

Production

Firms convert inputs such as capital, labor, energy, materials, etc. into outputs of goods and services.

The *production function* specifies the most output that can be produced with a given combination of inputs, based on the technology available to the firm.

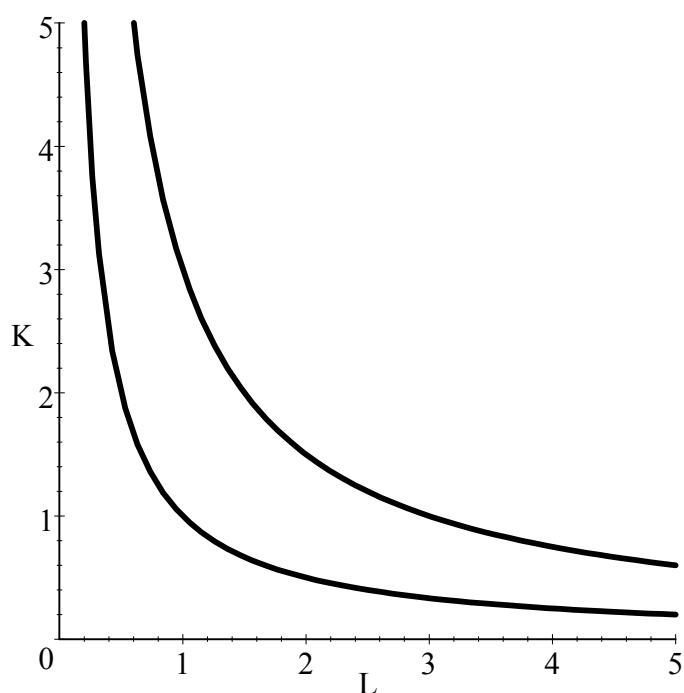
$$x = f(K, L)$$

Technological efficiency occurs if the firm is on its “production frontier.” That is, it is impossible to achieve more output with the same inputs. The firm is partially optimizing, by not wasting inputs. However, technological efficiency does not imply that the firm is choosing the right bundle of inputs. (We need to factor in prices to know that.)

Analogous to the more is better assumption for utility functions, we typically assume

$$\frac{\partial f(K, L)}{\partial K} > 0 \quad \text{and} \quad \frac{\partial f(K, L)}{\partial L} > 0.$$

A *production isoquant* (iso meaning equal and quant meaning quantity of output) is a curve describing the set of capital-labor combinations yielding the same output, according to the production function.



Production Isoquants

Production functions and isoquants are like utility functions and indifference curves, except output can be measured and traded, while utility cannot. (Cardinal vs. ordinal)

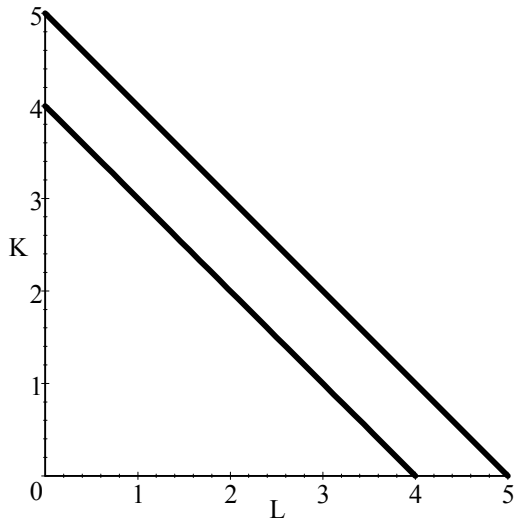
The *marginal rate of technical substitution* is defined to be the negative of the slope of the isoquant

