This document is an informal record compiled by me of most of the external (that is, not OSU-based) speakers in the OSU Math Logic Seminar from AY10–11 to AY19–20 in reverse chronological order of academic year. (As the seminar organizer, I had to keep good records only of speakers who received travel funding. As of this writing, local speakers still show up on a search of the Math Dept webpage for “logic seminar”.)

Chris Miller

OSU Math

September 17, 2024

AY 19–20

October 24, 2019 (rescheduled from Oct 22)
Title: An application of o-minimality in mathematical economics
Speaker: Charles Steinhorn - Vassar

Abstract: This talk deals with preference and utility theory in the context of o-minimal expansions of the ordered field of real numbers. We give a description of all preferences that can be defined in such a structure and when such preferences admit a utility function.

October 29, 2019, 1:50PM - 2:50PM
Title: Pre-H-fields and model companions

Speaker: Nigel Pynn-Coates - UIUC

Abstract: Pre-H-fields, introduced by Aschenbrenner and van den Dries in their work on the model theory of transseries and Hardy fields, provide a framework for studying the asymptotic behavior of real-valued functions at infinity from an algebraic and model-theoretic perspective. Together with van der Hoeven they showed that the theory of pre-H-fields has a model companion that is almost the theory of transseries, in particular, the model companion is "exponentially bounded" in a way that can be made precise. I will discuss a theory of pre-H-fields having a model companion that is not exponentially bounded.

November 26, 2019, 1:50PM - 2:50PM
Title: An Introduction to HT-fields
Speaker: Elliot Kaplan - UIUC

Abstract: H-fields were introduced by Aschenbrenner and van den Dries in their study of Hardy fields and fields of transseries. Together with van der Hoeven, they showed that the theory of H-fields has a model companion. In this talk, I will introduce the class of HT-fields, which are real closed H-fields equipped with additional compatible o-minimal structure. Key examples are Hardy fields of o-minimal expansions of the real field and the differential field of logarithmic-exponential transseries expanded by restricted analytic functions. I will conjecture what the model companion for the theory of HT-fields might be, and present some partial progress towards proving this conjecture.

December 3, 2019, 1:50PM - 2:50PM
Title: When does a subgroup of the real numbers embed nicely into a "generic" subgroup of the reals?

Speaker: Alexi Block Gorman - UIUC

Abstract: Model theorists have created examples and non-examples of interesting properties by thinking about "generic" sets, substructures, and reducts living in models whose theories are otherwise tame. Here, being generic means there is a witness from that set and from its complement for every formula that only uses existential quantifiers to define a "nice" (e.g. open and infinite) set. Intuitively, a set would only be called generic if both it and its complement intersect every infinite set in any arity of the ambient structure unless there is a really good reason it should not. In this talk, we will focus on expansions of o-minimal structures (for our purposes, an expansion of the reals in which all unary definable sets are a finite unions of points and intervals) and we will characterize when the theory of the reals expanded by a dense and codense divisible subgroup embeds nicely into a generic subgroup. The characterization is geometric in nature, and we can apply the characterization to both additive subgroups of the reals and multiplicative subgroups of the reals without zero. We will discuss an array of interesting examples and non-examples that highlight the versatility of the characterization.

January 28, 2020, 1:50PM - 3:00PM
Title: Model-theoretic techniques in query learning

Speaker: Hunter Chase - UI Chicago

Abstract: Several notions of complexity of set systems correspond both with model-theoretic dividing lines and notions of machine learning. We describe a new connection between equivalence query learning and stable formulas without the finite cover property. This is joint work with James Freitag.

February 25, 2020, 1:50PM – 3:00PM
Title: Characterizing some classes of rings via superstability
Speaker: Marcos Mazari-Armida - Carnegie-Mellon

Abstract: I will show how superstability of certain classes of modules can be used to characterize some well-studied classes of rings, among them noetherian rings. None of the classes of modules that I will consider are axiomatizable by a complete first-order theory and some of them are not even first-order axiomatizable, but they are all Abstract Elementary Classes. This new way of looking at classes of modules as AECs will be emphasized as I think it can have interesting applications.

March 3, 2020, 1:50PM – 3:00PM
Title: Borel structures on the space of left orderings

Speaker: Filippo Calderoni - University of Illinois at Chcago

Abstract: In this talk I will present some recent results on left-orderable groups and their interplay with descriptive set theory. We shall discuss how Borel classification can be used to analyze the space of left-orderings of a given countable group modulo the conjugacy action. In particular we shall see that if G is a countable nonabelian group, then the conjugacy relation on LO(G) is a universal countable Borel equivalence relation. This is joint work with A. Clay.

AY 18–19

September 18, 2018, 1:45PM - 2:45PM
Title: First-Order Logic with Isomorphism

Speaker: Dimitrios Tsementzis (Rutgers University)

Abstract: The Univalent Foundations (UF) is a proposed foundation for mathematics that takes as primitive a notion of space (rather than a notion of set). A logic for UF would be a formal system which expresses theories formalized in terms of spaces, just as first-order logic expresses theories formalized in terms of sets. After a quick introduction to UF, I will proceed to develop such a logic, and sketch a proof that it is sound and complete with respect to its spatial semantics.

December 4, 2018, 1:45PM - 2:45PM
Title: Characterizing Ordered Abelian Groups in the NIP Hierarchy

Speaker: Alf Dolich (CUNY)

Abstract: The independence property gives a rough dividing line between "wild" and "tame" theories. I will focus on the tame side of the divide, namely those theories without the independence property, generally referred to as the NIP theories. The class of NIP theories can further be divided into a hierarchy of notions, given by dp-rank, of increasing strength and hence increasing tameness. It is of considerable interest to characterize which ordered fields can be found in any of these levels. In attempting to address this problem one is quickly lead to the problem of attempting to characterize which ordered Abelian groups may be found in these various levels of the NIP hierarchy. In this talk I will give a simple algebraic description of the ordered Abelian groups in any level of the hierarchy.

