Details of the Extensive Form

Recall that an extensive-form game can be described by a game tree, consisting of *nodes* and *branches*.

The tree starts with an *initial node* and ends at the *terminal nodes*.

A branch has a direction associated with it (sometimes drawn with arrows), going from one node to the "next" node.

Definition: If there is a sequence of branches taking us from node A to node B, then node B is a *successor node* to A. If there is a single branch taking us from node A to node B, then node B is an *immediate successor node* to A.

Definition: If node B is a successor node (respectively, an immediate successor node) to node A, then node A is a *predecessor node* (respectively, an *immediate predecessor node*) to node B.

Here are some rules about trees that are required in order for it to describe an extensive-form game.

Tree Rule 1: Every node is a successor of the initial node, and the initial node is the only node with this property.

Tree Rule $1'$: (missing from the text) A node with no successor nodes is called a *terminal node*. Each terminal node must have associated with it a payoff for each player.

Definition: A *path* through the tree is a sequence of nodes such that (1) the sequence starts with the initial node, (2) the sequence ends with a terminal node, and (3) the second node in the sequence is an immediate successor of the first node in the sequence, the third node is an immediate successor of the second node, and so on.

The following rule shows that there is a unique path reaching any terminal node.

Tree Rule 2: Each node except the initial node has exactly one immediate predecessor. The initial node has no predecessors.

We label each branch with the name of the action that it represents.

Tree Rule 3: Multiple branches extending from the same node have different action labels.

Recall that an *information set* is a set of nodes between which a player cannot distinguish when making a decision.

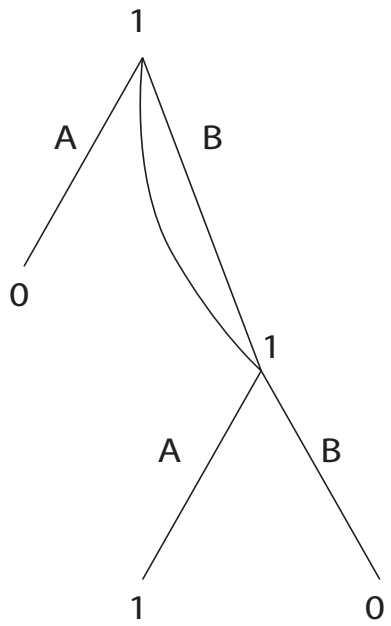
Tree Rule 4: Each information set contains decision nodes for only one of the players, and the player whose decision it is must be labeled.

Tree Rule 5: All nodes in a given information set must have the same number of immediate successors and they must have the same set of action labels on the branches leading to these successors.

Definition: If every information set in the game tree is a singleton node (no dashed lines), it is a game of *perfect information*. Otherwise, it is a game of *imperfect information*.

Most of the games we care about involve perfect recall, where each player anywhere in the game tree remembers the actions that he has already taken.

Some games with imperfect recall include the "Where did I park?" Game and the Forgetful Driver (or "Did I take my medicine?") Game.

