Introduction to Game Theory

–Cooperative Game Theory focuses on what coalitions of players can achieve, and assumes that players can meet and make binding agreements as to how to play. Even though the structure is cooperative, players are assumed to be interested in maximizing their own payoff.

–von Neumann and Morgenstern emphasize cooperative game theory.

–Nash's major contribution was to realize the importance of studying games where players cannot make binding agreements. More individualistic emphasis.

–This course is restricted to non-cooperative games.

A *non-cooperative game* is described by a set of players, a description of the strategies available to each player, and the payoffs or utility received by each player for each combination of strategies.

The interpretation is that each player independently chooses a feasible strategy, and that payoffs depend not only on her strategy but also on the strategy choices of the other players.

Two-player zero sum games are purely confrontational. Most games, however, have elements of conflict as well as elements of common interest.

There are two common ways to represent non-cooperative games: extensive form and normal form.

Extensive form games are explicit about the timing of actions taken by the players. The game is defined using a game tree.

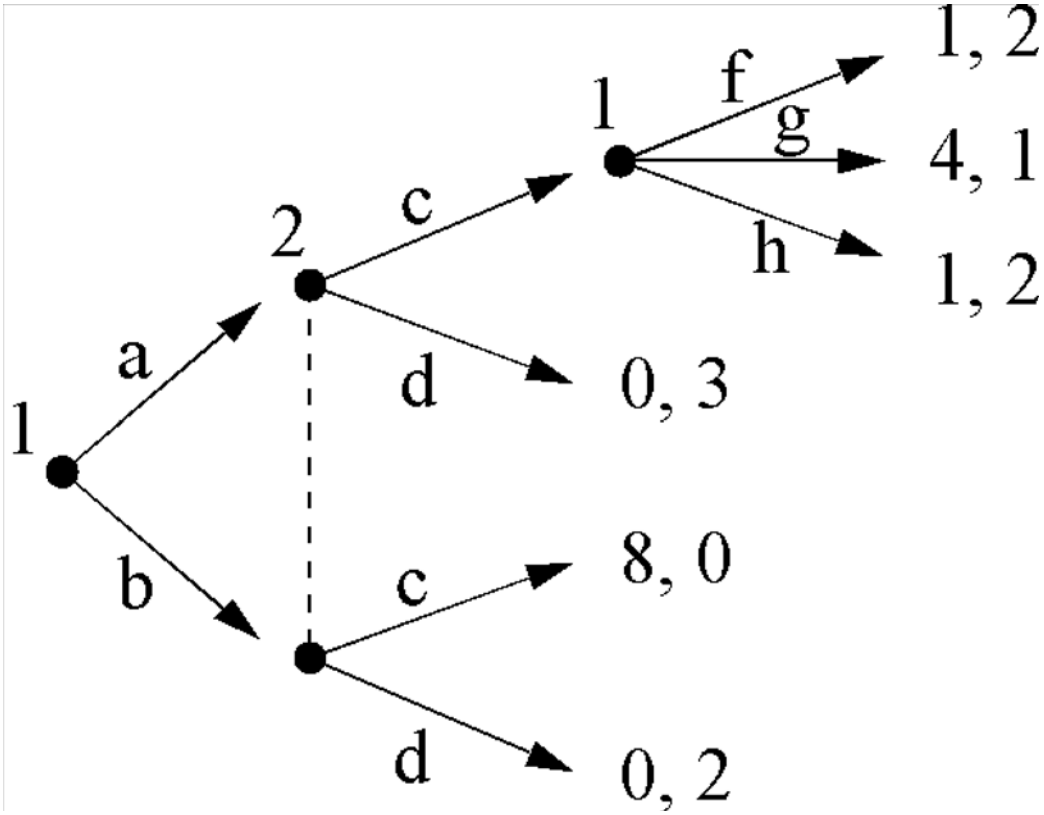
Normal form (sometimes called strategic form) games are not explicit about timing. Either the games are such that players simultaneously choose a single action as in rock-paper-scissors, or the timing is hidden by labeling each strategic plan (for what actions a player will take in all possible circumstances) as a different strategy.

A solution concept is a prediction of how the game will be played or a consistent theory of how the game might be played.

Modeling the game using either the extensive form or normal form representation is conceptually different from finding a solution.

Extensive Form Games

This is an example of a game tree.



A *game tree* has the following basic components: nodes and branches.

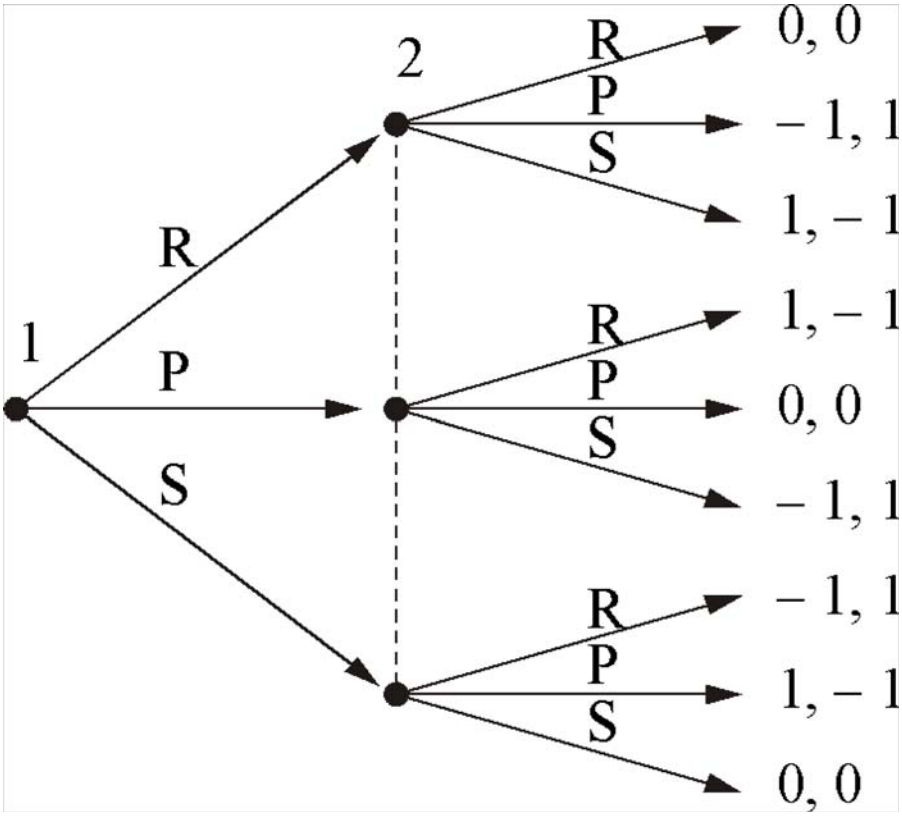
A node specifies where we are in the game, and is labeled by the player whose turn it is to move.