January 29, 2019, 1:45PM - 3:00PM
Title: Interpolative Fusions

Speaker: Alex Kruckman (University of Indiana Bloomington)

Abstract: Fix languages L and L' (possibly non-disjoint). An structure M in the union of these languages is said to be interpolative if whenever X is an L-definable set in M and X' is an L'-definable set in M, then X and X' intersect unless they are separated by disjoint definable sets in the intersection of L and L'. If T is an L theory and T' is an L' theory, we say that a theory T* is the interpolative fusion of T and T' if it axiomatizes the class of interpolative models of the union of T and T'. If morever T and T' are model-complete, this is exactly the model companion of the union theory. Interpolative fusions provide a unified framework for studying many examples of "generic constructions" in model theory. Some, like structures with generic predicates, or algebraically closed fields with several independent valuations, are explicitly interpolative fusions, while others, like structures with generic automorphisms or fields with generic operators, are bi-interpretable with interpolative fusions. In joint work with Erik Walsberg and Minh Tran, we study two basic questions: (1) When does the interpolative fusion exist, and how can we axiomatize it? (2) How can we understand properties of the interpolative fusion T* in terms of properties of the theories T, T', and their intersection?

February 5, 2019, 1:45PM - 3:00PM
Title: Impredicative encodings in homotopy type theory

Speaker: Steve Awodey (Carnegie Mellon)

Abstract: We present some recent results on impredicative encodings of inductive types (including higher inductive types) in HoTT. It is well-known that encoding inductives using higher-order quantification (i.e. quantification over all types) provides a potential theoretical and practical simplification of the type system. Using the further resources available in HoTT permits a sharpening of the usual System F style, impredicative encodings, but this begs the question of whether impredicativity is compatible with univalence. We give a realizability model using a combination of topos-theoretic, homotopical, and recent cubical methods. (Joint work with Jonas Frey and Pieter Hofstra.)

March 5, 2019, 1:45PM - 3:00PM
Title: Generalizing VC dimension to higher arity

Speaker: Henry Towsner (University of Pennsylvania)

Abstract: The notion of bounded VC dimension is a property at the intersection of combinatorics and probability. This family has been discovered repeatedly and studied from various perspectives - for instance, in model theory, theories with bounded VC dimension are known as NIP (the theories which do Not have the Independence Property). One useful property is that graphs with bounded VC dimension are the graphs that can be always be finitely approximated in a random-free way: graphs with bounded VC dimension satisfy a strengthening of Szemeredi's Regularity Lemma in which the densities between the pieces of the partition are either close to 0 or close to 1. The generalization of VC dimension to higher arity, known in model theory as k-NIP for various k, has been less well-studied. We summarize some known facts about this generalization, including a new result (joint with Chernikov) showing k-NIP hypergraphs have a similar kind of approximation with only lower order randomness.

March 19, 2019, 1:45PM - 3:00PM
Title: Schnorr randomness and Levy's Upward Convergence Theorem

Speaker: Sean Walsh (UCLA)

Abstract: Much recent work in algorithmic randomness has concerned characterizations of randomness notions in terms of the almost-everywhere behavior of suitably effectivized versions of functions from analysis or probability. In this work, we consider Levy's Upward Martingale Convergence Theorem from this perspective. Generalizing a result of Pathak-Rojas-Simpson from Euclidean space to Cantor space, we show that Schnorr randoms are precisely the points at which the conditional expectations of L^1-computable functions converge to their true value, as one passes along sub-sigma-algebras generated by longer and longer strings. This result has natural applications to formal epistemology and the philosophical interpretation of probability: for, the natural Bayesian interpretation of this result is that belief, in the form of an agent's best estimates of the true value of a random variable, aligns with truth in the limit, under appropriate effecitivity assumptions. We also consider other convergence theorems and other randomness notions. No prior knowledge of algorithmic randomness will be assumed, and we will remind the audience of the various martingale convergence theorems which we are effectivizing. This is joint work with Simon Huttegger (UC Irvine) and Francesca Zaffora Blando (Stanford).

March 26, 2019, 1:45PM – 3:00PM
Title: PAC fields with a strong notion of model completeness

Speaker: Greg Cousins (Notre Dame)

Abstract: A field K is pseudo-algebraically closed (PAC) if every absolutely irreducible variety defined over K has a K-rational point. I will show that if K is PAC, then simplicity of its theory (in the model-theoretic sense) is equivalent to a certain strong form of model completeness.

AY 17–18

Tuesday, October 24, 2017 - 1:45pm to 2:45pm

Title: Algebraically closed fields with a multiplicative cyclic ordering

Speaker: Chieu Minh Tran (University of Illinois Urbana Champaign)

Abstract: We study the model theory of structures (F,<) where F is the algebraic closure of the field of p elements and < is a cyclic ordering on the nonzero elements F* induced by an injective group homomorphism h: F*--> C*. Various model-theoretic properties of the structure turn out to be consequences of number-theoretic behaviors of the character map h. The results obtained loosely answer a question by van den Dries, Hrushovski, and Kowalski and form parts of a program to investigate the model-theoretic properties of structures where there is a presence of randomness.

Tuesday, November 7, 2017 - 1:45pm to 2:45pm

Title: Distal and non-distal expansions of the reals

Speaker: Travis Nell (University of Illinois Urbana-Champaign)

Abstract: Pierre Simon in "Distal and non-Distal NIP Theories" isolated a class of NIP (or dependent) theories that can be considered purely unstable. For example any o-minimal theory is distal and any stable theory is non-distal. Distal theories usually include some sort of ordering. However, algebraically closed valued fields are non-distal, as the structure on the residue field is stable. The class of distal theories is not closed under reducts, but some useful consequences of distality do pass to reducts. Thus, even for a non-distal structure, the question of whether it can be expanded to a distal structure is of interest. In this talk, I will survey what is known about distality and distal expansions in the context of expansions of the reals.

Tuesday, February 20, 2018 - 1:45pm to 2:45pm

Title: Hilbert's Tenth Problem for Subrings of the Rationals

Speaker: Russell Miller (CUNY)

Abstract: For a ring R, Hilbert's Tenth Problem HTP(R) is the set of all polynomials f in R[X_1,X_2,...] for which f=0 has a solution in R. In 1970, Matiyasevich completed work by Davis, Putnam, and Robinson to show that the original Tenth Problem of Hilbert, HTP(Z), is undecidable. On the other hand, the decidability of HTP(Q) remains an open question. We will examine this problem for subrings of the rational numbers, viewing these subrings as the elements of a topological space homeomorphic to Cantor space and connecting their Turing degrees and computability-theoretic properties to those of HTP(Q) itself.

Some of the work discussed is joint with Kramer, and some with Eisentraeger, Park, and Shlapentokh.