$$MRTS = -\frac{dK}{dL} \Big|_{f(K,L)=\text{constant}}$$

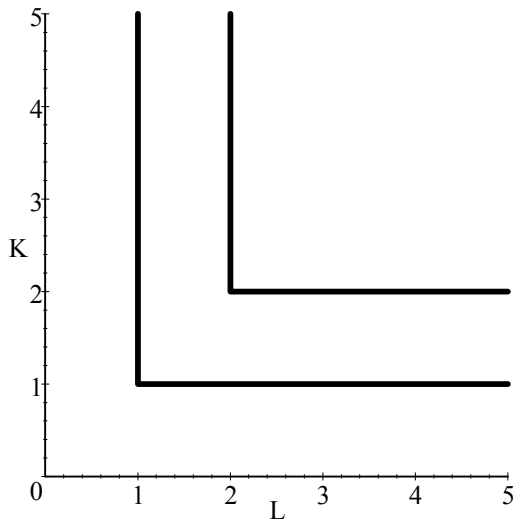
The MRTS is the rate at which the firm would be willing to give up capital in exchange for labor.

We assume diminishing MRTS, reflecting the idea that inputs will be allocated to their most essential uses first. The more labor we have, the less capital we would be willing to give up in exchange for more labor.

The curvature of the isoquant tells us the degree to which capital and labor are substitutes in the production process.



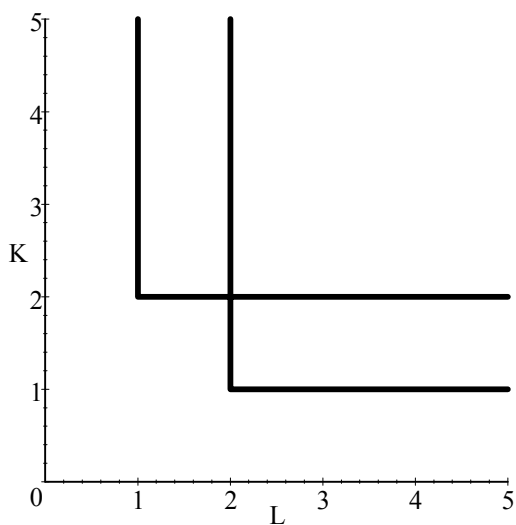
Perfect Substitutes



Perfect Complements

The case of perfect complements is noteworthy, because that is the case in which inputs must be combined in fixed proportions and there is no possibility of substitution across inputs. (For example, exact specifications for a fast food restaurant: design of restaurant, equipment, number of workers per shift, etc.)

Even if we cannot substitute inputs within a “recipe,” we can substitute across recipes.



Barnyard Breakthrough article

In the *long run*, all inputs are variable, so the firm can choose any combination of capital and labor. In the *short run*, at least one input is fixed and cannot be varied.

Returns to Scale

For long run decisions, we may be interested in what happens as we vary all of the inputs simultaneously.

The production function exhibits *decreasing returns to scale* if, for $\theta > 1$, we have

$$f(\theta K, \theta L) < \theta f(K, L).$$

The production function exhibits *constant returns to scale* if, for $\theta > 1$, we have

$$f(\theta K, \theta L) = \theta f(K, L).$$

The production function exhibits *increasing returns to scale* if, for $\theta > 1$, we have

$$f(\theta K, \theta L) > \theta f(K, L).$$

If the firm could just duplicate its existing operations by building more plants or stores, doubling the inputs and doubling the outputs, then we would have constant returns to scale.

Decreasing returns to scale come about when expanding the size of the firm requires resources to be spent to control the growing bureaucracy, or to control the increased information flows. Also, a firm would seem to have decreasing returns to scale if we were really holding some inputs fixed. For example, when an entrepreneur opens up a second and third restaurant, her attention gets stretched thinner and thinner.

Increasing returns to scale arise when there are inputs needed to get started, but which do not depend on how much output is produced. For example, writing a piece of software requires inputs from programmers, which do not depend on how many copies are sold. We could produce twice as many copies without having to double all of the inputs.

