Elasticity of Demand

How should one measure the sensitivity of demand to changes in prices or income?

$\dfrac{dx}{dp_x}, \dfrac{dx}{dp_y}$, and $\dfrac{dx}{dM}$ depend on the *units* in which x is measured.

Whether $\frac{dx}{dp_x}$ is -10 (millions of gallons) or $-10,000$ (thousands of gallons) obviously makes a big difference.

The *elasticity of demand* is the percentage change in demand, per percentage change in price or income.

The own-price elasticity of demand:

$$\varepsilon^d = \frac{\% \triangle x}{\% \triangle p_x} = \frac{\triangle x / x}{\triangle p_x / p_x}. \tag{1}$$

Equation (1) is the definition of the so-called arc elasticity, for a move from one point on the demand curve to another. By convention, we take x and p_x in (1) to be the averages, $(x_1 + x_2)/2$ and $(p_{x,1} + p_{x,2})/2$.

The own-price elasticity of demand at a single point is the limiting arc-elasticity:

$$\varepsilon^d = \frac{dx}{dp_x} \left(\frac{p_x}{x} \right).$$

Because demand is downward sloping, ε^d is negative.

If $\varepsilon^d < -1$, we say that demand is *elastic* (demand changes a lot or "stretches" in response to a price change).

If $\varepsilon^d = -1$, we say that demand has *unitary elasticity*.

If $\varepsilon^d > -1$, we say that demand is *inelastic*.

High elasticities (highly negative ε^d) occur when the good in question has close substitutes, since consumers will switch when the price goes up a little.

High elasticities tend to occur the more narrowly defined the commodity. For example, demand for first class seats might be inelastic, but demand for first class seats on American Airlines is highly elastic.

A *temporary* price change tends to have a higher elasticity than a *permanent* price change. For example, if Honda reduced the price on Civics by 30% for one week only, the percentage increase in demand for that week would be tremendous. If Honda permanently reduced the price by 30%, the increase in demand during the first week would be much less.

A price change has a higher elasticity in the *long run* than in the *short run*. People only gradually adjust their plans, due to fixed commitments. Think of (i) an Ohio State tuition increase, or (ii) an increase in gasoline prices.

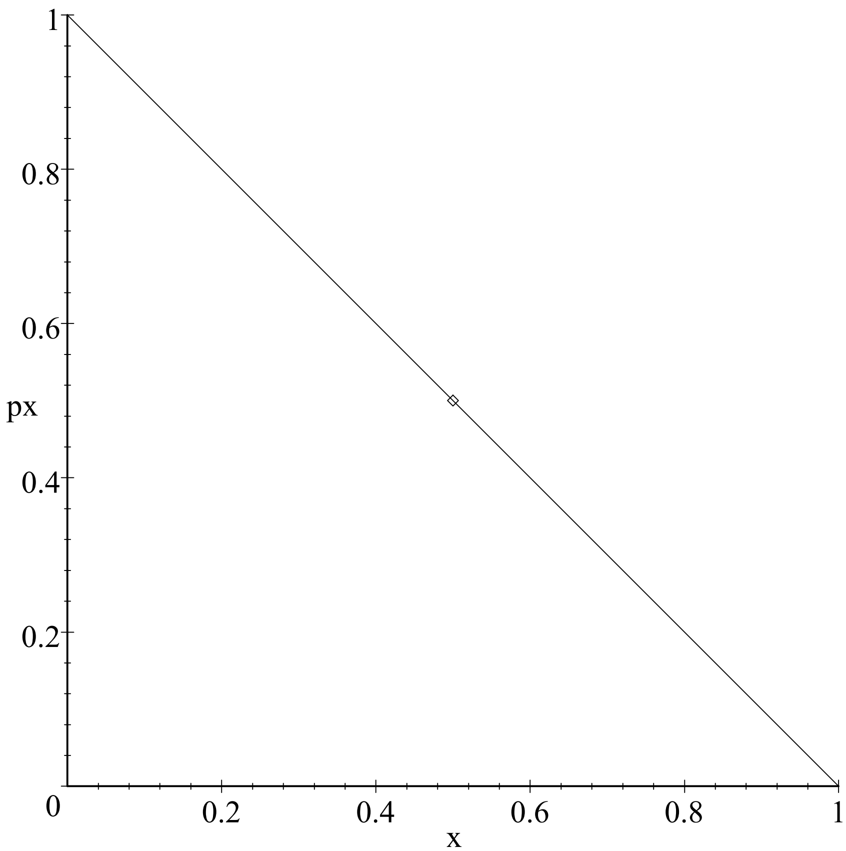
The elasticity depends on where we are on the demand curve.

Example: linear demand, $x = a - bp_x$

$$\frac{dx}{dp_x} = -b \quad \text{so} \quad \varepsilon^d = -b\left(\frac{p_x}{x}\right) = -b\frac{p_x}{a - bp_x}. \quad (2)$$

From (2), it is easy to show that the elasticity becomes higher (more negative) as the price increases. When the price is so high that the quantity demanded is near zero, at $p_x = a/b$, the elasticity approaches $-\infty$. When the price is near zero, the elasticity approaches zero.

The point of unitary elasticity is at the midpoint, where $p_x = \frac{a}{2b}$.



elastic range–unitary–inelastic range

Elasticity and Total Revenue

For any demand curve, we can write the inverse demand function as $p_x(x)$. Then the total revenue received, as a function of x, is $p_x(x)x$.

Marginal revenue is defined to be the derivative of total revenue, given by

$$MR(x) = p_x + x\frac{dp_x}{dx} = p_x\left[1 + \frac{1}{\varepsilon^d}\right]. \qquad (3)$$

From (3), we see that the point at which revenue is maximized, where $MR(x) = 0$, occurs when $\varepsilon^d = -1$.

Implications:

If we are on the elastic portion of the demand curve, revenues can be increased by reducing the price, because the increase in quantity demanded more than makes up for it. (lottery example)

If we are on the inelastic portion of the demand curve, revenues can be increased by reducing output. (This will lower the cost of production as well.)

Given that the demand for food is highly inelastic, farmers are better off when everyone has a poor harvest.

The Income Elasticity of Demand

$$\varepsilon_I^d = \frac{dx}{dM}\left(\frac{M}{x}\right)$$

If $\varepsilon_I^d > 0$, x is a normal good.

If $\varepsilon_I^d < 0$, x is an inferior good. (used cars)

If $\varepsilon_I^d > 1$, x is a luxury good.

The Cross Price Elasticity of Demand

$$\varepsilon_c^d = \frac{dx}{dp_y}\left(\frac{p_y}{x}\right)$$

If $\varepsilon_c^d > 0$, x is a gross substitute for y.

If $\varepsilon_c^d < 0$, x is a gross complement for y.