Tuesday, April 10, 2018 - 1:45pm to 2:45pm

Title: Realizability Semantics for Quantified Modal Logic

Speaker: Sean Walsh (UCLA, Dept. of Philosophy)

Abstract: In 1985, Flagg produced a model of first-order Peano arithmetic and a modal principle known as Epistemic Church's Thesis, which roughly expresses that any number-theoretic function known to be total is recursive. In some recent work ([1]), this construction was generalized to allow a construction of models of quantified modal logic on top of just about any of the traditional realizability models of various intuitionistic systems, such as fragments of second-order arithmetic and set theory. In this talk, we survey this construction and indicate what is known about the reduct of these structures to the non-modal language.

References: [1] B. G. Rin and S. Walsh. Realizability semantics for quantified modal logic: Generalizing Flagg's 1985 construction. The Review of Symbolic Logic, 9(4):752–809, 2016.

Tuesday, April 17, 2018 - 1:45pm to 2:45pm

Title: Limit cycles of planar vector fields, Hilbert's 16th problem and o-minimality

Speaker: Patrick Speissegger (McMaster University, Canada)

Abstract: Recent work links certain aspects of the second part of Hilbert's 16th problem (H16) to the theory of o-minimality. One of these aspects is the generation and destruction of limit cycles in families of planar vector fields, commonly referred to as "bifurcations". I will outline the significance of bifurcations for H16 and explain how logic--in particular, o-minimality--can be used to understand them well enough to be able to count limit cycles.

TWO CANCELLATIONS

Friday, March 2, 2018 - 12:30pm to 1:50pm

Title: Ultrapowers, asymptotic sequences, and classification

Speaker: Ilijas Farah (York University)

Abstract: Ultrapowers were discovered, apparently independently, by operator algebraists and logicians (in that order!) in the1950s. Since the early 1970s, the ultrapowers of separable operator algebras associated with nonprincipal ultrafilters on N have been one of the main tools in classification of von Neumann algebras and, more recently, C*-algebras. As all properties of such ultrapowers can be explained by two of their abstract properties, countable saturation and Los's theorem, it was to be expected that logic may play a role in the theory. I'll survey some of the progress in the past ten years and present some more recent results about the relation between the ultrapowers and asymptotic sequence algebras. (The latter are known to logicians as the reduced products associated with the Frechet ideal.)

Thursday, May 10, 2018 - 10:45am

Title: Impredicative encodings in homotopy type theory

Speaker: Steve Awodey (Carnegie-Mellon)

Abstract: We present some recent results on impredicative encodings of inductive types (including higher inductive types) in HoTT. It is well-known that encoding inductives using higher-order quantification (i.e. quantification over all types) provides a potential theoretical and practical simplification of the type system.

Using the further resources available in HoTT permits a sharpening of the usual System F style, impredicative encodings, but this begs the question of whether impredicativity is compatible with univalence. We give a realizability model using a combination of topos-theoretic, homotopical, and recent cubical methods. (Joint work with Jonas Frey and Pieter Hofstra.)

AY 16–17

Tuesday, November 29, 2016 – 1:50 to 2:50 pm

Title: Classifying expansions of the real field by complex subgroups

Speaker: Allen Gehret (University of Illinois Urbana-Champaign).

Abstract: The set of transseries is a large ordered valued differential field extending the reals that is equipped with additional operations such as integration, composition, exponentiation and taking logarithms. Transseries are of interest in several areas of mathematics, including asymptotic analysis, computer algebra, and surreal numbers. Aschenbrenner, van der Hoeven and van den Dries have shown that there is a good model theory; the proof is long and difficult. There is a substructure of interest, the "logarithmic" transseries. I will describe recent progress, as well as obstructions, toward proving model completeness (in an appproriate language) for this structure.

Tuesday, January 31, 2017 - 1:50pm to 2:50pm

Title: First order expansions of the ordered real additive group

Speaker: Erik Walsberg (University of Illinois Urbana-Champaign).

Abstract: I will describe the classification of first order expansions of the ordered real additive group according to the topology and geometry of their closed definable sets. We discuss two dividing lines: whether or not the structure defines an order with order type omega on a dense subset of an interval, and whether or not the structure can uniformly continuously surject small intervals onto [0,1]. Work in progress with Philipp Hieronymi.

Tuesday, February 28, 2017 - 1:50pm to 2:50pm

Title: Classifying expansions of the real field by complex subgroups

Speaker: Erin Caulfield (University of Illinois Urbana-Champaign)

Abstract: We construct two classes of finite rank multiplicative subgroups of the complex numbers such that an expansion of the real field by one such group is model-theoretically well-behaved. As an application we show that a classification of expansions of the real field by cyclic multiplicative subgroups of the complex numbers due to Hieronymi does not even extend to expansions by subgroups with two generators. We also discuss some progress towards a new classification of expansions of the real field by finitely generated multiplicative subgroups of the complex numbers.

Tuesday, March 7, 2017 - 1:50pm to 2:50pm
Title: Mining effective information from nonconstructive proofs in differential algebra

Speaker: William Simmons (University of Pennsylvania)

Abstract: Ultraproducts and other nonconstructive tools often yield existence results without giving explicit values. We examine the interplay of such arguments with "proof mining" techniques that systematically extract effective information even when it is not apparent. Our main application is to differential algebra, where the existence and nature of uniform bounds are more elusive than in the algebraic case. This is joint work with Henry Towsner.

Tuesday, April 11, 2017 - 1:50pm to 2:50pm
Title: Smooth parameterization in o-minimal structures

Speaker: Margaret Thomas (University of Konstanz)

Abstract: The counting theorem of Pila and Wilkie opened up one of the most important developments in model theory in recent years. It provides a bound on the density of rational points for sets definable in o-minimal expansions of the real field, a result which has had several stunning number-theoretic applications (e.g. to the Manin-Mumford and André-Oort Conjectures). Central to the proof of the theorem is an o-minimal version of Yomdin-Gromov parameterization, a type of `smooth parameterization', the decomposition of sets using functions with controlled higher-order derivatives. We will discuss various different directions of current research relating smooth parameterization and o-minimality. One direction is the pursuit of an effective version of the Pila-Wilkie Counting Theorem, and our first results in this direction are for certain surfaces described by so-called Pfaffian functions. Another direction is the study of `mild parameterization' in o-minimal structures. This is aimed towards a conjecture of Wilkie, which proposes a significant sharpening of the Pila-Wilkie bound for sets definable in the (o-minimal) real exponential field, and which would be established with sufficiently uniform mild parameterization in this setting.

