• Exam time: 50 minutes
• Books and notes are not allowed on this exam.
• There are ten questions on ten pages. Make sure that you have all sheets.
• Each question is worth 10 points.
• You must BOTH place the letter of your answer choice in the box provided and also circle the corresponding answering choice.
• There is no partial credit. Full credit will only be given if your circled answer choice matches the answer choice in the box and is correct.
• The following calculators are permitted: BA-35, BA II Plus, BA II Plus Professional, TI-30Xa, TI-30X II (IIS solar or IIB battery), TI-30XS MultiView (or XB battery)
• The fact $\lim_{x \to \infty} x^n e^{-x} = 0$ can be used when necessary without justification.
1. Let $X$ be a Poisson random variable with mean $\lambda$. If $P(X = 1|X \leq 1) = 0.8$, find the value of $\lambda$.

   (A) 4
   (B) $-\ln 2$
   (C) 0.8
   (D) 0.25
   (E) $-\ln 0.8$
2. The time between “Likes” for a post on Instagram has an exponential distribution with a variance equal to 9. Given that it has been more than 4 hours since the last “Like”, find the probability that the time between the last “Like” and the next “Like” will be more than 10 hours.

(A) $e^{-30}$

(B) $e^{-18}$

(C) $e^{-6}$

(D) $e^{-2}$

(E) $e^{-1}$
3. Your MA/STAT250 instructor’s three point shot attempts are independent and he has a remarkable 40% chance of success at NBA range. If selected for a contest during half-time of a Final Four game, find the probability that he misses exactly two shot attempts before his fourth successful shot.

\[
\begin{align*}
(A) & \quad 0.0027 \\
(B) & \quad 0.0922 \\
(C) & \quad 0.2074 \\
(D) & \quad 0.2239 \\
(E) & \quad 0.3583 
\end{align*}
\]
4. The normal random variable $X$ has mean 79 and a variance of 25. Let $a, b$ be constants such that $P(79 - a \leq X \leq 79 + b) = 0.6463$. Given $P(X \geq 79 + b) = 2P(X \leq 79 - a)$, find the value of $b$. (Round to the nearest tenth)

(A) 1

(B) 1.4

(C) 2.4

(D) 3.1

(E) 3.6
5. Let $X$ and $Y$ be independent random variables with marginal densities $f_X$ and $f_Y$ and conditional densities $f_{X|Y}$ and $f_{Y|X}$. Which of the following are necessarily true?

I. $Var_{Y|X}(Y|X = x) = Var(X)$

II. $\int_{-\infty}^{x} \int_{-\infty}^{y} f_{X|Y}(t_1|Y = y) f_Y(t_2) dt_1 dt_2 = \left( \int_{-\infty}^{x} f_X(t_1) dt_1 \right) \left( \int_{-\infty}^{y} f_Y(t_2) dt_2 \right)$

III. $\rho_{xy} \leq P(X < Y)$

(A) I Only
(B) II Only
(C) III Only
(D) I and II
(E) II and III
6. For random variables $X, Y,$ and $Z,$ if it is known that \( \text{Var}(X) = \text{Var}(Y) = 1, \) 
\( \text{Var}(Z) = 2, \) \( \text{Cov}(X, Y) = -1, \) \( \text{Cov}(X, Z) = 0, \) and \( \text{Cov}(Y, Z) = 1. \) 
Find the covariance between $X + 2Y$ and $Y + 2Z.$

(A) 2  
(B) 3  
(C) 5  
(D) 7  
(E) 8
7. Random variables $X$ and $Y$ are jointly distributed on the region $0 \leq y \leq x$, $y \leq 1$, $x \leq 2$. Which of the following represents $P(X + Y \leq 1)$?

(A) $\int_0^5 \int_y^{1-y} f(x, y) \, dx \, dy$

(B) $\int_0^5 \int_{1-y}^y f(x, y) \, dx \, dy$

(C) $\int_0^1 \int_{1-y}^y f(x, y) \, dx \, dy$

(D) $\int_0^5 \int_y^1 f(x, y) \, dx \, dy$

(E) $\int_5^1 \int_{1-y}^y f(x, y) \, dx \, dy$
8. Let $X$ and $Y$ be continuous random variables described by the joint probability density function
\[ f(x, y) = \begin{cases} 
\frac{1}{3}x - y + 1 & 1 \leq x \leq 2, 0 \leq y \leq 1, \\
0 & \text{otherwise}. 
\end{cases} \]
Find the probability that $P\left(\frac{1}{2} < Y < \frac{3}{4}\right)$.

(A) $\frac{17}{96}$

(B) $\frac{7}{32}$

(C) $\frac{17}{32}$

(D) $\frac{57}{96}$

(E) $\frac{27}{32}$
9. Let $X$ and $Y$ be discrete random variables with joint probability function

$$p(x, y) = \begin{cases} \frac{2x+y}{12} & \text{for } (x, y) = (0, 1), (0, 2), (1, 2), (1, 3) \\ 0 & \text{otherwise.} \end{cases}$$

Find the conditional expectation $E(Y^2|X = 1)$.

(A) \(\frac{43}{9}\)

(B) \(\frac{55}{9}\)

(C) \(\frac{61}{9}\)

(D) \(\frac{64}{9}\)

(E) \(\frac{79}{9}\)
10. Let the random variables $X$ and $Y$ have joint moment generating function $M(t_1, t_2) = \frac{1}{1-t_1-t_2+t_1t_2}$ for $t_1, t_2 < 1$. Compute $E(XY) - Var(X)$.

(A) $-1$

(B) $-\frac{2}{3}$

(C) 0

(D) $\frac{1}{2}$

(E) 1