ellipses overlap somewhat, but we could still easily reject both point estimates. Apparently one or the other, or both, regressions suffer substantially from the inconsistency Griliches describes. We therefore do not feel that any conclusions about the cyclicity or non-cyclicity of the time series investigated are warranted.

FIGURE 6

Estimates of c and d using annual real income data. "L" represents levels equation (20). "D" represents first differences equation (23). Ellipses are .95 confidence regions.



Assuming for the sake of argument that the regressions with the cyclic point estimates do represent underlying cyclic processes, it is interesting to compute the period using (14). This is shown in Table 5, together with the upper and lower bounds of a .95 confidence interval computed by applying the formula for asymptotic variances (Goldberger 1964, 122-25) to the cotangent of the transient frequency

$$\omega_T = \arccos[c/2(-d)^{1/2}] \tag{26}$$

on which (14) is based. This procedure implicitly assumes that the process is definitely cyclic, with a period in the permissible range of 2 years to infinity (or 6 months to infinity using quarterly data). Because of the non-linear transformation, the point estimate is not at the center of the interval. The asymptotic variance formula is accurate only when the variances in question are quite small. The derived confidence

TABLE 5

Values Derived From Regression Equations (20) and (23)

Variable	Period in years			Q	g
	lower bound	point estimate	upper bound		(%/yr)
Log Real Income	7.7	11.6	15.7	2.3	3.9
Δ Log Real Income	2.2	9.9	37.9	1.0	4.0
Log Real Investment	5.7	7.4	9.2	2.6	5.9
Δ Log Real Investment	3.7	5.3	7.0	1.5	4.8
Log Real Consumption	5.2	9.6	14.9	1.3	3.8
Δ Log Real Consumption		(non-cyclic point estimate)			3.6
Log Real Income (quarterly)		(non-cyclic point estimate)			3.7
Δ Log Real Income (quarterly)		(non-cyclic point estimate)			3.9

FIGURE 7

Estimates of c and d using annual real investment data. "L" represents levels equation (20). "D" represents first differences equation (23). Ellipses are .95 confidence regions.

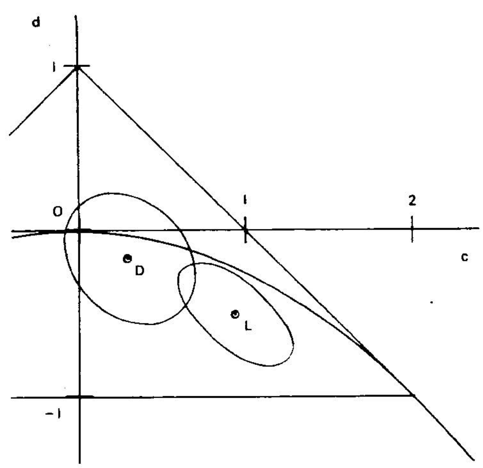


FIGURE 8

Estimates of c and d using annual real consumption data. "L" represents levels equation (20). "D" represents first differences equation (23). Ellipses are .95 confidence regions.

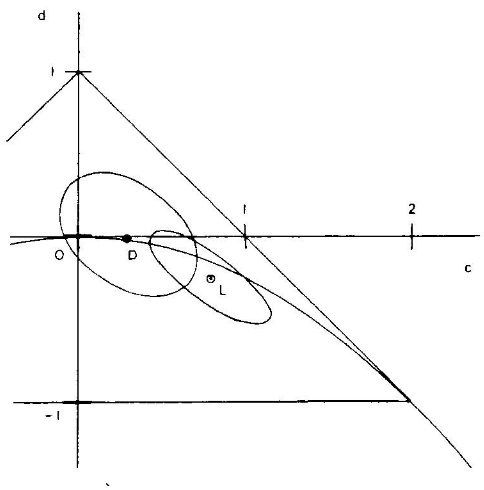
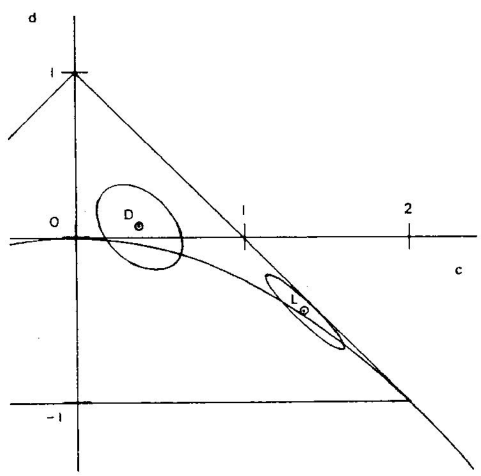


FIGURE 9

Estimates of c and d using quarterly real income data. "L" represents levels equation (20). "D" represents first differences equation (23). Ellipses are .95 confidence regions.



intervals are so broad that this assumption clearly must not hold, and therefore the true positions of the upper and lower bounds are not known. Nevertheless, the upper and lower bounds shown are sufficiently valid to indicate the utter inaccuracy of the point estimates.

A measure of the degree of damping of a harmonic oscillator that is often used in physics is its "Q" (Feynman et al. 1963, I, 23.5, 24.2). This is defined as the ratio of the oscillator's resonance frequency to its damping coefficient. Roughly speaking, it gives the number of radians the oscillator goes through before its amplitude is attenuated by a factor of $1/e$. In terms the economist can comprehend, a handball, which is relatively bouncy, has a Q of roughly 20 when dropped on a hard floor. A squash ball, which is relatively dead, has a Q of roughly 5.[14] If the business cycle is a respectable cycle, it should have a Q more like 20 than like 5.