Tuesday, April 18, 2017 - 1:50pm to 2:50pm
Title: Model theory and Painleve equations

Speaker: James Freitag (University of Illinois Chicago)

Abstract: Painleve equations are certain order two nonlinear differential equations which were isolated around the beginning of the last century by Painleve, Gambier, and Fuchs for reasons related to classical analytic problems. The equations arise in a variety of applications from physics to Diophantine geometry. In this talk, we will discuss how model theory can be used to prove transcendence results for solutions of Painleve equations. The talk will be accessible to a general audience.

Wednesday, June 7, 2017 - 2:30pm to 4:00pm

Title: Asymptotic classes, measurable structures, and beyond

Speaker: Charles Steinhorn (Vassar)

Abstract: Macpherson and I, together with several of Macpherson's students, initiated the study of asymptotic classes of finite structures and measurable structures in an effort to develop a model theory for classes of finite structures that reflects contemporary infinite model theoretic themes. In this talk,

we first review some of this work. Then we introduce current research that generalizes these concepts to what we call multidimensional asymptotic classes and generalized measurable structures. This most recent work is with Macpherson, S. Anscombe, and D. Wolf.

AY 15–16

Tuesday, October 13, 2015 - 1:45pm to 2:45pm
Title: Model theory of generalized Urysohn spaces

Speaker: Gabriel Conant, Notre Dame

Abstract: Many well known examples of homogeneous metric spaces and graphs can be viewed as analogs of the rational Urysohn space (for example, the random graph as the Urysohn space with distances {0,1,2}). In this talk, I consider the R-Urysohn space, where R is an arbitrary ordered commutative monoid. I will first construct an extension R* of R, such that any model of the theory of R-Urysohn space (in a relational language) can be given the structure of an R*-metric space. I will then characterize quantifier elimination in this theory by continuity of addition in R*. Finally, I will characterize various model theoretic properties of the R-Urysohn space (e.g. stability and simplicity) using natural algebraic properties of R.

Tuesday, October 27, 2015 - 1:45pm to 2:45pm
Title: Some new logical zero-one laws

Speaker: Caroline Terry (UI Chicago)

Abstract: Let L be a finite first-order language and, for each positive integer n, let F(n) be a set of L-structures with underlying set {1,..., n}. We say that the union of the F(n) has a zero-one law if, for every L-sentence, the limit as n goes to infinity of the proportion of elements in F(n) that satisfy the sentence is either zero or one. In this talk we give a brief overview of the history of this topic, then present some new examples of families with zero-one laws. This is joint work with Dhruv Mubayi.

Tuesday, November 3, 2015 - 1:45pm to 2:45pm
Title: The level-by-level strength of Borel determinacy

Speaker: Sherwood Hachtman (University of Illinois- Chicago)

Abstract: All infinite games with Borel winning condition are determined. By celebrated results of D.A. Martin and H. Friedman, this fact is provable in ZFC, but the base theory ZFC cannot be substantially weakened: any proof of Borel determinacy requires an appeal to the axioms of Power Set and Replacement. This correspondence holds "level-by-level": proving determinacy for sets in the level n+4 of the Borel hierarchy requires (roughly) n+1 uncountable infinities. But what ambient set theory is strictly necessary? We present a sharpest-possible refinement of the Martin/Friedman results by isolating a family of weak reflection principles whose strength matches up level-by-level with that of Borel determinacy. This furnishes an analysis of the complexity of the simplest winning strategies for Borel games in terms of the level of L at which these strategies are born. Formulating these results in the setting of higher-order reverse mathematics, we separate open and clopen determinacy for games with moves of large type, extending a result of Schweber.

Tuesday, December 1, 2015 - 1:45pm to 2:45pm
Title: Reducibilities utilizing incomplete or imperfect information

Speaker: Greg Igusa (Notre Dame)

Abstract: In complexity theory, there is an empirically observed phenomenon, that sometimes a problem might be easy to solve in practice despite being proved to have a high complexity in the worst case. This discrepancy has been frequently dismissed as an unfortunate situation in which practice is not accurately predicted by theory, but there has been recent work in complexity theory to rigorously study this phenomenon.

In 1986, Levin introduced "average-case complexity." Then, in 2003, Kapovich, Miasnikov, Schupp and Shpilrain introduced "generic-case complexity." Generic-case complexity looks at the majority of instances of a problem while completely ignoring the most difficult ones. In particular, an unsolvable problem can have a well-defined generic-case complexity as long as there is an algorithm that can solve most instances of the problem.

In 2012, Jockusch and Schupp introduced "generic computability" to study precisely the recursion-theoretic content of these sorts of algorithms. A generic computation of a real is a computation that usually halts: a partial recursive function whose domain has density 1 (in the sense of asymptotic density on the natural numbers) such that the partial recursive function correctly computes the real on its domain.

Jockusch and Schupp also introduced a related notion: "coarse computability." A real is coarsely computable if there is a total computable function that is correct about that real on a set of density 1. (An algorithm that always halts, and usually gives correct answers.) To study the degree structure of generic computability, they also define "generic reducibility," a notion of reducibility in which oracles, like computations, are only forced to halt on density 1.

We define coarse reducibility, as well as several other reducibility notions, and we use these to illustrate the differences between incomplete and imperfect computations, and also to illustrate the ways in which replacing "density 1" with other notions of largeness affects the final reducibility. We explore these reducibilities, the relationships between them, and links between them and hyperarithmeticity, reverse mathematics, and randomness.

Tuesday, March 1, 2016 - 1:45pm to 2:45pm
Title: A closure property of derived models
Speaker: Trevor Wilson (Miami University)

Abstract: Woodin's derived model construction uses forcing from a large cardinal hypothesis to produce a model of set theory plus the axiom of determinacy (and minus the axiom of choice). This extends Solovay's construction of a model of set theory where every set of reals is Lebesgue measurable. One of the differences between these constructions is that Solovay models are closed under countable sequences, whereas for derived models, this kind of closure seems harder to obtain because one must be much more conservative in adding sets to derived models. We show (under the appropriately mild large cardinal hypothesis of an inaccessible limit of Woodin cardinals) that derived models are in fact closed under countable sequences of sets of reals.

