Department of Economics
The Ohio State University
Midterm Exam Questions and Answers–Econ 8712

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1. (35 points)

A decision maker (DM) is a von Neumann-Morgenstern expected utility maximizer. The DM must predict the outcome of a random variable that is uniformly distributed over the unit interval, $[0, 1]$. Denoting the realization of the random variable as x and the prediction as P, the DM's Bernoulli utility function is given by

$$
\begin{aligned}
u(x, P) &= -(P - x) \quad \text{if} \quad P \geq x \\
u(x, P) &= -a(x - P) \quad \text{if} \quad P < x,
\end{aligned}
$$

where a is a positive parameter. (You can think of the DM as facing a "loss" associated with the gap between x and the prediction, but that the loss associated with an over-prediction is different from the loss associated with an under-prediction.)

Recall that the uniform distribution has a cdf given by $F(x) = x$ and a density function given by $f(x) = 1$.

(i) (10 points) Set up the optimization problem faced by the DM.
(ii) (25 points) Find the optimal prediction, P.

Answer:

Based on the uniform distribution and the given Bernoulli utility function, the DM's expected utility, as a function of P and the parameter a, is given by

$$
\begin{aligned}
U(P) &= -\int_{x=0}^{x=P} (P - x)dx - \int_{x=P}^{x=1} a(x - P)dx \\
&= (\frac{x^2}{2} - Px)|_0^P + a(Px - \frac{x^2}{2})|_P^1 \\
&= -\frac{P^2}{2} + a(P - \frac{1}{2}) - a(\frac{P^2}{2}) \\
&= a(P - \frac{1}{2}) - (1 + a)\frac{P^2}{2}.
\end{aligned}
$$

Notice that $U''(P) < 0$ holds, so this is a concave function. To find the optimal P, differentiate with respect to P, set the expression equal to zero, and solve for P. This yields the interior solution,

$$P = \frac{a}{1 + a}.$$

2. (30 points)

The following partial equilibrium economy has 2 consumers and 2 firms. Consumer 1 has the utility function,

$$u_1(x_1, m_1) = \log(x_1) + m_1,$$

the numeraire endowment $\omega_1^m = 1$, and owns firm 1. Consumer 2 has the utility function,

$$u_2(x_2, m_2) = 2\log(x_2) + m_2,$$

the numeraire endowment $\omega_2^m = 1$, and owns firm 2.

Firm 1 has the cost function given by

$$c_1(y_1) = (y_1)^2$$

and firm 2 has the cost function given by

$$c_2(y_2) = \frac{1}{5}(y_2)^2.$$

Solve for the competitive equilibrium price and allocation. The allocation should specify the output of each firm, the good consumption of each consumer, and the numeraire consumption of each consumer.

Answer:

First solve the firms' profit maximization problems. Equating marginal cost to the price for firm 1, we have

$$2y_1 = p,$$

so the supply function is $y_1 = \frac{p}{2}$ and profits are $\frac{p^2}{4}$. Similarly, for firm 2 we have

$$\frac{2}{5}y_2 = p,$$

so the supply function is $y_2 = \frac{5p}{2}$ and profits are $\frac{5p^2}{4}$.

Equating marginal utility to the price for consumer 1, we have

$$\frac{1}{x_1} = p,$$

yielding the demand function, $x_1 = \frac{1}{p}$. Equating marginal utility to the price for consumer 2, we have

$$\frac{2}{x_2} = p,$$

yielding the demand function, $x_2 = \frac{2}{p}$.

Equating demand equal to supply, we have

$$\begin{aligned}\frac{1}{p}+\frac{2}{p} &= \frac{p}{2}+\frac{5p}{2}, \text{ or}\\ \frac{3}{p} &= \frac{6p}{2}.\end{aligned}$$

Solving, we have $p = 1$. Thus, the consumption and production of the good is given by $x_1 = 1, x_2 = 2, y_1 = \frac{1}{2}, y_2 = \frac{5}{2}$. To find the numeraire consumptions, use the budget constraint and remember to account for profits. We have

$$\begin{aligned}m_1 &= \omega_1^m + \frac{p^2}{4} - px_1 = \frac{1}{4}.\\ m_2 &= \omega_2^m + \frac{5p^2}{4} - px_2 = \frac{1}{4}.\end{aligned}$$

3. (35 points)

The following pure exchange economy has 2 consumers and 3 goods. For $i = 1, 2$, consumer i has the utility function

$$u_i(x_i^1, x_i^2, x_i^3) = \log(x_i^1) + \log(x_i^2) + \log(x_i^3).$$

Consumer 1's initial endowment vector is $\omega_1 = (1, 0, 2)$ and consumer 2's initial endowment vector is $\omega_2 = (0, 1, 0)$.

Normalize the price of good 3 to be one, $p^3 = 1$. Calculate the competitive equilibrium price vector and allocation.

Answer:

To solve for the demand functions, we use two marginal rate of substitution conditions and the budget equation. For consumer 1, we have

$$\begin{aligned}\frac{x_1^3}{x_1^1} &= p^1\\ \frac{x_1^3}{x_1^2} &= p^2\\ p^1x_1^1 + p^2x_1^2 + x_1^3 &= p^1 + 2.\end{aligned}$$

Solving, we have

$$\begin{aligned}x_1^1 &= \frac{p^1+2}{3p^1}\\ x_1^2 &= \frac{p^1+2}{3p^2}\\ x_1^3 &= \frac{p^1+2}{3}.\end{aligned}$$

For consumer 2, we have

$$\begin{aligned}
\frac{x_2^3}{x_2^1} &= p^1 \\
\frac{x_2^3}{x_2^2} &= p^2 \\
p^1 x_2^1 + p^2 x_2^2 + x_2^3 &= p^2.
\end{aligned}$$

Solving, we have

$$\begin{aligned}
x_2^1 &= \frac{p^2}{3p^1} \\
x_2^2 &= \frac{1}{3} \\
x_2^3 &= \frac{p^2}{3p^1}.
\end{aligned}$$

To find the equilibrium prices, use two market clearing conditions. For good 1, we have

$$\begin{aligned}
\frac{p^1+2}{3p^1} + \frac{p^2}{3p^1} &= 1, \text{ or} \\
p^2 &= 2p^1 - 2.
\end{aligned}$$

For good 2, we have

$$\begin{aligned}
\frac{p^1+2}{3p^2} + \frac{1}{3} &= 1, \text{ or} \\
p^1 &= 2p^2 - 2.
\end{aligned}$$

Solving for the prices, we have $p^1 = p^2 = 2$.

Substituting the prices into the demand functions yields the allocation, $x_1 = (\frac{2}{3}, \frac{2}{3}, \frac{4}{3})$ and $x_2 = (\frac{1}{3}, \frac{1}{3}, \frac{2}{3})$.