Costs

Accounting Cost–stresses “out of pocket” expenses. Depreciation costs are based on tax laws.

Economic Cost–based on opportunity cost (the next best use of resources).

1. A self-employed entrepreneur's economic cost includes the opportunity cost of his time (the wage he could have received elsewhere).

2. A firm's economic cost includes its cost of capital, even if the firm owns the capital.

Example: Say you bought a building last year, and paid \$1,000,000. Suppose the current market value is \$800,000, the interest rate is i, and the rental rate is r. (ignore depreciation)

Then the economic cost of the building for this year is $i \cdot 800,000 = r \cdot 800,000$. One alternative to owning the building is selling it and buying a bond, receiving $i \cdot 800,000$. Another alternative is renting the building to someone else, and receiving $r \cdot 800,000$.

The price of capital is $r = i$. If $r > i$, everyone should borrow the money to buy their capital instead of renting. If $i > r$, everyone should sell their capital and rent it back.

Cost Minimization: the Importance of Marginal Analysis

Plant 1: 1000 workers, produce 10,000 widgets

Plant 2: 500 workers, produce 4000 widgets

A consultant is hired, who reasons as follows. $AP_L(1) = 10$ and $AP_L(2) = 8$. Therefore, the firm should send workers from plant 2 to plant 1, where they will be more productive.

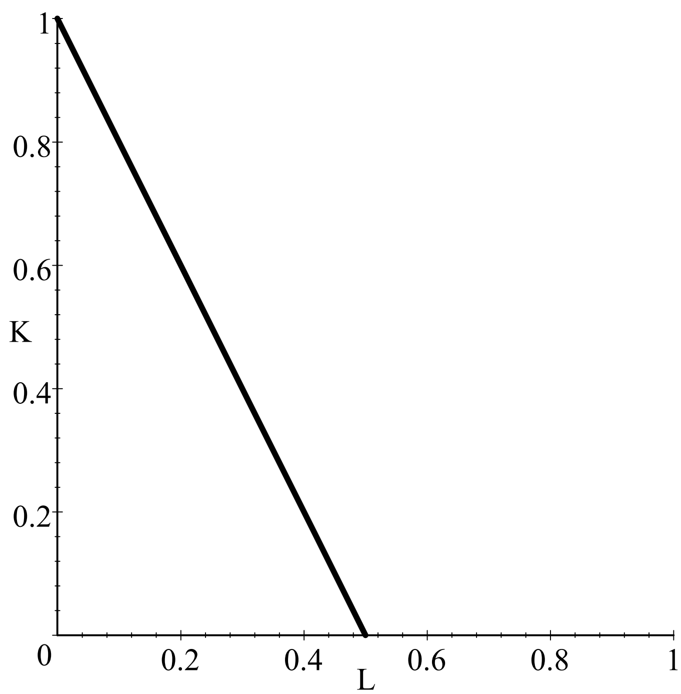
What should the firm do? Fire the consultant. To know whether workers are allocated efficiently, we need to know about marginal products. For example, if $MP_L(1) = 10$ and $MP_L(2) = 15$, send a worker from plant 1 to plant 2, for a net gain of 5 widgets.

Cost Minimization (Long Run)

Here we solve the problem of how to produce a given amount of output at the minimum cost.

