Department of Economics
The Ohio State University
Final Exam Answers–Econ 805

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Directions: *Answer all questions, carefully label all diagrams, and show all work.*

1. (30 points)

For the following matrix game, find a mixed strategy Nash equilibrium. Show your work, and make sure that your notation is clearly understandable.

		player 2		
		a	b	c
	w	$5,6$	$2,3$	$1,5$
player 1	x	$8,2$	$1,5$	$4,1$
	y	$0,3$	$6,1$	$7,2$

Answer:
First, we know that player 2 will choose c with probability zero, because it is strictly dominated by a. Next, although w is not dominated by any pure strategy, it is dominated by the mixed strategy assigning probability $\frac{2}{3}$ to x and $\frac{1}{3}$ to y. Thus, player 1's mixed strategy is of the form $\sigma_1 = (0, p, 1-p)$ and player 2's mixed strategy is of the form $\sigma_2 = (q, 1-q, 0)$.

Player 1 must be indifferent between x and y, yielding the equation

$$\begin{aligned} 8q + 1(1-q) &= 0q + 6(1-q), \text{ or} \\ q &= \frac{5}{13}. \end{aligned}$$

Player 2 must be indifferent between a and b, yielding the equation

$$\begin{aligned} 2p + 3(1-p) &= 5p + 1(1-p), \text{ or} \\ p &= \frac{2}{5}. \end{aligned}$$

2. (35 points)

This question refers to the extensive form game whose game tree is drawn on the last page of this exam. Nature selects player 1's type to be H with probability $\frac{1}{5}$ and L with probability $\frac{4}{5}$. Player 1 observes his type but player

2 does not observe player 1's type. At the terminal nodes, the top number is player 1's payoff and the bottom number is player 2's payoff.

Find all of the weak perfect Bayesian equilibria (WPBE) of this game. Explain why the definition of WPBE is satisfied, and make sure that I understand your notation.

Answer:

There is only one strategy profile consistent with a $WPBE$, (XX', M). One way to see this is to construct the 4×3 payoff matrix associated with the game and see that there is only one BNE. To see this directly, consider the cases:

1. Player 1 chooses XY'. Player 2 knows she is always at the right node in her information set, so the sequentially rational choice is T, but then Y' is not sequentially rational.

2. Player 1 chooses $X'Y$. Player 2 knows she is always at the left node in her information set, so the sequentially rational choice is B, but then Y is not sequentially rational.

3. Player 1 chooses YY'. Player 2 uses Bayes' rule to assign probability $\frac{1}{5}$ to being at the left node in her information set and $\frac{4}{5}$ to being at the right node, so the sequentially rational choice is T, but then Y' is not sequentially rational.

4. Player 1 chooses XX'. If player 2 chooses T, then X is not sequentially rational, and if player 2 chooses B, then X' is not sequentially rational. Thus, player 2 must choose M.

Having ruled out the other cases, let us find the beliefs μ, about the probability of being at the left node, conditional on reaching player 2's information set, such that the strategy profile (XX', M) and the belief system characterized by μ is a $WPBE$. Given player 2's strategy, player 1's choices X and X' are sequentially rational. Because player 2's information set is off the equilibrium path, Bayes' rule does not apply and any belief is consistent. For player 2's strategy to be sequentially rational, the expected payoff (given her beliefs) from M must be at least as high as the expected payoff from T and B:

$$\begin{aligned} 3 &\geq 0\mu + 4(1-\mu) \\ 3 &\geq 4\mu + 0(1-\mu). \end{aligned}$$

Together, these inequalities are both satisfied if and only if we have $\frac{1}{4} \leq \mu \leq \frac{3}{4}$. Thus, for any μ within this range, the strategy profile (XX', M) and the belief system characterized by μ is a $WPBE$.

3. (35 points)

A monopolist faces 100 potential customers over two time periods. For $i = 1, ..., 100$, consumer i has a valuation given by $v_i = i$. That is, consumer 1 has a valuation of \$1, consumer 2 has a valuation of \$2, and so on.

The timing of the market is as follows. First the monopolist sets the price for period 1, p_1. Then, all consumers whose valuation exceeds the price buy in period 1. That is, consumer i purchases if and only if $v_i > p_1$ holds. Next, the monopolist sets the price for period 2, p_2. Then, all consumers who did not purchase in period 1 but whose valuation exceeds p_2 buy in period 2. (Notice that consumers are not modeled as strategic players.)

The monopolist has a constant marginal cost of production equal to 10, and seeks to maximize the total profit summed over the two periods. Thus, letting q_1 denote the quantity sold in period 1 and q_2 denote the quantity sold in period 2, monopoly profits are given by

$$\pi^m = p_1 q_1 + p_2 q_2 - 10(q_1 + q_2).$$

What price will the monopolist charge in each period, and what will be the monopoly profit?

Answer:

Before answering the question, let me apologize for any confusion that might have arisen regarding whether prices had to be integers or not. I originally wrote this with the demand curve $D(p) = 100 - p$, but we did not talk about the concept of residual demand in class and I was trying to avoid the confusion of having a continuum of consumers. But for the solution with this continuous demand curve to work in the current problem, prices need to be integers. With a continuous price choice, the monopolist will want to charge ε less than an integer price, and technically speaking the monopoly problem has no solution (similar to Bertrand with heterogeneous costs). Anyway I gave credit to either interpretation of the problem.