Tuesday, March 22, 2016 - 1:45pm to 2:45pm
Title: Diophantine approximation, scalar multiplication and decidability
Speaker: Philipp Hieronymi (UIUC)

Abstract: It has long been known that the first order theory of the expansion (R,<,+,Z) of the ordered additive group of real numbers by just a predicate for the set of integers is decidable. Arguably due to Skolem, the result can be deduced easily from Buechi's theorem on the decidability of monadic second order theory of one successor, and was later rediscovered independently by Weispfenning and Miller. However, a consequence of Goedel's famous first incompleteness theorem states that when expanding this structure by a symbol for multiplication, the theory of the resulting structure (R,<,+,*,Z) becomes undecidable. This observation gives rise to the following natural and surprisingly still open question: How many traces of multiplication can be added to (R,<,+,Z) without making the first order theory undecidable? We will give a complete answer to this question when "traces of multiplication" is taken to mean scalar multiplication by certain irrational numbers. To make this statement precise: for b in R, let f_b: R -> R be the function that takes x to bx. I will show that the theory of (R,<,+,Z,f_b) is decidable if and only if b is quadratic. The proof rests on the observation that many of the Diophantine properties (in the sense of Diophantine approximation) of b can be coded in these structures.

Tuesday, May 17, 2016 - 1:45pm to 2:45pm

Title: Quasianalytic Ilyashenko algebras (continued)

Speaker: Patrick Speissegger (McMaster University)

Abstract: Taking up a topic I talked about at OSU a few years ago, I will recall the basic construction of a quasianalytic class whose functions have simple logarithmic series as asymptotic expansions. Motivated by a desired extension to several variables, I will then explain how this construction can be adapted to include series whose monomials are definable functions in the o-minimal structure R_an,exp. (Joint work with Zeinab Galal and Tobias Kaiser.)

AY 14–15

Tuesday, September 23, 2014 - 1:50pm to 2:50pm

Title: Dimensions of Definable Metric Spaces
Speaker: Erik Walsberg, UCLA

Abstract: I will discuss metric spaces which are definable in o-minimal expansions of the real field. I will describe a number of interesting examples of such spaces and discuss what I know and do not know about their topological, Hausdorff and Assouad dimensions.

Tuesday, October 28, 2014 - 1:50pm to 2:50pm
Title: Finite generating partitions for continuous actions of countable groups
Speaker: Anush Tserunyan (UIUC)

Abstract: Let a countable group G act continuously on a Polish space X. A countable Borel partition P of X is called a generator if the set of its translates {gA : g in G, A in P} generates the Borel sigma-algebra of X. For G=Z, the Kolmogorov-Sinai theorem gives a measure-theoretic obstruction to the existence of finite generators: they don't exist in the presence of an invariant probability measure with infinite entropy. It was asked by B. Weiss in the late 80s whether the nonexistence of any invariant

probability measure guarantees the existence of a finite generator. We show that the answer is positive for an arbitrary countable group G and sigma-compact X (in particular, for locally compact X). We also show that finite generators always exist for aperiodic actions in the context of Baire category (i.e. allowing ourselves to disregard a meager set), thus answering a question of A. Kechris from the mid-90s.

Tuesday, March 10, 2015 - 1:50pm to 3:20pm
Title: Second-order Reflection in Modal-Structural Interpretations (of set theories)
Speaker: Geoffrey Hellman (University of Minnesota)

Abstract: First, we review the man ideas of modal-structural (MS) interpretations of mathematical theories, especially in second-order formulations, e.g. Dedekind-Peano Arithmetic, and Real Analysis (with 2d-order LUB axiom), and set theories like Zermelo, ZFC, and extensions with (small) large cardinal axioms. Next we turn to the problem of introducing higher-order reflection principles into the MS versions of Z- (Zermelo less Infinity). (This choice will be explained). Even the usual motivation of such reflection has to be adjusted, since MS recognizes no “absolutely infinite” totalities such as “the universe of all sets" or "all the ordinals." Then we consider two attempts, one based on a translation scheme due to Putnam, but which runs into inconsistency; and a second which we claim is consistent relative to standard ZFC + e.g. an Erdös cardinal (borrowing from recent work of W.W.Tait and Peter Koellner). We close with discussion of significance of such results

Tuesday, April 14, 2015 - 1:50pm to 2:50pm
Title: Strong type theories: Their set and proof-theoretic sides
Speaker: Michael Rathjen (University of Leeds)

Abstract: There is tight fit between type theories à la Martin-Löf and constructive set theories such as CZF and its extension as well as classical Kripke-Platek set theory and extensions thereof. Moreover, the technology for determining their (exact) proof-theoretic strength was developed in the 1990s.

The situation is rather different when it comes to type theories (with universes) having the impredicative type of propositions Prop from the Calculus of Constructions that features in some powerful proof assistants. Aczel's sets-as-types interpretation into these type theories gives rise to rather unusual set-theoretic axioms: negative power set and negative separation. But it is not known how to determine the proof-theoretic strengths of intuitionistic set theories with such axioms via familiar classical set theories (though it is not difficult to see that ZFC plus infinitely many inaccessibles provides an upper bound).

The first part of the talk will be a survey of known results from this area. The second part will be concerned with the rather special proof-theoretic behavior of such theories.

Tuesday, April 21, 2015 - 1:50pm to 2:50pm
Title: Model theory and set theory
Speaker: John Baldwin (University of Illinois at Chicago)

Abstract: At least since Skolem’s formulation of his paradox, set theory and model theory have been intertwined. In contrast to Skolem, we investigate the methodological role of set theory in model theory. We will address several questions. In what ways is model theory entangled with cardinality, cardinal arithmetic, the replacement axioms, or large cardinal axioms?

This analysis responds to Maddy’s injunction, “The Second Philosopher sees fit to adjudicate the

methodological questions of mathematics – what makes for a good definition, an acceptable axiom, a dependable proof technique?–". This talk concentrates on the methodology of model theory as an aspect of the following more general program:

1. Formalization of specific mathematical areas is a tool for studying issues in the philosophy of mathematics (methodology, axiomatization, purity, categoricity and completeness);

2. The systematic comparison of local formalization of distinct areas is a useful tool for organizing and doing mathematics and the analysis of mathematical practice.

We will give some evidence of item ii) and illustrate that the acceptance of definitions in model theory was influenced by connections with other areas of mathematics.

Tuesday, May 5, 2015 - 2:00pm to 3:30pm
Title: A tame Cantor set
Speaker: Philipp Hieronymi (UIUC)

Abstract: Let R denote the real ordered field. Our focus here is on expansions of R by Cantor sets. For our purposes, a Cantor set is a non-empty, compact subset of the real line that has neither interior nor isolated points. We consider the following question due to Friedman, Kurdyka, Miller and Speissegger: is there a Cantor set K and a natural number N such that every set definable in (R,K) is Sigma_N^1? I will answer this question positively. In addition to using techniques from model theory, o-minimality and descriptive set theory and previous work of Friedman et al., the work presented in this talk depends crucially on well known results about the monadic second order theory of one successor due to Buechi, Landweber and McNaughton.

