Profit Maximization by a Competitive Firm

Having derived the total cost function (either long run or short run), we can now solve for the profit-maximizing output level, x^*. Given x^*, we can then compute the unconditional demand for inputs such as capital and labor.

The profit function is total revenue minus total cost,

$$\pi(x) = TR(x) - TC(x).$$

For an *interior* choice of x that maximizes profit, set the derivative equal to zero:

$$\frac{d\pi(x)}{dx} = \frac{dTR(x)}{dx} - \frac{dTC(x)}{dx} = MR(x) - MC(x) = 0.$$

Thus, a profit maximizing firm (either competitive or one with market power) chooses x so that marginal revenue equals marginal cost.

A perfectly competitive firm cannot influence the price, p_x. Therefore, marginal revenue is just the price, since one more unit receives additional revenues of p_x.

A perfectly competitive firm's profit function is then $\pi(x) = p_x x - TC(x)$, and the condition for (interior) profit maximization is

$$p_x = MC(x). \tag{1}$$

To make sure that we have a profit maximum (and not a minimum!), use the second-order condition

$$\frac{d^2\pi}{dx^2} = -\frac{dMC(x)}{dx} < 0,$$

which says that the marginal cost curve should be upward sloping (increasing in x).

The interpretation of equation (1), $p_x = MC(x)$, is not that the firm is choosing the price.

Rather, the firm takes the price as determined in the market, and chooses x at the point where marginal cost equals the price.

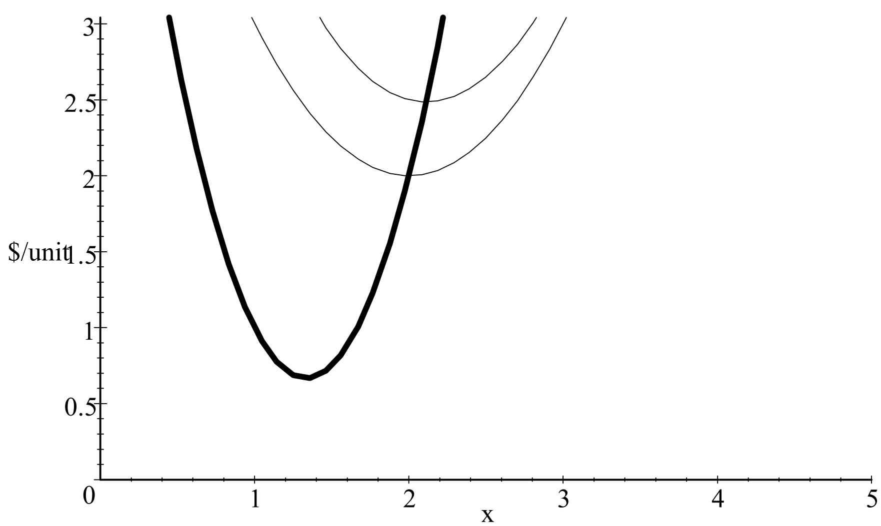
Perfect competition assumes that the firm has no market power to influence the price it can charge. All firms in the market for good x produce the exact same product, and all consumers know about all the firms and what they are charging. A firm can sell as much as it wants at the market price. No explanation for Marketing Departments.

We can think of the firm as free to choose its price, but if it charges a penny above p_x it loses all its customers. There is no reason to charge less, either, since you can sell all you want at p_x.

This idealization should shed light on actual markets that are fairly competitive.

At any price, the firm will choose x so that $p_x = MC(x)$. Thus, a competitive firm's marginal cost curve is its supply curve.

The Short Run Supply Curve



Competitive Firm's Short Run Supply Curve

If p_x is greater than the minimum SRATC, the firm receives positive economic profits by selecting the profit maximizing x.

If p_x is greater than the minimum SRAVC but less than the minimum SRATC, the firm receives negative profits, but still should produce.

If p_x is less than the minimum SRAVC, the firm cannot even cover its variable costs at any level of output, and should shut down.

