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Econ. 805 Prof. James Peck
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Extra Problems in General Equilibrium Theory (Do not hand in)

(1) An allocation is called **envy free** if every consumer prefers his/her allocation to the allocation of any other consumer. That is, x is envy free if for all h and i, we have $u_i(x_i) \geq u_i(x_h)$.

(a) Will every envy free allocation be Pareto optimal? Explain why or why not.

(b) Explain why this economy has at least one envy free allocation that is Pareto optimal. Specify any theorems that your argument relies upon. (Hint: First, redistribute resources so that everyone has the same endowment. Now can you find an allocation that is both envy free and Pareto optimal?)

(2) Give an example of a pure exchange economy with two consumers and two commodities, where: (i) each consumer has a utility function that is continuous and strictly increasing, and (ii) the conclusion of the second fundamental theorem of welfare economics is false (that is, there is a Pareto optimal allocation, strictly positive in all components, that cannot be achieved as a competitive equilibrium).

A carefully drawn Edgeworth box diagram that is clearly explained is sufficient to answer this problem.

(3) Consider an exchange economy with I different "types" of agents, and M agents of each type. In other words, the total number of agents is IM. Any two agents of the same type have the same utility function and the same strictly positive endowments. Assume that all utility functions are strictly monotonic, continuous, and strictly quasiconcave. **For each of the following two statements, either show why it is true or find a counterexample.**

(a) Any competitive equilibrium allocation must involve all agents of the same type receiving the same consumption vector.
(b) Any Pareto optimal allocation must involve all agents of the same type receiving the same consumption vector.

(4) Consider the following economy with 2 goods, x and y, and 2 consumers.

$u_1(x_1,y_1) = \log(x_1) + \log(y_1)$ endowment: 1 unit of good x, 0 units of good y

$u_2(x_2,y_2) = \log(x_2) + a \log(y_2)$ endowment: 0 units of good x, 1 unit of good y

(a) Define a competitive equilibrium for this economy. Calculate the competitive equilibrium price, as a function of the parameter, a. What is the equilibrium utility derived by consumer 2?

(b) Now suppose that the value of a in consumer 2's actual utility function is a = 5. Also suppose that consumer 2 could "pretend" to have a parameter of a' (for $1 \leq a' \leq 10$) instead of 5. That is, she could replace her true excess demand function with the demand function of a consumer with parameter, a'. Assuming that the final allocation will be the competitive equilibrium resulting from consumer 1's actual utility function and consumer 2's reported utility function (a'), what value of a' will consumer 2 choose? What will be consumer 2's actual utility?

(c) Is the allocation you found in part (b) Pareto optimal? Why or why not?

(5) Consider the following economy with 2 goods, x and y, and 2 consumers. Utility functions and endowments (the initial allocation is denoted as ω) are as follows:

$u_1(x_1,y_1) = \log(x_1) + \log(y_1)$ $\omega_1 = (2,1)$

$u_2(x_2,y_2) = \log(x_2) + \log(y_2)$ $\omega_2 = (1,2)$.

(a) Define a competitive equilibrium for this economy.
(b) Calculate the competitive equilibrium.
(c) Find an allocation that Pareto dominates ω but is not Pareto optimal.
(d) Find the allocation, X, that satisfies all 3 of the following conditions: (i) X is Pareto optimal. (ii) X Pareto dominates ω. (iii) Of all the allocations satisfying (i) and (ii), X provides the highest utility to consumer 2.

(6) In the following economy, there are 2 goods and 2 consumers. For i = 1,2, consumer i has a utility function given by

$$u_i(x_i^1, x_i^2) = \min[\, x_i^1, x_i^2 \,].$$

Consumer 1 has the endowment vector $\omega_1 = (101,0)$ and consumer 2 has the endowment vector

$\omega_2 = (0,100)$.

(a) Calculate all of the Pareto optimal allocations for this economy. Are there any Pareto optimal allocations outside the Edgeworth Box?
(b) Define a competitive equilibrium for this economy.
(c) Calculate all of the competitive equilibria.
(d) Can you apply the first fundamental theorem of welfare economics (FFTWE) to conclude that every competitive equilibrium of this economy is Pareto optimal?
(e) For this economy, is every Pareto optimal allocation a competitive equilibrium allocation for some redistribution of endowments? Why or why not?

Hint: Do not take any derivatives. Just use the definitions and your logic.

(7) Consider the following pure exchange economy with two consumers and two goods, x and y. For i = 1,2, consumer i has the utility function $u_i(x_i , y_i)$ given by

$$u_i(x_i , y_i) = \begin{cases} 2 & \text{if } y_i \geq 1 \\ 1 & \text{if } y_i < 1 \text{ and } x_i \geq 1 \\ 0 & \text{if } y_i < 1 \text{ and } x_i < 1. \end{cases}$$

The aggregate endowment for this economy is 1 unit of good x and 1 unit of good y.

(a) Does this utility function satisfy local nonsatiation? Continuity?
(b) Describe as carefully as you can the set of strongly Pareto optimal allocations (where there is no other allocation making at least one consumer strictly better off and all consumers at least as well off).
(c) Suppose each consumer's endowment vector is (1/2 , 1/2). Does this economy have a competitive equilibrium? Explain your answer.
(d) Is the conclusion of the second fundamental theorem of welfare economics true? In other words, starting with any strongly Pareto optimal allocation as the endowment, is that allocation a competitive equilibrium allocation? Explain your answer.

(8) Consider the following economy with one consumer, 2 firms, and 3 goods. The three goods are two consumption goods, x and y, and leisure, ℓ. The consumer receives no utility or disutility from leisure, as long as we have $\ell \geq 0$. The utility function is: $u(x,y,\ell) = \log(x) + \log(y)$. The consumer is endowed with 1 unit of leisure (convertible into labor on a 1 to 1 basis) and none of goods x or y: $\omega = (0,0,1)$.

Firm f, producing good x, has the production function

$$x_f = (L_f/8)^{1/2}$$

where L_f is the labor input used by firm f in producing good x. Firm g, producing good y, has the production function

$$y_g = (L_g/8)^{1/2}$$

where L_g is the labor input used by firm g in producing good y. The consumer owns both firms.

(a) What is the economy's production possibilities frontier? That is, give an expression for the maximum amount of commodity y that can be produced, as a function of the amount of commodity x produced.

(b) Define a competitive equilibrium for this economy.

(c) Calculate the competitive equilibrium prices, allocation, and profits.