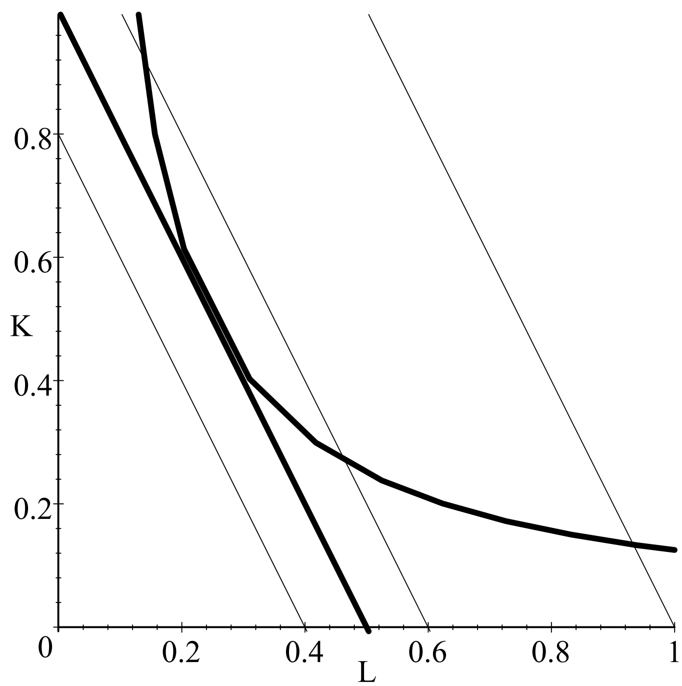
Letting w be the price of labor (dollars per labor hour) and r be the price of capital (dollars per machine hour), the equation for the *isocost* line corresponding to a total cost of TC is

$$wL + rK = TC.$$



An Isocost Line (w=2, r=1)

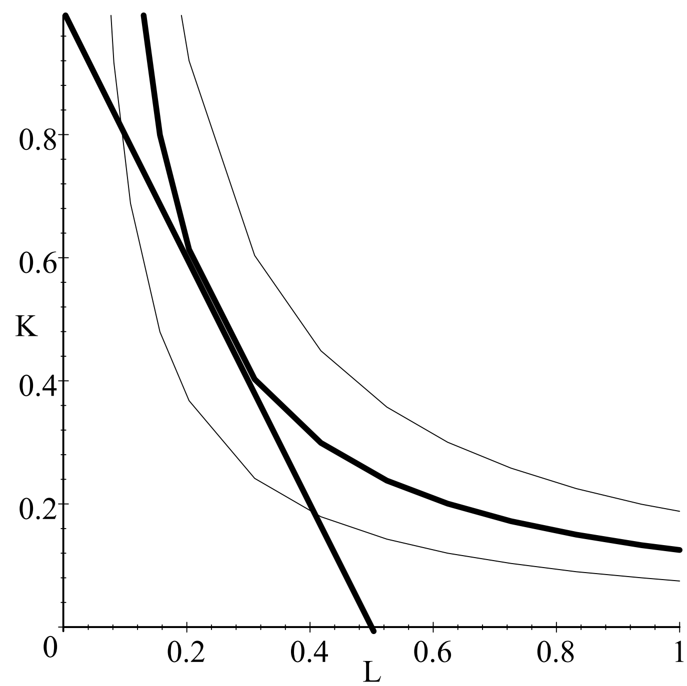
Graphically, we can see that the input bundle that minimizes the cost of producing a given amount of output is where the isoquant is tangent to an isocost line.



The Cost Minimizing Input Bundle (w=2, r=1)

In the above figure, the cost of getting to this isoquant is minimized by choosing L=1/4 and K=1/2, at a total cost of 1.

"Duality": Cost minimization subject to a minimum output constraint (finding the lowest isocost intersecting an isoquant, shown above) is really the same as output maximization subject to a budget constraint (finding the highest isoquant intersecting an isocost, shown below).



The Cost Minimizing Input Bundle (w=2, r=1)

The slope of the isoquant is $-MRTS$, and the slope of the isocost is $-\frac{w}{r}$. Therefore, the optimal input bundle satisfies

$$MRTS = \frac{MP_L}{MP_K} = \frac{w}{r}. \tag{1}$$

To derive condition (1) formally, we fix output at x and solve

$$\begin{aligned} & \min wL + rK \\ \text{subject to } f(K, L) &= x \end{aligned}$$

Set up the Lagrangean,

$$Lagr. = wL + rK + \lambda[x - f(K, L)]$$

The first order conditions are

$$\frac{\partial Lagr.}{\partial L} = 0 = w - \lambda \frac{\partial f}{\partial L} \tag{2}$$

$$\frac{\partial Lagr.}{\partial K} = 0 = r - \lambda \frac{\partial f}{\partial K} \tag{3}$$

$$\frac{\partial Lagr.}{\partial \lambda} = 0 = x - f(K, L) \tag{4}$$

Solving (2) and (3) for λ, we have

$$\lambda = \frac{w}{MP_L} = \frac{r}{MP_K}. \tag{5}$$

Equation (5) is the same as equation (1). Equation (1) says that the ratio of marginal products should equal the ratio of input prices. The interpretation is that the internal willingness to trade K for L (the ratio of marginal products) should equal the external rate at which K can be traded for L.

