# Preferences and Utility Functions

Underlying the demand function is consumers' choice of goods and services that are most preferred.

Early economists thought that *utility* is a physical property that can be measured and compared across consumers. "utils" for utility, like degrees of temperature. This is an exteme form of *cardinal utility*, sometimes used to justify the existence of a leisure class. (The people good at producing should work, and the people good at creating utility should consume.)

Around 1900, the *Marginalists* believed that utility cannot be meaningfully compared across people, but that an individual could assign meaningful numbers to the utility of consuming the first orange, the second orange, and so on. **diminishing marginal utility**

## Ordinal Utility

Even for an individual, the happiness derived from consuming a particular good (cookies) depends on what other goods are being consumed (milk). Utility should be based on combinations of consumption goods, called *consumption bundles*. **for simplicity, we usually assume two goods, x and y**

Under *ordinal utility*, we can only give a rank ordering of different consumption bundles (which bundle is preferred). In other words, consider a utility function that assigns a number to every possible consumption bundle. A bundle with a higher utility number is preferred to a bundle with a lower number. Under ordinal utility, the *magnitude* of the utility numbers has no meaning, only the ranking.

You cannot say that one bundle gives twice as much utility as another, or by how much one bundle is preferred. Since a consumer chooses the affordable bundle that is most preferred, only the ordinal properties of a utility function are important.

# Axioms of Consumer Preference

Axiom 1: Preferences are complete: for any two bundles, A and B, exactly one of the following is true. (1) A is preferred to B, $A^P B$, (2) B is preferred to A, $B^P A$, or (3) A is indifferent to B, $A^I B$. (You can always make a comparison. For a counterexample, think of $A^P B$ meaning that team $A$ beat team $B$ in a tournament.)

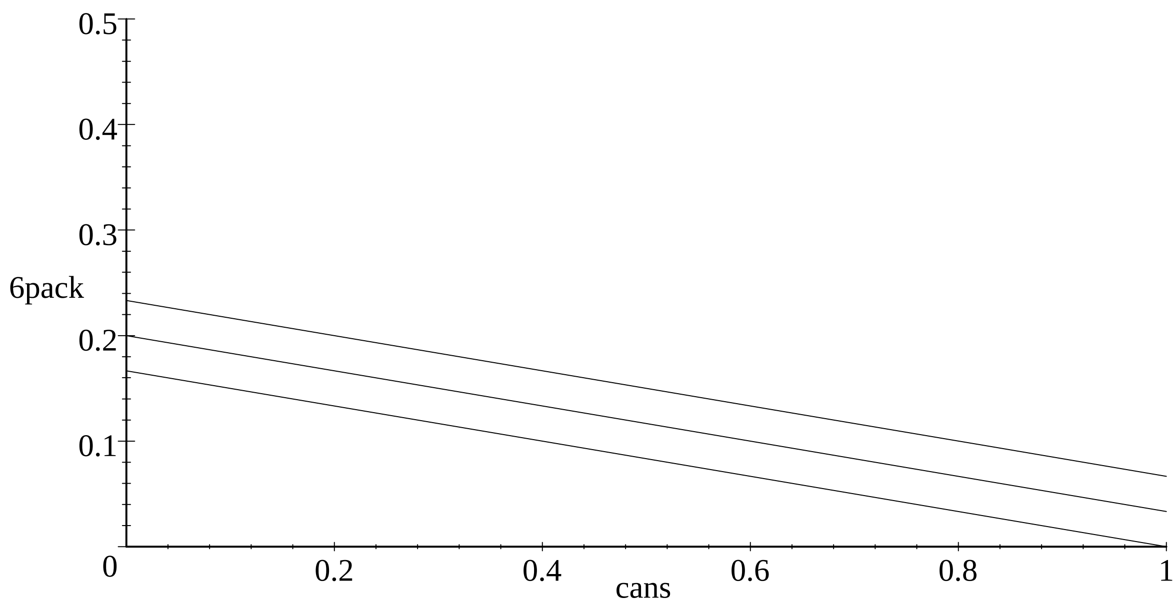
Axiom 2: Preferences are reflexive: $A^I A$. (For a counterexample, think of $A^P B$ meaning that individual $A$ dislikes individual $B$.)

Axiom 3: Preferences are transitive: $A^P B$ and $B^P C \Rightarrow A^P C$ (rules out inconsistent behavior).

Note: group preferences might not be transitive. Suppose preferences between candidates x,y, and z are as follows: Allison prefers x to y to z; Bob prefers y to z to x; and Chris prefers z to x to y. Then group preferences over candidates, as measured by majority rule in pairwise contests, are given by: $x^P y, y^P z, z^P x$.

Axiom 4: Preferences are continuous. If $A^P B$, and if $C$ is sufficiently close to $B$, then $A^P C$ . (For a counterexample, lexicographic preferences.)

Axioms 1-4 allow preferences to be represented graphically by *indifference curves*, and by a utility function, $u$. $A^P B$ if and only if $u(x^A, y^A) > u(x^B, y^B)$.