In order to compute the Q of a difference equation like (20) or (23), we need its resonance frequency ω_R, the frequency the corresponding second order differential equation would have if its first order (damping) term were eliminated. Several conventions exist for going from difference equations to differential equations. Following one of them, we have

$$\omega_R = \arccos[c/(1-d)] . \tag{27}$$

As $Q \rightarrow \infty$ $(d \rightarrow -1)$, the resonance frequency approaches the transient frequency of equation (26).

After t units of time, the cycles will be attenuated by a factor of

$$(-d)^{t/2} = e^{-\log(-1/d)t/2} = e^{-\lambda t} \tag{28}$$

where

$$\lambda = \frac{1}{2}\log(-1/d) . \tag{29}$$

Combining (26) and (29) we have

$$Q = \omega_T/\lambda = 2\arccos[c/(1-d)]/\log(-1/d). \tag{30}$$

Values of Q for our autoregression equations are shown in Table 5. They are very low, ranging from 1.0 to 2.6. This means that the economy rebounds on the first bounce only as much as a squashball does on the second, third, fourth, or even fifth bounce. Even if we are willing to accept the cyclic estimates, the cycles are so highly damped as to have little practical consequence.

In a classic study, Guy H. Orcutt [1948] found that the equation

$$y_t = y_{t-1} + 0.3(y_{t-1} - y_{t-2}) + u_t \tag{31}$$

14. These illustrative figures were based on the attenuation in the first bounce, allowing that it represented 2π radians. Strictly speaking, however, a bouncing ball does not have the constant period of a true harmonic oscillator.

fits the 52 series in Tinbergen's study of business cycles fairly well. Equation (31) implies $c = 1.3$ and $d = -0.3$, a combination of parameters which lies outside our confidence ellipses shown in Figures 6-9. This inconsistency is explained by the fact that Orcutt did not allow the possibility of a trend term such as we admitted in (20). We tried forcing $a = b = 0$ in (20), and obtained estimates of c and d not significantly different from Orcutt's. However, since it is plausible that there is a trend, and since a and b entered significantly into (20), we must reject Orcutt's equation.[15]

SPECTRAL ANALYSES

Another approach to investigating cyclicity in time series is spectral analysis. Two studies along this line are especially noteworthy.

Adelman (1965) specifically investigates the so-called long swing or Kuznets cycle, which is supposed to have duration 10 to 20 years. She finds no evidence of these long swings.

Howrey (1968), using a different procedure for eliminating trend than Adelman did, also finds no evidence of the Kuznets cycle. However, he does find significant spectral peaks at 3.4 and 5.6 years in GNP, provided the number of lags used to compute the correlogram is greater than 20. For a few disaggregated series, he finds significant peaks even with only 20 lags.

Howrey's findings of multiple peaks in GNP is interesting in view of the fact that our likelihood ratio test tends to be confounded by multiple periodicities. The same is true of the second order autoregression approach, unless it can be appropriately disaggregated. It is further interesting that both his peaks lie outside the confidence interval for the cycle period shown in Table 5 for the annual income levels regression.

No econometric technique is foolproof, and spectral analysis, like the two approaches we have applied above, has its own problems. One is the fact that it works properly only on stationary time series, so that it is very sensitive to how trend is eliminated. It requires a very large sample to work well. It is also sensitive to any smoothing, interpolation or seasonal adjustment the data may have undergone.[16]

15. The issue of trend was raised in a comment by M. H. Quenouille published along with Orcutt's paper (Orcutt 1948, 51-52). However, Orcutt did not address this comment in his reply to the other commentators. Perhaps Orcutt did not allow for trend because over Tinbergen's sample period, 1919 to 1932, most of the series actually end up lower than they begin (Tinbergen 1939, II, 208-9). Nelson (1972, 905) estimates an income equation similar to Orcutt's which incorporates a trend term, but still forces $c + d = 1$, which precludes cycles. Nelson's nondurable consumer expenditure equation (1972, 915) is of the same form as (23), and gives a non-cyclic, stable point estimate. See also Evans (1974, A-8).

16. I am grateful to Irma Adelman for pointing out this last problem. Ranson (1974, 6.29-6.32) finds little evidence of systematic cycles in the correlogram of quarterly real output data, 1947-I to 1972-III. Anderson and Duffy [1972] find strong spectral peaks in the 2½ to 3 year and 8 year range using 19th century U.S. Balance of Payments statistics, but do not test for significance.

CONCLUSIONS

We have introduced a technique for testing whether economic activity exhibits systematic cycles or is essentially random. Its chief virtue is its simplicity, since it requires only turning point data and is relatively easy to compute. Using it we were unable to reject the Monte Carlo cycle hypothesis. Apparently the business "cycle" exists only in the eye of the beholder.

For comparison, we also investigated a second order autoregression with trend. Some of the point estimates indicated cycles, and were significantly outside the non-cyclic region. However, this sort of regression is subject to undetectable inconsistency, so the results must be interpreted with caution. Indeed, contradictory results were obtained using the data in levels and first differences form. Furthermore, even if we accept that there are such cycles, their period is so uncertain and they are so highly damped that they would be of little practical importance.

Of greater potential interest is the spectral analysis literature, which we mention, but to which we have nothing to add at the present time. This literature suggests that there may be multiple periodicities.

We hope our findings will sustain interest in the question of whether business fluctuations are in any meaningful sense "cyclic." Until this issue is settled, the expression "business cycle" should be avoided, since it prejudges the question. We suggest that "business fluctuation" be used instead to denote the general ups and downs of the economy.

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