This document is my attempt to archive the OSU Math Logic seminars for AY 21–22. (I was on sabbatical all that year and got behind on my records.) If anyone notices any omissions and wants them corrected, please feel free to let me know.

Chris Miller

OSU Math

September 4, 2024

August 31, 2021, 1:45PM - 2:45PM
Title: Towards Finding a Lattice that Characterizes the > w^2-Fickle Recursively Enumerable Turing Degrees

Speaker: Liling Ko (OSU)

Abstract: Given a finite lattice L that can be embedded in the recursively enumerable (r.e.) Turing degrees hRT, ≤Ti, we do not in general know how to characterize the degrees d ∈ RT below which L can be bounded. The important characterizations known are of the L7 and M3 lattices, where the lattices are bounded below d if and only if d contains sets of “fickleness” > ω and ≥ ω ω respectively. We work towards finding a lattice that characterizes the levels above ω 2 , the first non-trivial level after ω. We introduced a lattice-theoretic property called “3-directness” to describe lattices that are no “wider” or “taller” than L7 and M3. We exhaust the 3-direct lattices L, but they turn out to also characterize the > ω or ≥ ω ω levels, if L is not already embeddable below all non-zero r.e. degrees. We also considered upper semilattices (USLs) by removing the bottom meet(s) of some 3-direct lattices, but the removals did not change the levels characterized. This leads us to conjecture that a USL characterizes the same r.e. degrees as the lattice on which the USL is based. We discovered three 3-direct lattices besides M3 that also characterize the ≥ ω ω-levels. Our search for a > ω2 -candidate therefore involves the lattice-theoretic problem of finding lattices that do not contain any of the four ≥ ω ω-lattices as sublattices.

September 7, 2021, 1:45PM – 2:45PM

Title: An introduction to pre-H-fields

Speaker: Nigel Pynn-Coates (OSU)

Abstract: The elementary class of pre-H-fields was introduced by Aschenbrenner and van den Dries as part of their work on the model theory of transseries. Transseries are built rather intricately from formal power series over the reals using exponentials and logarithms, and they arise when trying to expand solutions to real differential equations at infinity and to compare their asymptotics. Such structures were introduced by Dahn--Goering and Ecalle, the latter's work in connection with Hilbert's 16th problem. Aschenbrenner, van den Dries, and van der Hoeven isolated a theory that has the differential field of transseries as a natural model and axiomatizes the class of existentially closed pre-H-fields. (An existentially closed pre-H-field has as many solutions to asymptotic differential equations as possible in a pre-H-field. For comparison, a field is existentially closed iff it's algebraically closed, and an ordered field is existentially closed iff it's real closed.) In this talk, I will introduce pre-H-fields and mention some of my work on a subclass of pre-H-fields called pre-H-fields with gap 0, including an axiomatization of the class of existentially closed pre-H-fields with gap 0. I will describe a pre-H-field with gap 0 that is obtained from a transexponential extension of transseries.

October 26, 2021, 1:40PM - 2:40PM
Speaker: Ken Supowit (OSU)

Title: A (somewhat) new proof of the Sachs Density Theorem

Abstract: The Sacks Density Theorem states that, for c.e. setsA<TC, there exists a c.e. setBsuch thatA<TB<TC. It is considered the pearl of recursive function theory (also known as computabilitytheory). The original proof (1964) is very hard to understand. A somewhat easier (but still noteasy) proof was given by Soare in 1987. Both of those proofs are infinite-injury priority arguments,using “hatted” computations and “true stages.” The more modern and robust approach to copingwith infinite-injury uses “priority trees.” Downey and Hirschfeldt (2010) sketched a tree-based proofof this theorem, but it is just a sketch and contains some slippery errors. I will describe a tree-basedproof with a few innovations to enhance its readability and (hopefully) its correctness.

November 9, 2021, 1:40PM - 2:40PM
Title: Undefinability of Multiplication in Presburger Arithmetic with Sets of Powers
Speaker: Christian Schulz (University of Illinois at Urbana-Champaign)
Abstract: In a forthcoming paper, we have shown that the expansion of Presburger arithmetic by a k-recognizable set and an l-recognizable set will always have an undecidable theory. This strengthens not only the Cobham-Semënov theorem but also a family of results by Villemaire and Bès. However, in contrast to Villemaire and Bès's results, which were proven via defining multiplication in the structure, in this talk we demonstrate that some such expansions give rise to a dichotomy which precludes the definability of certain arithmetic sets. Hence we produce the first proven example of an expansion of Presburger arithmetic by a k-recognizable set and an l-recognizable set whose first-order theory is undecidable but that does not define multiplication. This talk is on joint work with my advisor, Philipp Hieronymi.