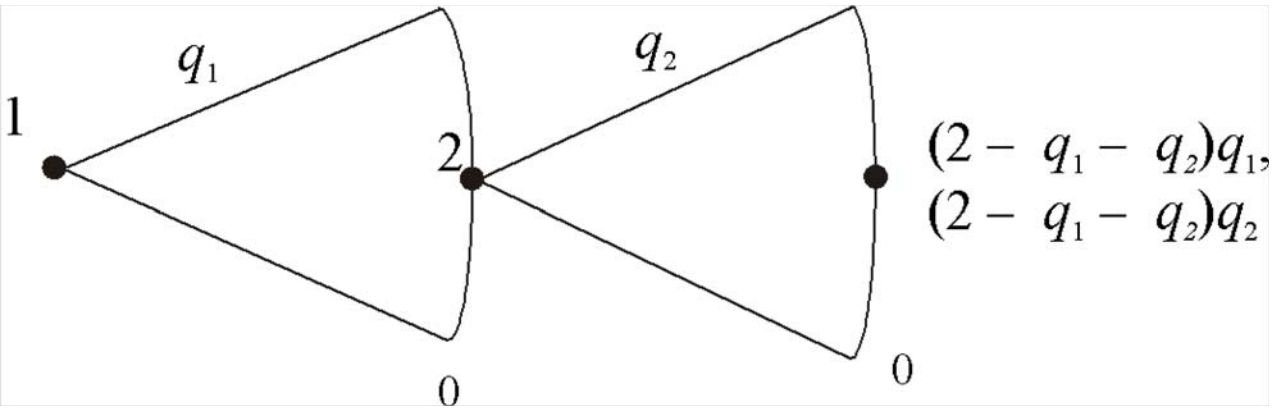


Games with imperfect recall raise interesting issues about what it means to randomize, but they are tricky and of limited usefulness.

To model random events in extensive-form games, we will introduce moves by nature at a later point.

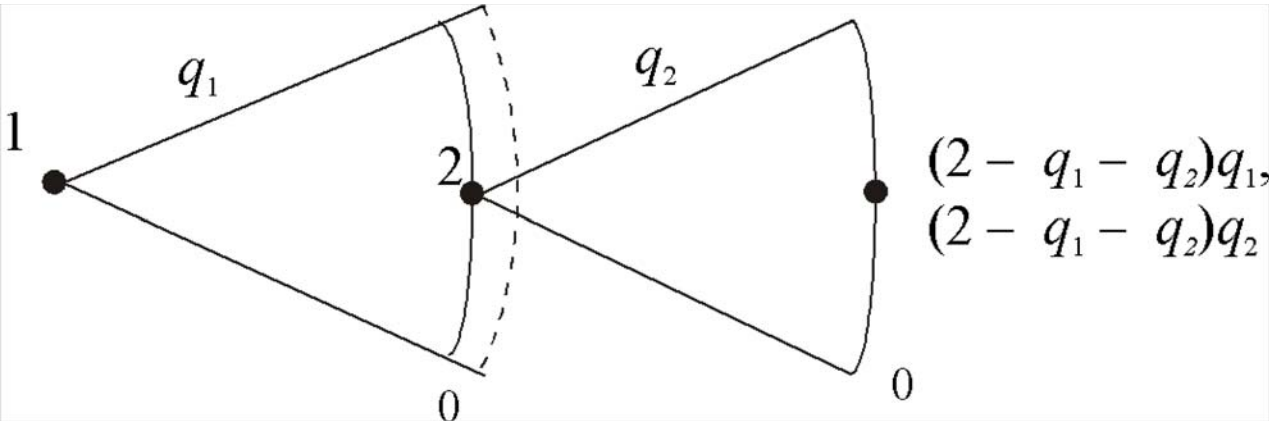
For extensive-form games with a **continuum of actions**, we can draw a game tree with a branch corresponding to the "lowest" action and a branch corresponding to the "highest" action. Then connect these two branches with an arc to represent the possibility of choosing any action in between. Label the branches by a variable representing the action, like q_1.

Then, in the interior of the arc, draw a typical node that continues the tree.



A Stackelberg Game

For games with a continuum of actions and information sets, make a dotted arc.



A Cournot Game

Recall that a strategy for a player is a complete contingent plan of how to play the game. For each information set at which a player must decide which action to make, that action must be specified. (The branch or branches corresponding to the chosen action are highlighted.)

A pure strategy profile (one pure strategy for each player) determines a unique path from the initial node to a terminal node. The payoffs at that terminal node are the payoffs resulting from that strategy profile.

Definition: A strategy profile for an extensive-form game is a Nash equilibrium if and only if when we convert the game to its normal form, the strategy profile is a Nash equilibrium.