Other examples of increasing returns to scale occur when the firm must set up a network to be in business, because connecting more customers to the network requires few additional inputs. (bus service, cable TV)

example: the Cobb-Douglas production function

$$x = K^\alpha L^\beta$$

where α and β are positive constants. Multiplying both inputs by the factor, θ , we have

$$f(\theta K, \theta L) = (\theta K)^\alpha (\theta L)^\beta = \theta^{\alpha+\beta} K^\alpha L^\beta = \theta^{\alpha+\beta} f(K, L).$$

Thus, if $\alpha + \beta < 1$, we have decreasing returns to scale,

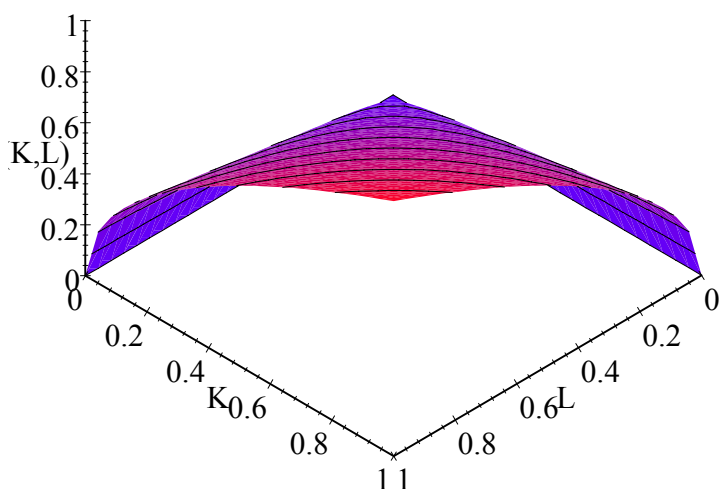
if $\alpha + \beta = 1$, we have constant returns to scale,

if $\alpha + \beta > 1$, we have increasing returns to scale.

Total, Average, and Marginal Physical Products

Hold all but one of the inputs fixed (say, fix $K = \bar{K}$). Perhaps we are in a short run situation, or perhaps we are just focusing on the effect of changing L .

The *total product of labor* is given by the function, $x = f(L; \bar{K})$. We can graph this as a cross-section of the production function.



Cobb-Douglas Production Function

The average product of labor is defined as

$$AP_L = \frac{f(L; \bar{K})}{L}.$$

The marginal product of labor is defined as

$$MP_L = \frac{\partial f(L; \bar{K})}{\partial L}.$$

Cobb-Douglas example: $x = K^\alpha L^\beta$

$$AP_L = \frac{K^\alpha L^\beta}{L} = K^\alpha L^{\beta-1}$$

$$MP_L = \frac{\partial K^\alpha L^\beta}{\partial L} = \beta K^\alpha L^{\beta-1}$$

$$AP_K = \frac{K^\alpha L^\beta}{K} = K^{\alpha-1} L^\beta$$

$$MP_K = \frac{\partial K^\alpha L^\beta}{\partial K} = \alpha K^{\alpha-1} L^\beta$$

Connection between MRTS and Marginal Products

We can get an expression for the MRTS by treating output as fixed and totally differentiating the equation

$$f(K, L) = \bar{x}.$$

By holding x fixed, we are remaining on the same isoquant as we vary K and L . This yields

$$\frac{\partial f}{\partial L}dL + \frac{\partial f}{\partial K}dK = 0.$$

Rearranging, we have

$$-\frac{dK}{dL} \Big|_{f(K,L)=\bar{x}} = \frac{\frac{\partial f}{\partial L}}{\frac{\partial f}{\partial K}}. \quad (1)$$

The left side of (1) is the MRTS, and the right side is the ratio of marginal products.

$$MRTS = \frac{MP_L}{MP_K}. \quad (2)$$

Diminishing Marginal Returns

Diminishing marginal returns (to labor) occur when the marginal product (of labor) eventually falls as L increases.

$$\frac{\partial MP_L}{\partial L} < 0$$

That is, labor is less and less productive at the margin, as L increases. It can be shown that with CRS or DRS, we must have diminishing marginal returns to each input.

Cobb-Douglas example: $x = K^\alpha L^\beta$

$$MP_L = \frac{\partial K^\alpha L^\beta}{\partial L} = \beta K^\alpha L^{\beta-1}$$
$$\frac{\partial MP_L}{\partial L} = (\beta - 1)\beta K^\alpha L^{\beta-2}$$

Thus, we have diminishing marginal returns to labor when $\beta < 1$. Constant returns to scale and diminishing marginal returns can easily coexist.