The interpretation of (5) is that the *marginal cost* of producing output using labor should equal the *marginal cost* of producing output using capital. For example, if w $= 10$ and $MP_L = 2$, then one more labor hour yields 2 more units of output and costs $10, so the marginal cost of one more unit of output is $5.

The multiplier, λ, has an economic significance, corresponding to the marginal cost of output. In any Lagrangean problem, the multiplier has the interpretation of the marginal (objective) of (constraint). Here, the objective is cost and the constraint is output.

Equations (4) and (5) can be used to solve for the generalized (conditional) input demand functions, $L^*(w, r, x)$ and $K^*(w, r, x)$.

Cobb-Douglas Example: $x = AK^{1/3}L^{2/3}$ (where A is a positive constant)

From (5), we have

$$\lambda = \frac{w}{\frac{2}{3}AL^{-1/3}K^{1/3}} = \frac{r}{\frac{1}{3}AL^{2/3}K^{-2/3}}. \tag{6}$$

Equation (6) can be simplified to

$$L = \frac{2rK}{w}. \tag{7}$$

Now plug (7) into (4), $x = AK^{1/3}L^{2/3}$, which yields

$$x = AK^{1/3}(\frac{2rK}{w})^{2/3}. \tag{8}$$

We are interested in the demand for K and L, so solve (8) for K:

$$K^* = (\frac{2r}{w})^{-2/3} A^{-1} x. \tag{9}$$

Plugging (9) into (7), we have the generalized demand function for L:

$$L^* = (\frac{2r}{w})^{1/3} A^{-1} x. \tag{10}$$

Deriving the Total Cost Function

Since (9) and (10) tell us the amounts of K and L to choose in order to produce x units of output, we can derive the total cost of producing x, assuming the firm chooses its inputs optimally to minimize costs.

$$TC^* = wL^* + rK^* \tag{11}$$

For our example, (11) becomes

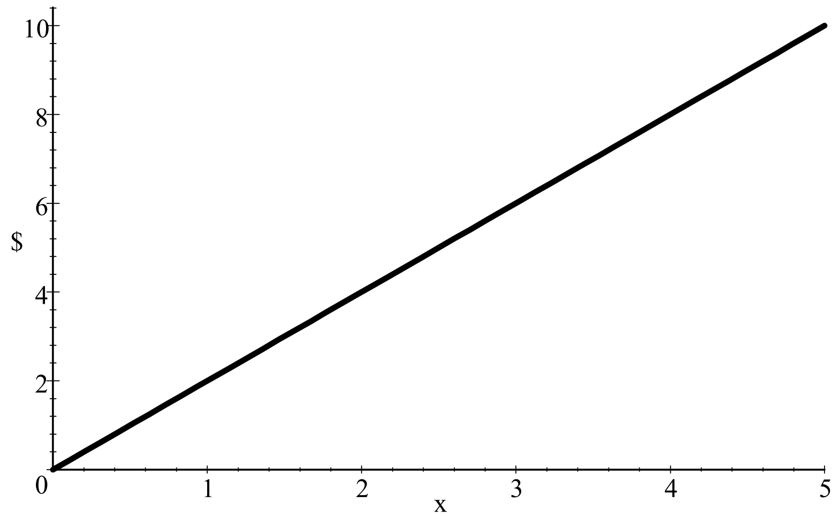
$$\begin{aligned} TC^* &= w(\frac{2r}{w})^{1/3}A^{-1}x + r(\frac{2r}{w})^{-2/3}A^{-1}x \\ &= [2^{1/3} + 2^{-2/3}]w^{2/3}r^{1/3}A^{-1}x \end{aligned}$$

Notice that total cost is proportional to x. This is a property of any constant-returns-to-scale production function. Since all inputs are variable for this calculation, TC^* is sometimes called the *long run total cost function*, LRTC.