Perfect Substitutes

Indifference curves to the north and east represent higher utility. If we change the function only by relabeling the utility values corresponding to the indifference curves, we do not change the ordinal properties of the preferences.

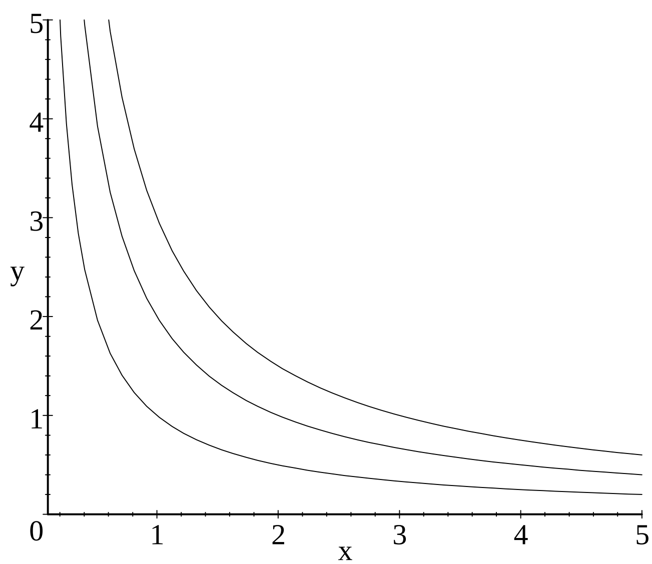
Under two more axioms, consumer demand will be well-behaved:

Axiom 5: More is better. Starting with the bundle $A = (x^A, y^A)$, then increasing any of the goods in $A$ yields a new bundle that is preferred to $A$. For any positive $\varepsilon$, $u(x^A + \varepsilon, y^A) > u(x^A, y^A)$. This rules out the possibility of having as much consumption of some good as you would want.

Axiom 6: All indifference curves exhibit diminishing *marginal rates of substitution.* The MRS is the absolute value of the slope of the indifference curve. The indifference curve flattens as you move along the indifference curve to the right.

# The Marginal Rate of Substitution

The interpretation of the marginal rate of substitution is the amount of good $y$ that the consumer is willing to give up for 1 unit of good $x$, or the internal rate of trade between the two goods. Diminishing marginal rate of substitution occurs when, the more good $x$ you have, the less good $y$ you are willing to give up.




Diminishing Marginal Rate of Substitution

# Mathematical Derivation of the Marginal Rate of Substitution

Consider a utility function $u(x, y)$ over bundles of two goods, $x$ and $y$. The set of bundles $(x, y)$ that yield utility of $\overline{U}$ are the solutions to the equation $u(x, y) = \overline{U}$. These bundles form an indifference curve, so for example $u(x, y) = 5$ gives us the equation of the indifference curve corresponding to a utility of 5.

We define the *marginal rate of substitution* of $y$ for $x$ as the negative of the slope of this indifference curve:

$$MRS_{yx} = -\frac{dy}{dx} \mid_{du=0} . \tag{1}$$

## MRS and marginal utilities

Given a utility function, $u(x,y)$, the *marginal utility* of x is defined by

$$MU_x = \frac{\partial u}{\partial x}$$

and the marginal utility of y is defined by

$$MU_y = \frac{\partial u}{\partial y}.$$

It can be shown that the marginal rate of substitution equals the ratio of marginal utilities:

$$MRS_{yx} = -\frac{dy}{dx} \mid_{du=0} = \frac{MU_x}{MU_y}. \tag{2}$$

Technical derivation of (2): Along an indifference curve, we have $\overline{U} = u(x,y)$. Taking the "total differential" of both sides yields: $0 = du = \frac{\partial u}{\partial x}dx + \frac{\partial u}{\partial y}dy$. Solving for $\frac{\partial u}{\partial x}/\frac{\partial u}{\partial y}$ and using the definition of MRS in (1) yields (2).

Ordinal and Cardinal Utility (revisited)

$MRS_{yx}$ reflects the direction of preference in x-y space, and is an ordinal property.