Tuesday, Nov. 16 2021, 1:40-2:40pm

Title: The inverse Kemperman problem

Speaker: Minh Chieu Tran (University of Notre Dame)

Abstract: Let G be a locally compact group with a left Haar measure μ, and let $A,B \subseteq G$ be nonempty and compact. In 1964, Kemperman showed that if G is unimodular (i.e., μ is also the right Haar measure, e.g., when G is $\mathbb{R}/\mathbb{Z}$, $\mathrm{SL2}(\mathbb{R})$, or $\mathrm{SO3}(\mathbb{R})$), then

$$\mu(AB) \geq \min\{\mu(A)+\mu(B), \mu(G)\}.$$

The inverse Kemperman problem (proposed by Griesmer, Kemperman, and Tao) asks when the equality happens or nearly happens. I will discuss the recent solution of this problem by Jinpeng An, Yifan Jing, Ruixiang Zhang, and myself highlighting the role played by model-theoretic group theory.

February 8, 2022, 1:40PM - 2:40PM
Title: 'Explosion v. Implosion'
Speaker: Neil Tennant (The Ohio State University)
Abstract: We set out five basic requirements for a logical system to be adequate for the regimentation of deductive reasoning in mathematics and science. We raise the question whether there are any further

requirements, not entailed by these five, that ought to be imposed. One possible reply is dismissed: that the logical system should allow one to infer any proposition at all from an inconsistent set. We then propose that deductive logic could be designed so as to be implosive rather than explosive. It could be furnished with a semantical relation of relevant logical consequence according to which no inconsistent (equivalently: unsatisfiable) set of sentences can have any consequences at all other than absurdity. We anticipate and dispose of several objections that traditionalists might raise against the explosionist's proposal. We close by examining how this coheres with what we know about Goedel's second incompleteness theorem.

March 8, 2022, 1:45PM - 2:45PM
Title: Introduction to effective randomness
Speaker: Li Ling Ko (The Ohio State University)
Abstract: In this introductory talk, I will provide three notions of randomness of reals --- Martin-Lof randomness, Solovay randomness, and Levin-Chaitin randomness. These definitions are well studied in computability theory and are known to be equivalent. I will also introduce a fundamental tool in the study of randomness --- Van Lambalgen's theorem, which formalizes the intuition that two subsequences in a random real should be highly independent, and also conversely, that two sequences that are random relative to each other should join to a random real.

March 22, 2022, 1:40PM - 2:45PM
Title: Separating Stochasticity and Randomness
Speaker: Justin Miller (Dartmouth University)
Abstract: The law of large numbers says that the average value of random variables tends towards the expected value as the number of trials increases. In computability, the corresponding notion for algorithmically random sets is stochasticity. Random sets are stochastic, but not all stochastic sets are random. We shall separate the computational strength of algorithmically random sets and the injection stochastic sets using the Into and Within set operations. We shall also discuss the application of these techniques to other notions of stochasticity.

March 29, 2022, 1:40PM - 2:45PM
Title: Higher-order stability arithmetic regularity
Speaker: Caroline Terry (The Ohio State University)
Abstract: We precent recent work, joint with J. Wolf, in which we define a natural notion of higher-order stability and show that subsets of $\mathbb{F}np$

that are tame in this sense can be approximately described by a union of low-complexity quadratic subvarieties, up to linear error. This extends joint work with Wolf on arithmetic regularity lemmas for stable subsets of $\mathbb{F}np$ to the realm of higher order Fourier analysis.

May 3, 2022, 10:30AM - 11:40AM
Title: Caley [sic] graphs and Finite Axiomatizability
Speaker: Léo Jimenez (The Fields Institute and The University of Waterloo)
Abstract: In model theory, finding stable, superstable, or omega stable finitely axiomatizable theories required some combinatorial creativity. Curiously, no strongly minimal, finitely axiomatizable theory is known. It was shown by Hrushovski that an example must be locally modular, which allows the problem to be divided into the trivial and non-trivial case. In the 90s, Ivanov made a lot of progress regarding the trivial case by examining the specific case of Caley graphs. The Caley graph of a finitely

generated group is always strongly minimal, and Ivanov identified a necessary and sufficient condition for its finite axiomatizability. Unfortunately, the existence of groups satisfying this condition is unknown, and connected to difficult group theory problems, such as the existence of a finitely presented group with finitely many conjugacy classes. In this talk, I will give an alternative exposition and proof of Ivanov's result, and discuss adjacent problems. This is joint work with David Meretzky, Rachel Skipper and Caroline Terry.

May 3, 2022, 1:40PM - 2:45PM
Speaker: Henry Towsner (University of Pennsylvania)
Title: Topology, Model Theory, and Hypergraph Regularity
Abstract: Szemeredi's Regularity Lemma tells us that binary relations can be viewed as a combination of a "roughly unary" part and a "quasirandom" part. When the binary relation is tame in the model theoretic sense - specifically, when it has finite VC dimension (that is, is NIP) - the quasirandom part disappears. When the relation is stable, a stronger theorem holds in which the "roughly unary" part has a particularly nice structure. We discuss a "topological" perspective on these results inspired by the Lebesgue differentiation theorem, showing that we can prove these results by looking at the behavior of "points of density".

We then discuss the generalization to hypergraphs - that is, to k-ary relations - where the same topological perspective gives us a short proof that a k-ary relation has no quasirandom part exactly when the relation satisfies the high arity generalization of finite VC dimension, finite VC_{k-1} dimension.