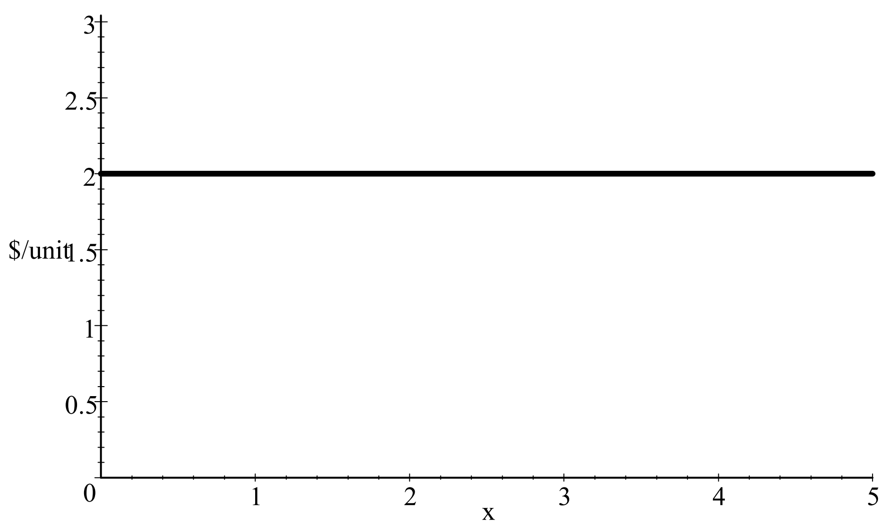
We can also define *long run average cost and long run marginal cost*:

$$LRAC = \frac{LRTC}{x} \quad \text{and} \quad LRMC = \frac{d(LRTC)}{dx}$$

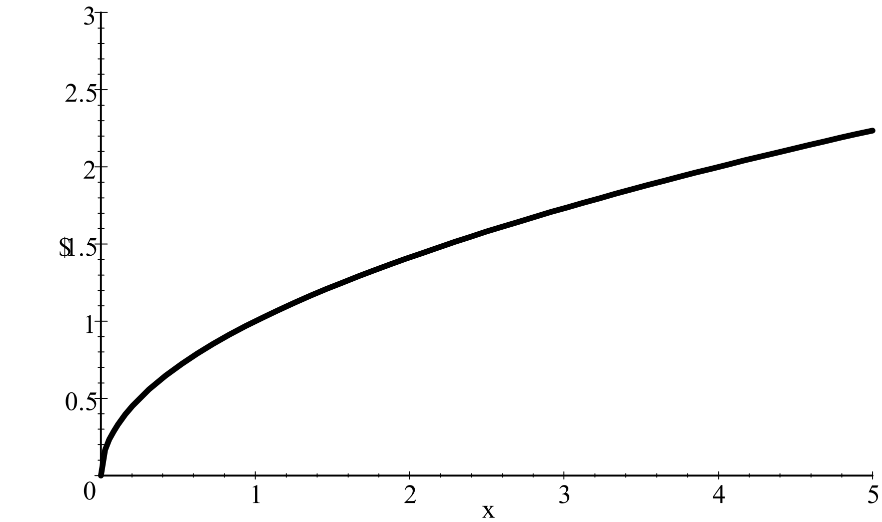
For constant returns to scale, these cost functions are constant, independent of x



