Department of Economics
The Ohio State University
Midterm Questions and Answers–Econ 8712

Prof. James Peck
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1. (30 points)

A decision maker (DM) is a von Neumann-Morgenstern expected utility maximizer. There are two states of nature, and the DM assigns a subjective probability to each state. The DM has a Bernoulli utility function over final wealth x given by the strictly concave function $u(x) = x^{1/2}$. This function does not vary with the state. Let (x_1, x_2) denote the state-contingent allocation in which the DM consumes x_1 in state 1 and x_2 in state 2.

Suppose the DM is indifferent between the state-contingent allocation $(1, 25)$ *and the state-contingent allocation* $(4, 9)$*. What are the DM's subjective probabilities, and what is the certainty equivalent of the state-contingent allocation* $(4, 16)$*? In other words, what constant state-contingent allocation* (x, x) *will the DM find indifferent to* $(4, 16)$*?*

Answer:

Denote the probability of state 1 as π. Because the DM is indifferent between $(1, 25)$ and $(4, 9)$, we have

$$\begin{aligned} \pi(1)^{1/2} + (1-\pi)(25)^{1/2} &= \pi(4)^{1/2} + (1-\pi)(9)^{1/2} \\ \pi + 5(1-\pi) &= 2\pi + 3(1-\pi) \\ \pi &= \frac{2}{3}. \end{aligned}$$

The DM's subjective probabilities are $(\frac{2}{3}, \frac{1}{3})$. The certainty equivalent is the solution to

$$\begin{aligned} x^{1/2} &= \frac{2}{3}(4)^{1/2} + \frac{1}{3}(16)^{1/2} = \frac{8}{3} \\ x &= \frac{64}{9}. \end{aligned}$$

2. (30 points)

The following partial equilibrim economy has 1 firm, with the cost function $c(y) = (y)^2$. There are 2 consumers, each of whom has an initial endowment of money (numeraire), $\omega_i^m = 1$ and a zero endowment of the good. Also, each consumer owns a share, $\frac{1}{2}$, of each firm's profits. Letting x_i and m_i denote consumer i's consumption of the good and money, respectively, for $i = 1, 2$, consumer i has the utility function, $\log(1 + x_i) + m_i$.

(a) (20 points) Calculate the competitive equilibrium price and allocation.

(b) (10 points) Calculate the Marshallian Surplus associated with output X, given that this output is efficiently allocated to consumers.

Answer:

(a) To find the firm's supply function we equate marginal cost to the price, so the supply function is $y = \frac{p}{2}$. Profits are given by

$$py - y^2 = \frac{p^2}{4}.$$

The demand function for consumer i ($i = 1, 2$) is computed by the marginal rate of substitution condition,

$$\begin{aligned} \frac{1}{1+x_i} &= p \\ x_i &= \frac{1}{p} - 1. \end{aligned}$$

Money consumption is determined by the budget equation,

$$m_i = \frac{1}{2}(\frac{p^2}{4}) + 1 - p(\frac{1}{p} - 1) = \frac{1}{2}(\frac{p^2}{4}) + p.$$

To find the price we equate demand and supply:

$$\begin{aligned} 2(\frac{1}{p} - 1) &= \frac{p}{2} \\ p^2 + 4p - 4 &= 0 \\ p &= \frac{-4 \pm \sqrt{32}}{2}. \end{aligned}$$

Since obviously we want the positive root, we have $p = 2\sqrt{2} - 2$. Substituting into the demand and supply functions yields the allocation

$$\begin{aligned} y &= \sqrt{2} - 1 \\ x_i &= \frac{\sqrt{2}-1}{2} \\ m_i &= \sqrt{2} - \frac{1}{2}. \end{aligned}$$

Note: there are correct answers whose expressions look different, but are equivalent.