AY 13–14

Note: I was on sabbatical this AY. Tim Carlson filled in as seminar organizer, but he also had other significant administrative work. The result was fewer seminars than usual.

Degrees of relative provability

Mingzhong Cai (University of Wisconsin, Madison)

Abstract. We study the proof-theoretic strength of true Pi^0_2 sentences, which is equivalent to totality of recursive functions. Given two total recursive functions f and g with fixed algorithm, we say f is provably reducible to g if the totality of g proves the totality of f over some base theory. Using recursion-theoretic methods, we show various results, for example the density theorem (between every two degrees there is always a new one between), or more intuitively, there is no "minimal" unprovable Pi^0_2 sentence. We can also show that this degree structure is a distributive lattice with two possible "jump" operators. Part of the work is joint with Andrews, Diamondstone, Lempp and Miller.

Computable Algebra: A Personal Perspective

Chris Conidis (Vanderbilt)

Abstract. I will survey some of my recent results in Computable Algebra pertaining to computable aspects of Artinian rings, infinite dimensional vector spaces, and Euclidean domains.

March 18, 2014, 1:50PM - 2:50PM
Title: Computability Theoretic Reduction between Π12 Principles
Speaker: Denis Hirschfeldt, University of Chicago
Abstract: Many mathematical principles can be stated in the form "for all X such that $C(X)$ holds, there is a Y such that $D(X,Y)$ holds", where X and Y range over second order objects, and C and D are arithmetic conditions. We think of such a principle as a problem, where an instance of the problem is an X such that $C(X)$ holds, and a solution to this instance is a Y such that $D(X,Y)$holds. Examples of particular relevance to this talk are versions of Koenig's Lemma (such as KL and WKL) and of Ramsey's Theorem (such as $RTn2$). We'll discuss several notions of computability theoretic reducibility between such problems, and their connections with reverse mathematics. Among other things, I will explain how recasting the idea of "every ω-model of P is a model of Q" in terms of games allows us to define a notion of uniform reducibility from Q to P that permits the use of multiple instances of P to solve a single instance of Q. This is joint work with Carl Jockusch.

AY 12–13

Oct 2 2012 - 1:50pm - 2:50 pm

Remarks on locally-finite model theory

Cameron Hill (Notre Dame)

Abstract. In bringing ideas and results from "geometric" model theory to bear on questions from finitary discrete mathematics, there are two reasonably well-known approaches -- pseudo-finite theories (ultraproducts of finite structures) and generic structures (limits of amalgamation classes of finite structures). I will argue that the confluence of these -- theories of generic structures that are pseudo-finite via the same class -- is the "natural" setting for bringing discrete mathematics into model theory. As evidence, I will discuss (a selection from among) the appearance of well-quasi-orderings in amalgamation classes, quasi-finite axiomatizability, zero-one laws, and Ramsey theorems for classes of finite structures.

Oct 19 2012 - 1:50pm - 2:50 pm

From combinatorial complexity to triangulations of monotone families

Saugata Basu (Purdue)

Abstract. I will explain how to extend the combinatorial parts of certain well known bounds on the topology (the Betti numbers) of semi-algebraic sets to the general o-minimal setting and mention some applications of such bounds in discrete geometry. In the second part of the talk I will explain a result of Gabrielov and Vorobjov which reduces the problem of bounding the topology of arbitrary definable sets to that of compact ones, and show how it leads to the problem of proving the existence of triangulations compatible with monotone definable families. I will mention some partial results in this direction.

(The last part of the talk is joint work with A. Gabrielov and N. Vorobjov.)

Nov 6 2012 - 1:50pm - 2:50 pm

Projecting Prikry type forcings

Dima Sinapova (UI Chicago)

Abstract. The singular cardinal problem is the project to find a complete set of rules for the operation kappa-->2^\kappa for singular cardinals kappa. Consistency results about singular cardinals are obtained using Prikry type forcings. In this talk I will describe properties of these forcings. Then I will present a recent result about projecting Prikry forcings and a consistency result. This is joint work with Spencer Unger.

Nov 20 2012 - 1:50pm - 2:50 pm

Three tales about Weak Konig's Lemma

Jeff Hirst (Appalachian State University)

Abstract. This talk will explore results in reverse mathematics taken from three recent papers. Although the results are all related to Konig's Lemma restricted to 0-1 trees, the settings are very different. One result concerns dichotomy for real numbers. Another has to do with embeddings of algebraic field extensions, and the last is purely combinatorial in nature. The results come from collaborations with François Dorais, Damir Dzhafarov, Joseph Mileti, and Paul Shafer.

Jan 29 2013 - 1:50pm - 2:50 pm

The theory of tracial von Neumann algebras does not have a model companion

Isaac Goldbring (UI Chicago)

Abstract. In this talk, we will show that the theory of tracial von Neumann algebras does not have a model companion. In addition, we will show that a positive solution to the Connes Embedding Problem implies that there is no model complete theory of tracial von Neumann algebras. All functional analytic notions and most model-theoretic notions will be defined. This is joint work with Bradd Hart and Thomas Sinclair.

Apr 2 2013 - 1:50pm - 2:50 pm

Computability and uniformity in analysis

Jeremy Avigad (Carnegie-Mellon)

Abstract. Countless theorems of analysis assert the convergence of sequences of numbers, functions, or elements of an abstract space. Classical proofs often establish such results without providing explicit rates of convergence, and, in fact, it is often impossible to compute the limiting object or a rate of convergence from the given data. This results in the curious situation that a theorem may tell us that a sequence converges, but we have no way of knowing how fast it converges, or what it converges to.

On the positive side, it is often possible to "mine" quantitative and computational information from a convergence theorem, even when a rate of convergence is generally unavailable. Moreover, such information can often be surprisingly uniform in the data. In this talk, I will discuss examples that illustrate the kinds of information that can and cannot be obtained, focusing on results in ergodic theory.