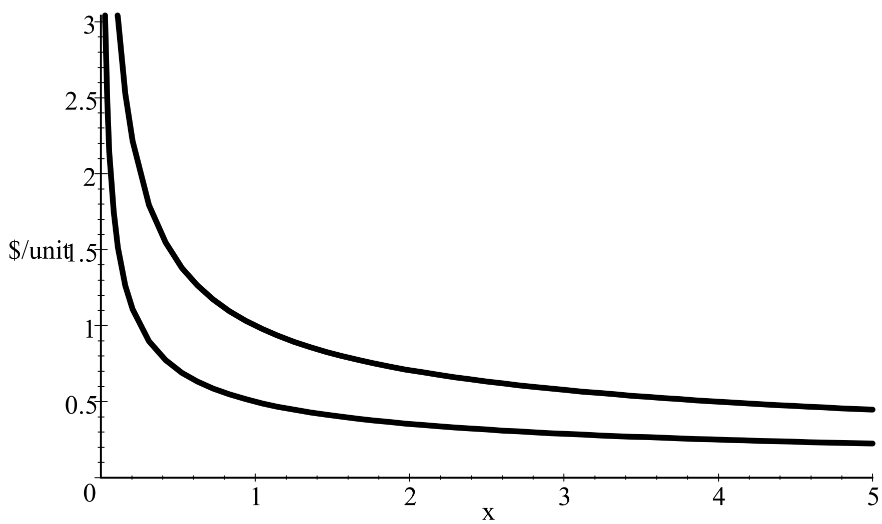
LRTC (constant returns)



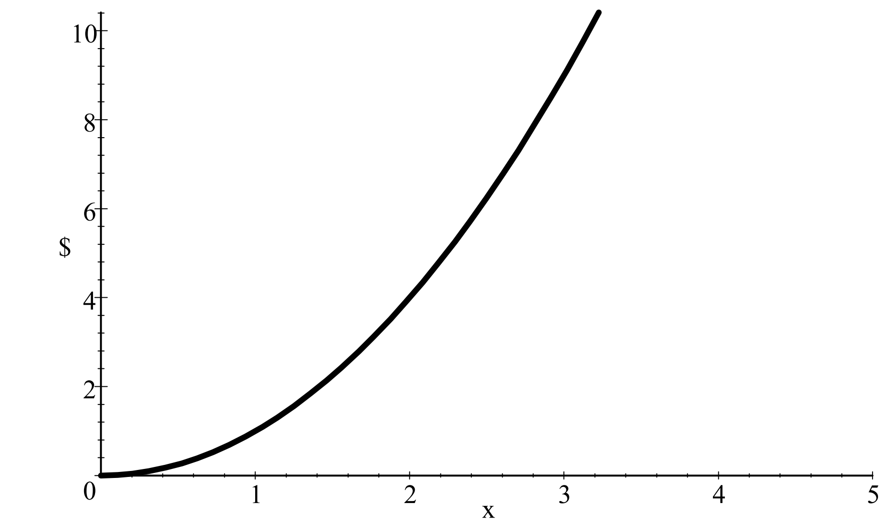
LRAC and LRMC (constant returns)



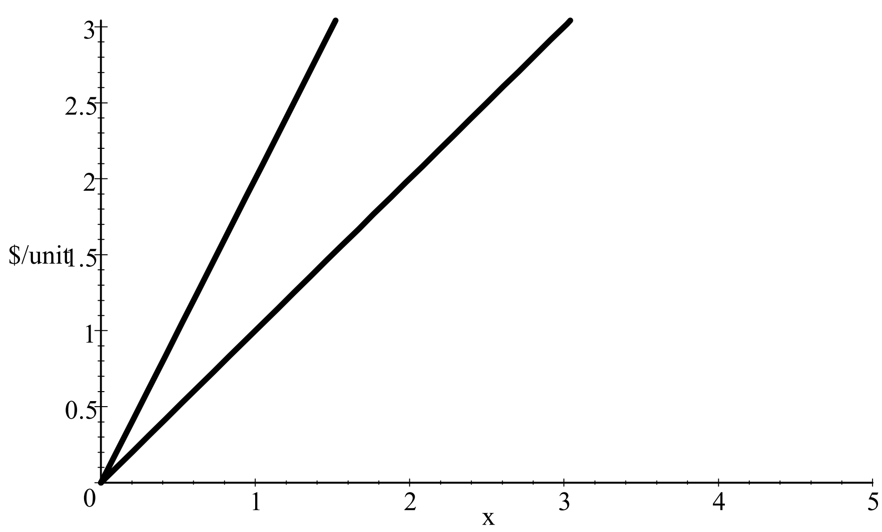
LRTC (increasing returns)