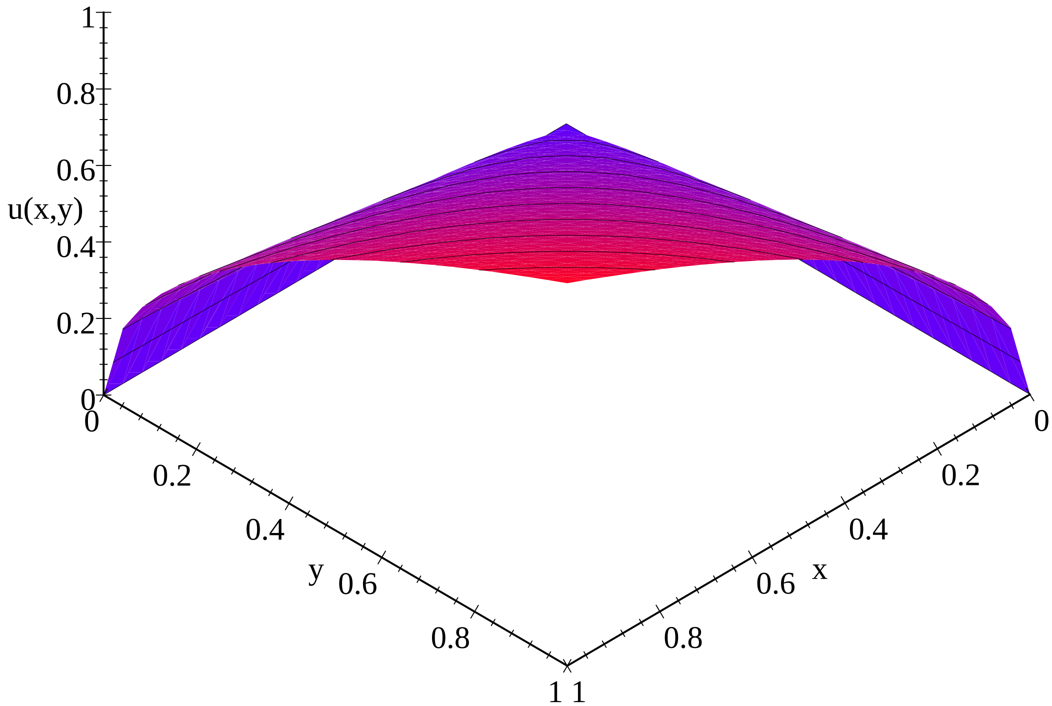
$MU_x$ and $MU_y$ reflect the rate at which utility increases, and depend on the cardinal utility function that represents the consumer's preferences.

example: (i) $x^{1/2}y^{1/2}$ and (ii) $x^2y^2$ both represent the same preferences and have the same indifference curves.

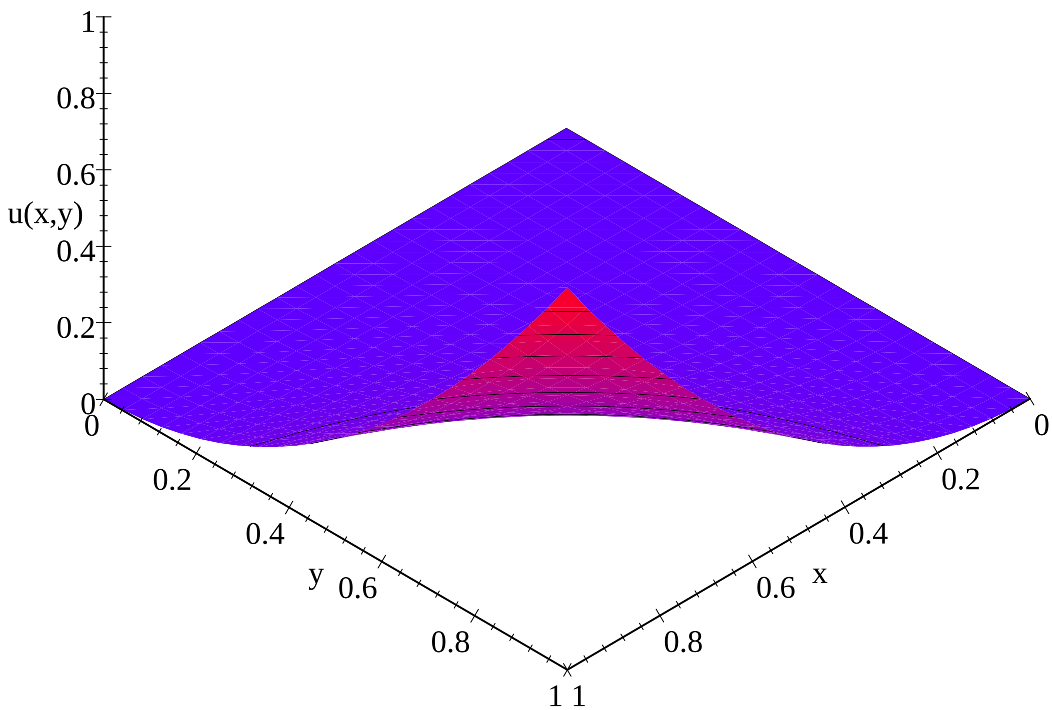
(i) $\frac{\partial u}{\partial x} = (1/2)x^{-1/2}y^{1/2}$, $\frac{\partial u}{\partial y} = (1/2)x^{1/2}y^{-1/2}$, and $MRS_{yx} = \frac{y}{x}$.

(ii) $\frac{\partial u}{\partial x} = 2xy^2$, $\frac{\partial u}{\partial y} = 2x^2y$, and $MRS_{yx} = \frac{y}{x}$.

$x^{1/2}y^{1/2} = \overline{U}$ and $x^2y^2 = \overline{U}^4$ are equations for the same indifference curve.




$x^{1/2}y^{1/2}$




$x^2y^2$