Apr 9 2013 - 1:50pm - 2:50 pm

Model completeness of o-minimal fields with convex valuations

Jana Marikova (Western Illinois U)

Abstract. Let R be an o-minimal field and V a convex subring. The structure (R,V) is well-understood when R is a pure real closed field (Cherlin, Dickmann), and also, more generally, when V is a T-convex subring (van den Dries, Lewenberg). We continue here the investigation of a strictly larger class of structures (R,V), namely the ones where the corresponding residue field expanded by all residues of R-definable sets is o-minimal. This class of structures (R,V) includes for example all cases where V is the convex hull of the rationals in R. We make heavy use of the notion of separation (introduced by Baisalov and Poizat) in Morley sequences to prove a model completeness result for (R,V).

This is joint work with Clifton Ealy.

May 31 2013 - 1:30pm - 3:30 pm

Model theory of (partial) differential fields with an automorphism

Omar Leon Sanchez (U Waterloo)

Abstract. Suppose T is a first order theory. It is an interesting, and rather difficult, question to determine if the theory of models of T with a distinguished automorphism has a model companion TA. It is known that if T is stable, and the model companion TA exists, then TA has good dimension properties (in model theoretic lingo "TA is simple"). In this talk, we will specialize to the case when T is the theory of partial differentially closed fields, show that in this case TA exists and present some of its model theoretic properties.

AY 11–12

Nov 15 2011 - 3:30pm - 4:30 pm

Computational Complexity of Automatic Structures

Jacob Carson (Notre Dame)

Abstract. An automatic structure is one whose universe an relations can be recognized by a deterministic finite automaton. Although these objects are substantially simpler than Turing Machines, the structures they recognize can be surprisingly complicated. This talk outlines some of the previous results on automatic structures, and expands on some prior work on automatic equivalence relations.

Nov 22 2011 - 3:30pm - 4:30 pm

Reverse mathematics and a packed Ramsey's theorem

Stephen Flood (Notre Dame)

Abstract. Ramsey's theorem states that each coloring has an infinite homogeneous set, but these sets can be arbitrarily spread out. Paul Erdos and Fred Galvin proved that for each coloring f, there is an infinite set which is not "too spread out" that is not given "too many" colors by f. In this talk, I will give the precise statement of this packed Ramsey's theorem and discuss my work on its reverse mathematical strength. In particular, I have shown that this theorem is equivalent to Ramsey's theorem for each exponent n other than 2, and that it implies Ramsey's theorem for n=2.

Nov 29 2011 - 3:30pm - 4:30 pm

Proving that every Artinian ring is Noetherian without proving that every Artinian ring has finite length

Chris Conidis (Waterloo)

Abstract. It is well-known that every Artinian ring is Noetherian (1), and every proof of this fact actually proves the stronger statement that says every Artinian ring has finite length as a module over itself (2). We will present a new proof of (1) that does not filter through a proof of (2) and use our new proof to separate the strengths of (1) and (2) in the context of Reverse Mathematics. In other words, we will construct a mathematical model (of RCA_0) in which every Artinian ring is Noetherian, but not every Artinian ring is of finite length.

Jan 24 2012 - 3:30pm - 4:30 pm

Orbit Relation and Isomorphism Type for Computable Trees Under Predecessor

Rebecca Steiner (CUNY)

[Abstract not available.]

Mar 27 2012 - 3:30pm - 4:30 pm

The homeomorphism group of the Cantor set, ample generics, and the projective Fraisse limit

Aleksandra Kwiatkowska (UIUC)

Abstract. Akin, Hurley, and Kennedy and independently Glasner and Weiss showed that H(K), the homeomorphism group of the Cantor set, has a dense conjugacy class. This result was strengthened by Kechris and Rosendal who proved that H(K) has a comeager conjugacy class. A topological group G has ample generics if it has a comeager conjugacy class in every dimension, more precisely, if for every m, the diagonal conjugacy action of G on G^m has a comeager orbit. The existence of ample generics in a Polish group has many important consequences for its structure (the small index property, an automatic continuity property). Answering a question of Kechris and Rosendal, we show that H(K) has ample generics. The main tool we use in the proof is the projective Fraisse theory, which is a dualization of the classical Fraisse theory from model theory.

Apr 3 2012 - 3:30pm - 4:30 pm

Graev metrics on free products and HNN extensions of groups with two-sided invariant metrics

Kostyantyn Slutskyy (UIUC)

Abstract. M. Graev back in the 40's introduced a construction of two-sided invariant metrics on free groups over metric spaces. Free groups with Graev metrics are used in descriptive set theory as a good source of "large" groups with two-sided invariant metrics. We will show that a similar construction can be carried out for free products (possibly with amalgamation) of groups with two-sided invariant metrics.

Apr 10 2012 - 3:30pm - 4:30 pm

VC-Density over Indiscernible Sequences

Vincent Guingona (Notre Dame)

Abstract. We begin with a discussion of VC-density in the general realm of set systems and its application to model theory. We then talk specifically about VC-density over indiscernible sequences and show how it relates to dp-rank. We also explore the relationship of these ranks to uniform definability of types over finite sets.

Apr 24 2012 - 3:30pm - 4:30 pm

New results on the strength of Ramsey's theorem for pairs

Damir Dzafarov (Notre Dame)

Abstract. The investigation of the logical content of mathematical theorems in the framework of reverse mathematics is today an active and well-developed area. Its originators saw the subject as the classification of mathematical principles into one of several natural categories according to the set-existence assumptions required for their proof. While this view was affirmed by the strengths of the vast majority of mathematical statements, a growing number of principles are being discovered that fall outside its scope. A notable example is Ramsey's theorem for pairs, which has been the focus of an extensive research program in computability theory and reverse mathematics over the past twenty years. Last year, a longstanding open question regarding this principle was solved by Chong, Slaman, and Yang, who showed that it is not implied by the so-called stable Ramsey's theorem over RCA_0. However, their proof employed a highly customized model with non-standard first-order part, leaving open the question of whether or not the implication holds in omega models. We will present some new results progressing towards a negative answer, showing that there is no Muchnik reduction from instances of Ramsey's theorem to instances of the stable Ramsey's theorem. Time permitting, we will also give a brief survey of the zoo of irregular principles lying below Ramsey's theorem for pairs, and their relationships to other areas of mathematics.

May 8 2012 - 3:30pm - 4:30 pm

Partial and total orders in o-minimal structures with an application to economics

Charles Steinhorn (Vassar)

Abstract. We describe our work (with others) analyzing partial and total orders first-order definable in o-minimal structures. This work appears to have an interesting application in mathematical economics.

