Game Theory

A game in normal form (strategic form) is composed of:

1. A set of players, $i = 1, ..., n$

2. For each player, a set of possible strategies, S_i

3. An outcome function, assigning an outcome to every possible combination of strategies

$$g : S_1 \times \cdots \times S_n \rightarrow X$$

4. For each player, a utility function, which assigns utility to outcomes,

$$u_i : X \rightarrow \Re$$

Notes:

The specification of the possible strategies, and the outcome function g, are what we usually think of as the rules of the game.

If we do not specify the utility function, we have a *game form* instead of a game. For example, it is common to think of an auction in terms of who are the bidders, how the winner is determined, and what price is paid. This is an example of a game form. If we also specify the utility functions of the bidders (what information they have about the quality of the object, how much they would value consuming an object of a given quality, etc.), then we would have a game.

It is common to combine the outcome function and the utility function into a payoff function,

$$\pi_i : S_1 \times \cdots \times S_n \to \Re$$

$S \equiv S_1 \times \cdots \times S_n$

$s \equiv (s_1, ..., s_n)$

For $s \in S$, the payoff resulting from the collection of strategies, s, is $\pi(s)$.

Notice that a player's payoff can depend on the strategies chosen by all the players. Strategic interdependence.

When the number of possible strategies is finite, we can define a normal form game as a matrix.

Example: Matching Pennies. This game is a zero-sum game, purely confrontational.

		player 2	
		heads	tails
player 1	heads	$1, -1$	$-1, 1$
	tails	$-1, 1$	$1, -1$

Example: Prisoner's Dilemma. This is not a two-person zero sum game. Elements of competition and elements of gains from cooperation.

		player 2	
		cooperate	defect
player 1	cooperate	$3, 3$	$0, 4$
	defect	$4, 0$	$1, 1$

Extensive Form Games

The normal form representation does not keep track of some potentially important aspects of the game. A strategy is a complete description of what a player will do in all circumstances, so all timing considerations are implicit. Extensive form games are explicit about the timing of who moves when.

An *extensive form game* is characterized by

1. A set of nodes, a set of possible actions (different from strategies), and a set of players.

2. A *game tree*, which specifies the initial or root node, and the unique predecessor node for all other nodes. A node may have one or more successor nodes; if not, it is a terminal node.

3. A function specifying the action that takes a node to each of its successor nodes.

4. A collection of information sets, with each information set containing one or more nodes. All nodes in an information set must have the same set of available actions, and specify the same player who chooses the action. If nature moves, the probabilities of the various actions are specified.

5. Payoff functions, specifying utilities for each player at each terminal node. (Assume players seek to maximize their expected utility or payoff.)