To see this, profits from producing x are: $p_x x - SRVC - FC$.

Profits from producing zero are: $-FC$.

Therefore, shut down if, for all x, $p_x x - SRVC < 0$, or $p_x < SRVC/x = SRAVC$.

Notice that the decision of what to produce, and whether to shut down, does not depend on the level of fixed costs.

Movies: (1) losing your tickets (2) whether to walk out.

Sunk costs vs. fixed costs: Sunk costs cannot be recovered, and are totally irrelevent to any profit maximizing decision. All fixed costs are sunk in the short run, but some fixed costs may not be sunk in the long run.

For example, an airline may have the size of its fleet fixed in the short run, but much of the cost can be recovered by selling the planes. In the short run, the fixed cost of the planes is irrelevant to the decision of how many flights to offer, or whether to temporarily shut down. In the long run, the firm has the option of adjusting its fleet by selling planes, so these costs (fixed in the short run but not sunk) can matter.

A quasi-fixed cost arises when a firm does not purchase any of the input when output is zero, but purchases a fixed quantity of the input for any positive output. For example, a bar must purchase a liquor licence no matter how many drinks it plans to serve, but it can avoid this cost if it shuts down before purchasing the licence.

Importance of marginal cost, practical issues: isolate only those costs that depend on output

(1) separate depreciation that depends on time from depreciation that depends on output

(2) separate fixed overhead costs from overhead costs that depend on output

(3) costs that depend on today's output but arise later should be charged to today (delayed maintenance and repairs)

Complete solution of the firm's problem in the *short run*: an example

$f(K,L) = K^{1/2}L^{1/2}$, where K is fixed at $\overline{K}$.

Approach #1: choose L to maximize profits.

$$\pi = p_x\overline{K}^{1/2}L^{1/2} - wL - rK$$

The first order condition is

$$\frac{\partial \pi}{\partial L} = 0 = \frac{1}{2}p_x\overline{K}^{1/2}L^{-1/2} - w.$$

Solving for L, we have

$$L^{-1/2} = \frac{2w}{p_x\overline{K}^{1/2}}.$$

Taking both sides to the -2 power, we have

$$L^* = \frac{p_x^2\overline{K}}{4w^2}. \qquad (2)$$

Plugging (2) into the production function, we have the optimal output choice:

$$x = \overline{K}^{1/2} L^{1/2} = \overline{K}^{1/2} (\frac{p_x^2 \overline{K}}{4w^2})^{1/2} = \frac{\overline{K} p_x}{2w}.$$

This gives us the equation for the supply curve, where x depends only on $\overline{K}$ and prices.

The firm's profits will be

$$\pi = p_x (\frac{\overline{K} p_x}{2w}) - w(\frac{p_x^2 \overline{K}}{4w^2}) - r\overline{K.}$$

Approach #2: Set SRMC equal to the price.

From $x = \overline{K}^{1/2} L^{1/2}$, we solve for L:

$$L^{1/2} = \frac{x}{\overline{K}^{1/2}}, \text{ which implies } L = \frac{x^2}{\overline{K}}.$$

Therefore, we have

$$SRTC = w\frac{x^2}{\overline{K}} + r\overline{K.}$$

Taking the derivative to get SRMC, and setting it equal to the price, we have

$$SRMC = \frac{2wx}{\overline{K}} = p_x.$$

Notice that $SRAVC = wx/\overline{K}$, so the minimum SRAVC occurs at x=0. We do not have to worry about shutting down.

Changes in the Supply Curve

–We move along the supply curve (in $x - p_x$ space) when we change p_x. At higher prices, the quantity supplied increases.

–The supply curve shifts when we change the price of variable inputs. Here, an increase in w increases marginal cost and shifts the supply curve up and steeper, so less output is supplied at any price.

–The supply curve depends on $\overline{K}$. Higher $\overline{K}$ lowers marginal cost and shifts the supply curve down and flatter, so more output is supplied at any price.

–Technological change can shift the supply curve, usually leading to more output at any price.