AY 10–11

Oct 21 2010 - 3:30pm - 4:30 pm

Feedback ITTMs and \Sigma^0_3 Determinacy

Robert Lubarsky (Florida Atlantic University)

Abstract. AQI is the theory which posits the existence of outputs of all infinite time Turing machine computations, whether writable, eventually writable, or accidentally writable. Philip Welch [2] showed that the least ordinal satisfying AQI is less than the least ordinal satisfying \Sigma^0_3 Determinacy. We extend his argument to show that the ordinals writable by the much more powerful feedback

ITTMs of [1] are also less than the least model of \Sigma^0_3 Determinacy.

[1] Robert Lubarsky, ITTMs with feedback, Ways of Proof Theory (Ralf Schindler, editor), Ontos, Eichenweg 25, Ortenberg 63683, Germany, http://www.ontos-verlag.de, http://www.ontoslink.com/, 2010, to appear.

[2] Philip Welch, Weak systems of determinacy and arithmetical quasi-inductive definitions, Journal of Symbolic Logic, to appear; also available at arXiv: 0905.4412; also available at http://www.maths.bris.ac.uk/ mapdw/.

Nov 4 2010 - 3:30pm - 4:30 pm

Analytic sets and the diagonal method

Vladimir Uspenskiy (Ohio University)

Abstract. Goedel's incompleteness theorem, Tarski's undefinability theorem, and the existence of non-Borel analytic sets can all be viewed as applications of Cantor's diagonal method. We'll discuss this point of view, as well as early history of analytic sets, including Lebesgue's error noted by Suslin, the Alexandrov -- Hausdorff theorem on the cardinality of Borel sets, and related developments in topology.

Nov 18 2010 - 3:30pm - 4:30 pm

Universally measurable sets

Paul Larson (Miami University)

Abstract. A subset of a topological space is said to be universally measurable if it is measured by the completion of each finite, countably additive Borel measure on the space, and universally null if it has measure zero for each such atomless measure. In 1908, Hausdorff proved that there exist universally null sets of cardinality ℵ1, and thus that there exist at least $2^{\aleph_1}$ such sets. Laver showed in the 1970's that consistently there are just continuum many universally null sets. The question of whether there must exist more than continuum many universally measurable sets was asked by Mauldin in 1978, and answered negatively by Larson, Neeman and Shelah in 2008. The speaker noticed in 2009 that the Filter Dichotomy, a statement proved consistent by Blass and Laflamme in the 1980's, implies that every universally measurable filter has the property of Baire. This statement implies the nonexistence of universally measurable probability measures on the integers (so-called medial limits). Several interesting open questions remain, including the question of whether consistently every universally measurable set has the property of Baire.

Apr 14 2011 - 3:30pm - 4:30 pm

Characterization of NIP theories by generalized indiscernible sequences

Lynn Scow (UI Chicago)

Abstract. Indiscernible sequences and sets have been used widely in model theory to solve classification theory problems. The class of stable of theories is fairly low in the hierarchy of

complexity of theories. A result of Shelah states that a theory T is stable just in case any infinite indiscernible sequence in a model of T is an indiscernible set. The class of NIP theories strictly contains the class of stable theories. In this talk I will give a generalization of Shelah's result to characterize the class of NIP theories.

Apr 28 2011 - 3:30pm - 4:30 pm

Spectra of structures and theories

Joseph Miller (Madison)

Abstract. The spectrum of a countable structure M is the set of all Turing degrees that compute a presentation of M. The spectrum of a complete theory T is the set of all degrees that present models of T. Although many of the spectra that are known to be possible for structures are also spectra of theories, we give an example of a structure spectrum that is not a theory spectrum. In the other direction, we show that the PA degrees (the degree of complete consistent extensions of Peano Arithmetic) are a theory spectrum but not a structure spectrum, providing a new example of a spectrum that is impossible for structures and showing that neither type of spectrum subsumes the other.

Theory spectra are new to our work and very basic questions remain. Our examples of theory spectra that are not structure spectra (including the PA degrees) are impossible for atomic theories, leaving open the question of whether the spectrum of an atomic theory is always a structure spectrum. (Joint work with Uri Andrews.)

Apr 29 2011 - 1:30pm - 2:30 pm

Idealized forcing and its applications

Marcin Sabok (Urbana, visiting)

AbstractIdealized forcing is a modern technique of applying forcing and absoluteness results to descriptive set theory. It is the study of the posets of Borel sets modulo a fixed sigma-ideal on a Polish space. During the talk I will introduce the basic notions in this theory and present a couple of applications. I will show how natural problems from descriptive set theory can be translated into the language of forcing. The applications will include descriptive set theoretical "perfect set theorems", Ramsey-type results and problems in the study of Borel equivalence relations.

May 12 2011 - 3:30pm - 4:30 pm

Definability in henselian valued fields

Joseph Flenner (Notre Dame)

Abstract. Beginning with Robinson's 1956 model completeness and particularly flourishing in the last decade, the theory of algebraically valued fields has been well loved by model theorists. The p-adics have also offered many nice results, including Cohen's decidability and Macintyre's quantifier elimination. However, the general class of henselian valued fields, which includes both of these, presents an inherent obstacle: a henselian valued field can be constructed which interprets an arbitrary field. We outline an approach to studying definability in henselian valued fields, in a sense, relative to certain, potentially very complicated, associated structures, and give henselian analogs to some classical theorems on algebraically closed valued fields and the p-adics.

May 26 2011 - 3:30pm - 4:30 pm

Ordered groups where every infinite definable set has interior

Alf Dolich (East Stroudsberg)

Abstract. We consider expansions of divisible ordered Abelian groups under the assumption that every infinite definable subset of the line has interior. Such structures arise naturally when considering ordered structures satisfying various forms of the independence property. We analyze properties of such structures and give a method to construct a host of examples.

Jun 2 2011 - 3:30pm - 4:30 pm

A new, simpler finitary construction of the real closure of a computable ordered field

Charles Delzell (LSU)

Abstract. We give a new, simple, finitary construction of the real closure R of a computable ordered field (K,>), as the set of equivalence classes of (iota-terms involving) uniquely satisfiable formulae with one free variable in the first-order language of ordered rings (+,-,.,0,1,>) with equality, augmented by a constant symbol c_r for each element r in K. It is routine to verify, finitarily, that this R satisfies the axioms of real closed, ordered fields, with the exception of the axiom "0 does not equal 1," for which the verification is difficult, and depends on (and is equivalent to) a finitary proof of the consistency of the theory of real closed ordered fields augmented by the (atomic) diagram of (K,>).