A branch takes us from one node to another, and is labeled by the action corresponding to that branch.

A game tree starts with a single initial node, and all paths from node to node (connected by branches) ends with a terminal node. The terminal node specifies payoffs to each of the players.

If a player whose turn it is to move does not know where he/she is in the game tree, then two or more nodes are connected by a dotted line (or in some textbooks, the nodes are circled). This is called a non-singleton information set.

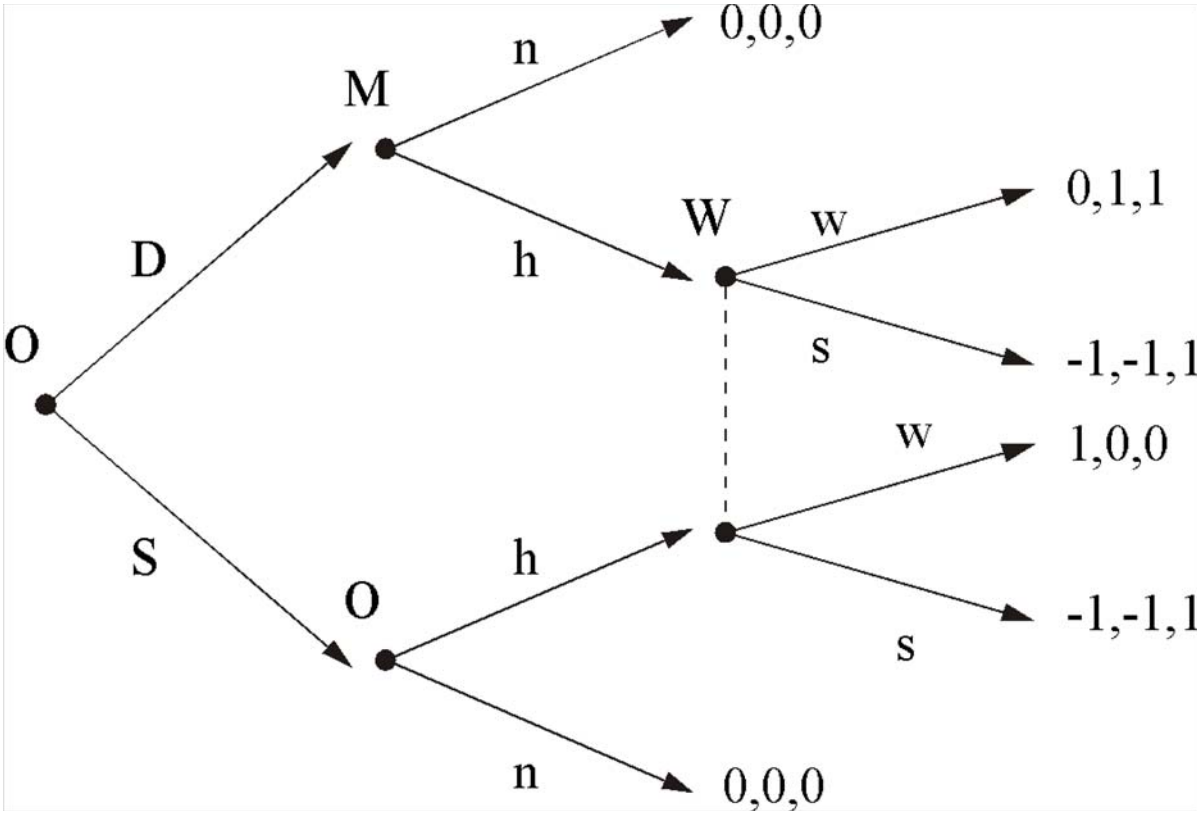
"Non-singleton" information sets occur if a player does not observe a move made earlier in the game. This can be a way of modeling simultaneous play, as in the rock, paper, scissors game:



Here we work out Chapter 2, exercise 2. Consider the following strategic situation:

An owner decides whether to hire a worker, refuse to hire the worker, or let the manager decide. If the worker is hired, she must decide whether to work hard or shirk, but does not know whether it was the owner or manager that hired her. If the worker is not hired, everyone receives a payoff of 0. If the worker is hired and shirks, she receives a payoff of 1, but the owner and manager receive a payoff of -1. If the worker is hired and works hard, whoever hired her receives a payoff of 1, and she and the other player receive a payoff of 0.

Formulate this as an extensive form game by drawing and labeling the game tree.



Payoff numbers represent the preferences of the players. In games where players receive monetary payoffs, the payoffs received in the game should be the utility of the money received. If player 1 cares about what happens to other players in the game, that should also be reflected in player 1's payoff number.