Nash Equilibrium

For many games, common knowledge of rationality does not pin down a unique strategy profile.

A *Nash equilibrium* is a strategy profile in which each player is best responding to the other players' strategies. Thus, this solution concept entails a certain amount of coordination in the players' strategy choices. [Watson calls this "congruity."]

Definition: A strategy profile $s \in S$ is a Nash equilibrium (in pure strategies) if $s_i \in BR_i(s_{-i})$ for each player i.

An equivalent condition for Nash equilibrium is that

$$u_i(s_i, s_{-i}) \geq u_i(s_i', s_{-i})$$

holds for all $s_i' \in S_i$ and each player i.

Notice that a Nash equilibrium is a property of an entire strategy profile. It does not make sense to say that a strategy by just one player is an equilibrium.

When we look at some of the classic games defined earlier, we see that

1. Some games do not have a Nash equilibrium (matching pennies)

2. Some games have multiple Nash equilibria

3. There can be a Nash equilibrium that is less efficient than another Nash equilibrium (stag hunt and qwerty typewriter keyboard).

It should not be too surprising that a Nash equilibrium may not always be a compelling prediction of how the game will be played. Rationality in a group situation is trickier than in an individual decision situation.

Interpretations of Nash equilibrium:

1. A prediction of how the game will be played.

2. A consistent theory of how the game might be played, which can serve as a benchmark to compare with actual play.

3. A stable strategy profile. If because of pre-game communication, social norms, or the game being played over and over, once this profile becomes anticipated all players have an incentive to play their part of a Nash equilibrium.

4. A profile satisfying "rational expectations." If players expect others to play according to the NE and best respond to their beliefs, each player's choice will confirm the beliefs others hold about him/her.

In matrix games, it is easy to verify whether a strategy profile is a NE. Compare player 1's payoff with all of his other payoffs along the same column, and compare player 2's payoff with all of her other payoffs along the same row.

One technique for finding the Nash equilibria is to guess and check. Look at each strategy profile.

Another technique is to go through the payoff matrix one player at a time, underlining the payoffs for that player that are best responses to each strategy of the other player. If both payoffs are underlined, the profile is a NE.

Consider the following game, Figure 9.2 in the text.

		player 2		
		X	Y	Z
	J	5, 6	3, 7	0, 4
	K	8, 3	3, 1	5, 2
player 1	L	7, 5	4, 4	5, 6
	M	3, 5	7, 5	3, 3

Underline the best responses of player 1 to each strategy of player 2:

		player 2		
		X	Y	Z
	J	5, 6	3, 7	0, 4
	K	$\underline{8}$, 3	3, 1	$\underline{5}$, 2
player 1	L	7, 5	4, 4	$\underline{5}$, 6
	M	3, 5	$\underline{7}$, 5	3, 3

Now also underline the best responses of player 2 to each strategy of player 1:

		player 2		
		X	Y	Z
	J	$5, 6$	$3, \underline{7}$	$0, 4$
	K	$\underline{8}, \underline{3}$	$3, 1$	$\underline{5}, 2$
player 1	L	$7, 5$	$4, 4$	$\underline{5}, \underline{6}$
	M	$3, \underline{5}$	$\underline{7}, \underline{5}$	$3, 3$

The Nash equilibria are: (K, X), (L, Z), and (M, Y).

Discussion of Experimental Game Theory

As we have discussed, Nash equilibrium does not always offer an accurate prediction of how a game will actually be played. One approach is to gather data from real world markets and compare with the theoretical predictions. A problem with this approach is that the real world is complicated and it might be difficult to come up with a game that is simple enough to study but captures the strategic environment.

Do departures from Nash equilibrium reflect lack of coordination or strategic sophistication, or is the actual game being played different from what we think is being played?

Another approach is to run a lab experiment. Here the experimenter can control the game that the subjects are playing. Economists pay the subjects, and are careful never to deceive the subjects about any aspect of the game. (We need the game being played to be common knowledge.)

Problems with this approach include:

–The games being played in the lab are real games, but maybe not the exact "real world" games that we are interested in understanding. To what extent can we extrapolate to the real world?

–While the experimenter can control the monetary payoffs, it is impossible to control the utility functions. Utility depends on events outside of the lab (how rich they are), and a player might care about spite, generosity, fairness, vengence, etc.

But experiments can help us understand regularities in non-monetary motives. Consider the Ultimatum Game and the Dictator Game.