(b) The Marshallian Surplus for this problem is given by

$$\log(1 + x_1) + \log(1 + x_2) - X^2,$$

where $x_1 + x_2 = X$. Since X is distributed efficiently, we must have $x_1 = x_2 = \frac{X}{2}$, so the Surplus is equal to

$$2\log(1 + \frac{X}{2}) - X^2.$$

Another way to solve for the surplus is to compute the area between the (inverse) supply and demand curves. Since market demand is $X(p) = 2(\frac{1}{p} - 1)$, the inverse demand curve is

$$P(X) = \frac{2}{X+2}.$$

The inverse supply curve is marginal cost, so

$$C'(X) = 2X.$$

Using t as the dummy index in the integrals, Surplus is given by

$$\begin{aligned} & \int_0^X \frac{2}{t+2} dt - \int_0^X 2t dt \\ = \quad & 2\log(X+2) - 2\log(2) - X^2, \end{aligned}$$

which is the same as the above expression.

3. (40 points)

Consider the following pure-exchange economy with n consumers and two goods. Each consumer i has the utility function

$$u_i(x_i^1, x_i^2) = \log(x_i^1) + \log(x_i^2).$$

Consumer 1 has the initial endowment vector, $(A, 0)$, and consumers $2, 3, ..., n$ have the initial endowment vector, $(0, B)$, where A and B are positive numbers.

(a) (10 points) Define a competitive equilibrium for this economy.

(b) (20 points) Compute the competitive equilibrium price vector and allocation as a function of the endowment parameters.

(c) (10 points) Is it possible for a decrease in A to benefit consumer 1? Justify your answer.

Answer:

(a) In defining a C.E., I will normalize prices to be $(p, 1)$, and use monotonicity to conclude that budget constraints and market clearing hold with equality.

A C.E. is a price vector, $(p, 1)$, and an allocation, (x_i^1, x_i^2) for $i = 1, ..., n$, such that

(i) (x_1^1, x_1^2) solves

$$\begin{aligned} & \max \log(x_1^1) + \log(x_1^2) \\ & \text{subject to} \\ p x_1^1 + x_1^2 &= pA \\ x_1 &\geq 0, \end{aligned}$$

(ii) for $i = 2, ..., n$, (x_i^1, x_i^2) solves

$$\begin{aligned} & \max \log(x_i^1) + \log(x_i^2) \\ & \text{subject to} \\ p x_i^1 + x_i^2 &= B \\ x_i &\geq 0, \end{aligned}$$

(iii) markets clear:

$$\begin{aligned} \sum_{i=1}^{n} x_i^1 &= A \\ \sum_{i=1}^{n} x_i^2 &= (n-1)B. \end{aligned}$$

(b) The demand function for consumer 1 is characterized by the budget equation and the marginal rate of substitution condition,

$$\frac{x_1^2}{x_1^1} = p,$$

yielding

$$\begin{aligned} x_1^1 &= \frac{A}{2} \\ x_1^2 &= \frac{pA}{2}. \end{aligned}$$

For $i = 2, ..., n$, the demand function for consumer 2 is characterized by the budget equation and the marginal rate of substitution condition,

$$\frac{x_i^2}{x_i^1} = p,$$

yielding

$$\begin{aligned} x_i^1 &= \frac{B}{2p} \\ x_i^2 &= \frac{B}{2}. \end{aligned}$$

Market clearing for good 2 implies

$$\begin{aligned}\frac{pA}{2} + (n-1)\frac{B}{2} &= (n-1)B \\ p &= \frac{(n-1)B}{A}.\end{aligned}$$

Substituting the price into the demand functions yields the equilibrium allocation,

$$\begin{aligned}x_1 &= (\frac{A}{2}, \frac{(n-1)B}{2}) \\ x_i &= (\frac{A}{2(n-1)}, \frac{B}{2}) \text{ for } i = 2, ..., n.\end{aligned}$$

(c) Looking at the equilibrium consumption for consumer 1, it is clear that a decrease in A decreases his/her consumption of good 1, and it does not affect his/her consumption of good 2. Therefore, consumer 1's utility cannot increase if A decreases.