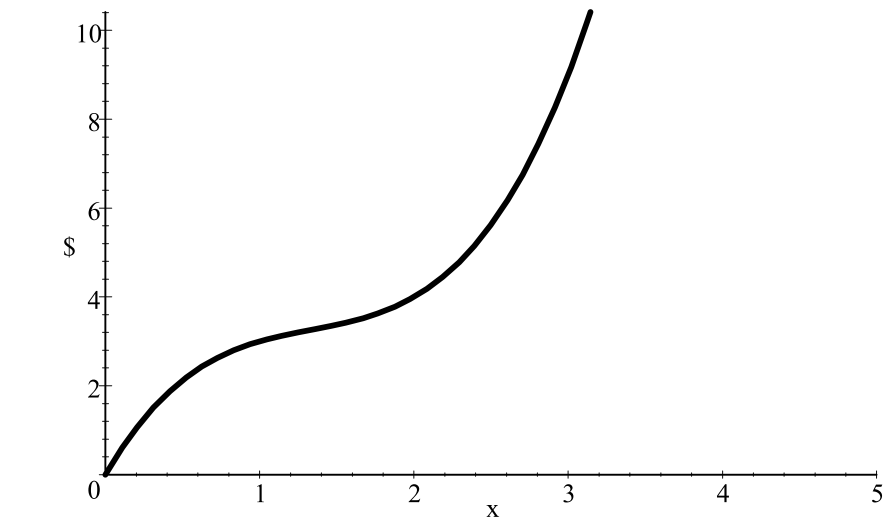
LRAC and LRMC (increasing returns)



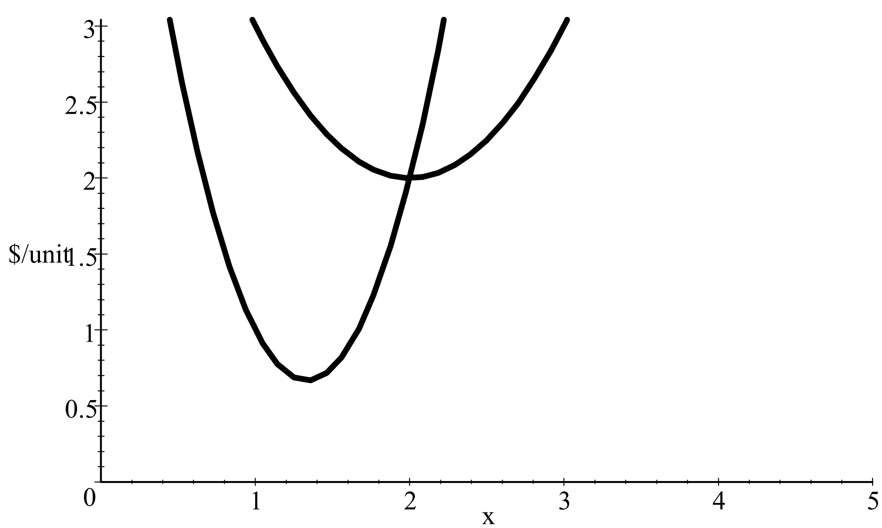
LRTC (decreasing returns)



LRAC and LRMC (decreasing returns)



LRTC (S-shaped)



LRAC and LRMC (U-shaped LRAC)

There are IRS for $x < 2$ and DRS for $x > 2$.

If the LRAC curve is falling, then at the margin, we are bringing down the average as we increase x. Thus, the marginal cost must be below average cost (in order to be bringing down the average).

If the LRAC curve is increasing, then marginal cost must be above average cost.

For a U-shaped LRAC curve, then at the minimum point, the curve is flat. Since one more unit is not changing the average, it must be that marginal cost is equal to average cost.

Returns to Scope and Joint Production (We will not treat this topic formally.) Sometimes there can be cost savings by expanding the set of products produced, rather than expanding the amount of a given good produced.

Examples include: (1) airlines, spoke and hub networks. By adding the Milwaukee to Chicago route, more passengers will want to fly the Chicago to New York route. Federal Express. (2) Gas station and convenience store. (3) Time-share condos, hotels. (4) Produce intermediate as well as final products.