It is without loss of generality to assume that $p_2 \leq p_1$ holds, because otherwise there are no sales in period 2 and profits are the same as if the price in period 2 was set equal to the price in period 1. If the monopolist sets the integer prices, p_1 and p_2, then we have $q_1 = 100 - p_1$ and $q_2 = p_1 - p_2$. This implies

$$\begin{aligned} p_1 &= 100 - q_1 \\ p_2 &= 100 - q_1 - q_2 \end{aligned}$$

Profits are then given by

$$\pi^m = (100 - q_1)q_1 + (100 - q_1 - q_2)q_2 - 10(q_1 + q_2).$$

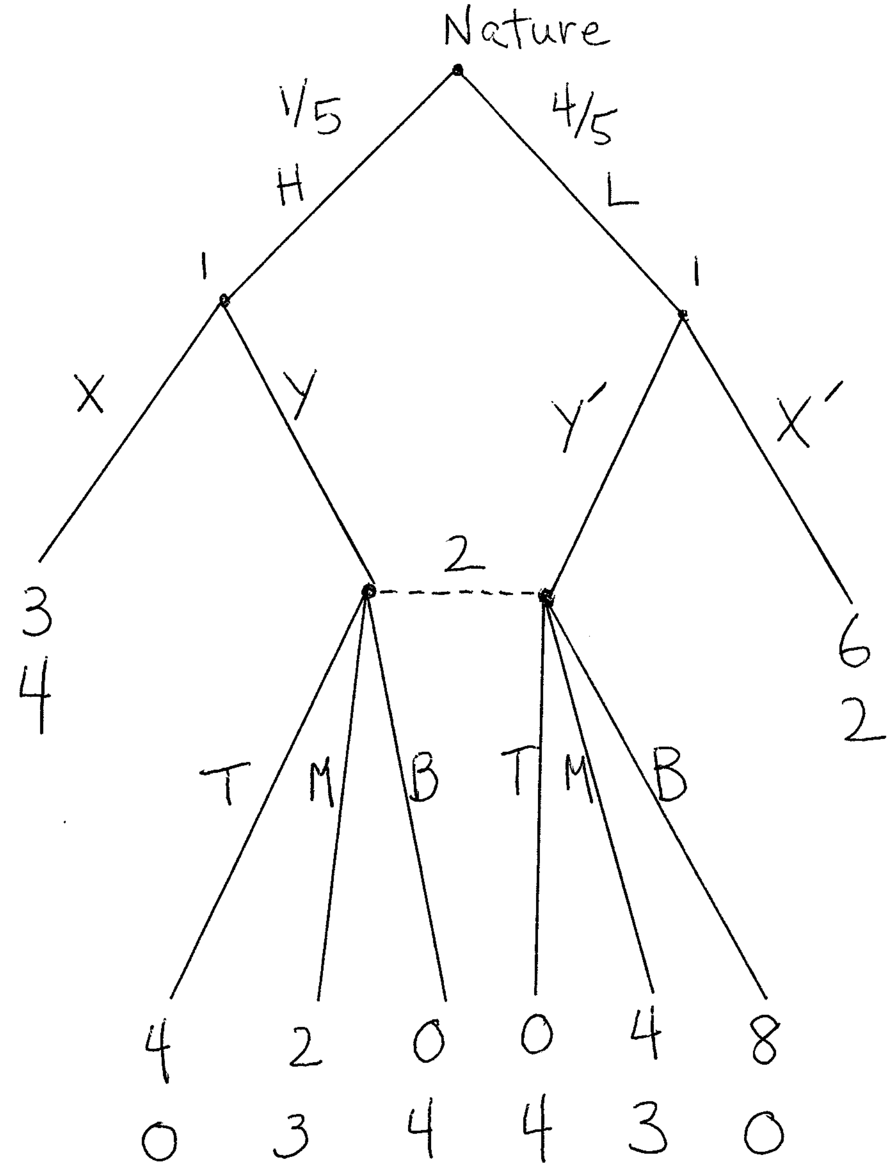
Differentiating with respect to q_1 and q_2, we have the first order conditions

$$\begin{aligned} 90 - 2q_1 - q_2 &= 0 \\ 90 - q_1 - 2q_2 &= 0. \end{aligned}$$

Solving, we have $q_1 = q_2 = 30$, which implies $p_1 = 70, p_2 = 40, \pi^m = 2700$.

If prices do not have to be integers, then by setting $p_1 = 71 - \varepsilon$ and $p_2 = 41 - \varepsilon$, the monopolist can sell the same quantities but receive profits arbitrarily close to 2760. Alternatively, by setting $p_1 = 70 - \varepsilon$ and $p_2 = 40 - \varepsilon$, the monopolist can sell 31 in period 1 and 30 in period 2, and also receive profits arbitrarily close to 2760.

As a final note, if you found the prices $p_1 = 55$ and $p_2 \simeq 22.5$, then you set the monopoly price of the one-period model in period 1, and then set the monopoly price of the one-period model (with the consumers who did not buy in period 1) in period 2. This is incorrect, because you did not take into account that raising p_1 to 70 sacrifices profits in period 1, but this is more than compensated by the additional profits in period 2. Basically, the monopolist practices a form of price discrimination.



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