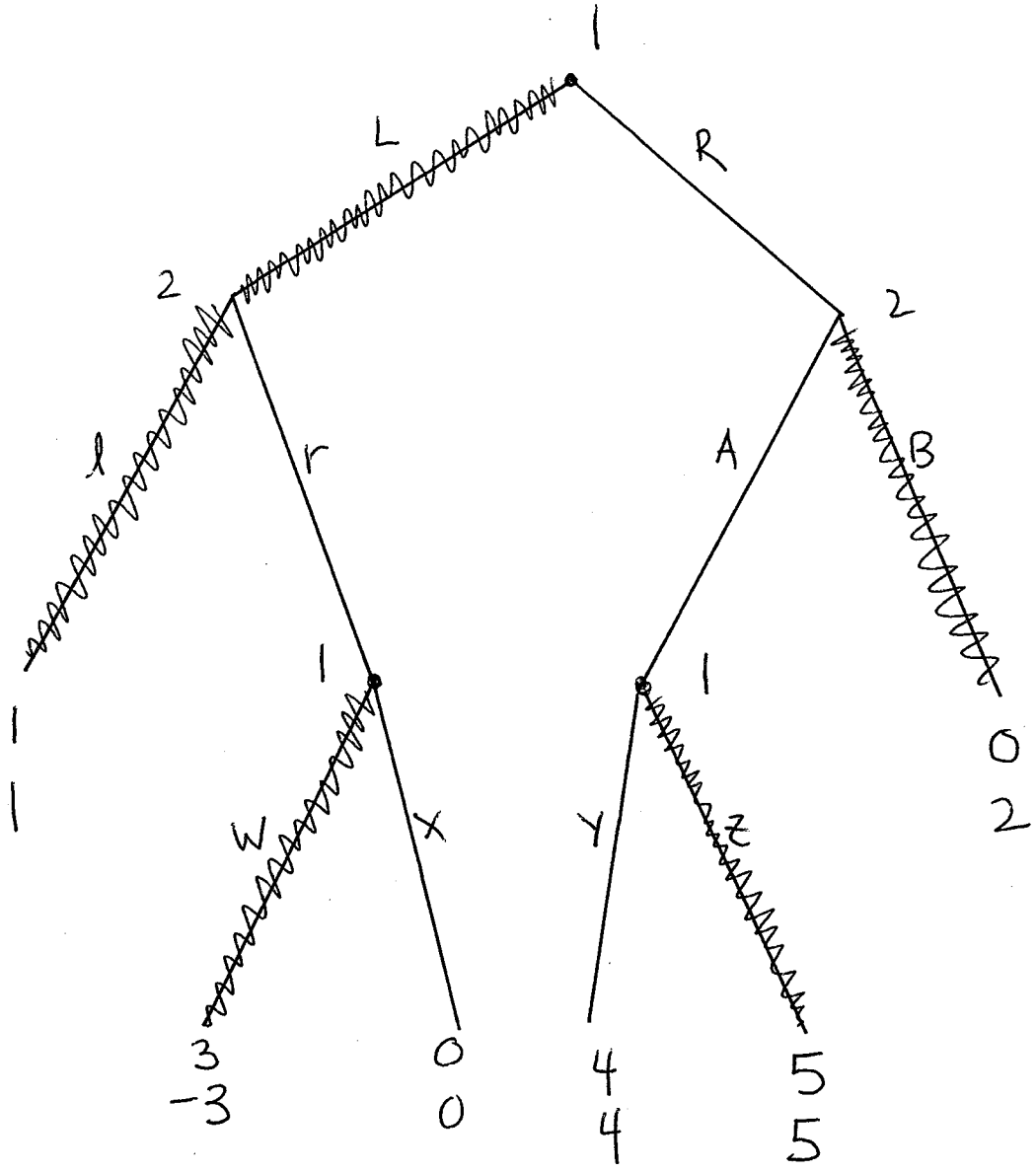
It follows that any two game trees with the same normal-form representation have the same set of Nash equilibrium strategy profiles.

One way to find the Nash equilibria of an extensive-form game is to convert the game into its normal-form.

It is usually easier to guess and check. Highlight the strategy profile you think might work, and verify that no player can do better by changing her strategy and steering the path to a terminal node giving her a higher payoff.

[do example here]

Notice that if we observe the sequence of actions taken or "path" of the game, we will generally not know the complete strategies chosen by the players. Therefore, observing the path of the game is generally not enough information to determine whether the actual strategies form a NE.



$S_1 = LWZ$

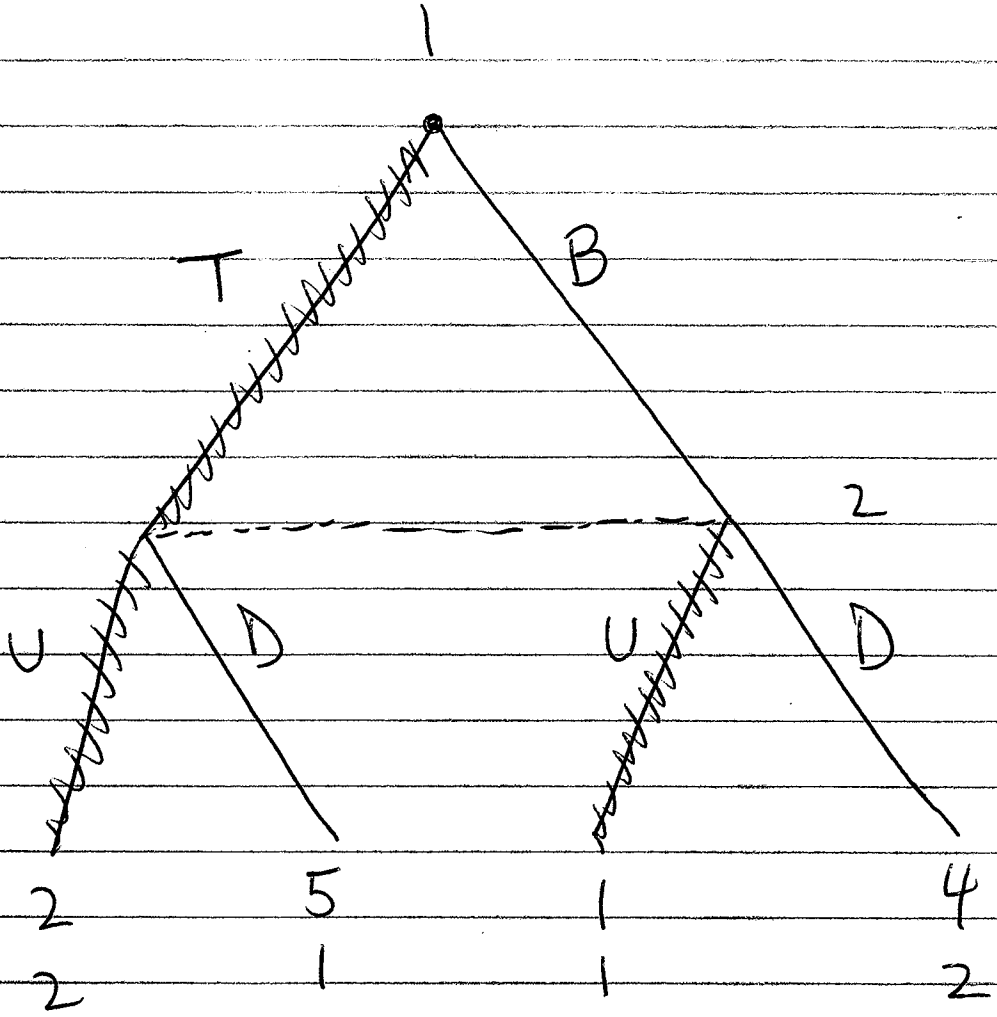
$S_2 = lB$

This is a NE. There are several.

path: L, l

NE payoffs: (1,1)

From the path (L,l), we do not know the strategy profile or whether the strategies chosen form a NE.



(T, U) is a NE

(B, D) is <u>not</u> a NE