Department of Economics
The Ohio State University
Econ 8817–Advanced Game Theory

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Homework #2 Answers

1. O-R, exercise 56.4.

Answer: By the symmetry of the game, the set of rationalizable pure actions is the same for both players. Call it Z. Consider $m \equiv \inf(Z)$ and $M \equiv \sup(Z)$. Any best response of player i to a belief about player j (whose support is a subset of Z) maximizes $E(a_i(1 - a_i - a_j))$, or equivalently, it maximizes $a_i(1 - a_i - E(a_j))$. Thus, player i's best response to a belief about player j depends only on $E(a_j)$, which can be written as $B_i(E(a_j)) = (1 - E(a_j))/2$. Because $m \leq E(a_j) \leq M$ must hold, $a_i \in B_i(E(a_j))$ implies $a_i \in [(1-M)/2, (1-m)/2]$. By the best response property of the rationalizable set, we have $m \in [(1-M)/2, (1-m)/2]$ and $M \in [(1-M)/2, (1-m)/2]$. Therefore, we have

$$m \geq \frac{1-M}{2} \text{ and} \tag{1}$$

$$M \leq \frac{1-m}{2}. \tag{2}$$

It follows from (1) and (2) that $m \geq M$ holds, which can only occur if $m = M$. From (1) and (2), we have $m = M = 1/3$. Therefore, the only rationalizable strategy is the unique Nash equilibrium strategy, $a_i = 1/3$.

2. O-R, exercise 76.1.

Answer: The simplest example, in which it is common knowledge that two players have different posteriors about some event A, is the following. There are two states, with prior probability 1/2 for each state. $\Omega = \{1, 2\}$ and $p(1) = p(2) = 1/2$. Player 1 cannot distinguish between the two states, $\wp_1 = \{\{1, 2\}\}$, and player 2 can distinguish between the two states, $\wp_2 = \{\{1\}, \{2\}\}$. Therefore, the meet of the two information structures is $\wp_1 \wedge \wp_2 = \{\{1, 2\}\}$. Let $A = \{1\}$. At $\omega = 1$, player 1's posterior is 1, and player 2's posterior is 1/2. At $\omega = 2$, player 1's posterior is 0, and player 2's posterior is 1/2. Because posteriors are different at all states, it is common knowledge that posteriors are different.

Let $E = \{\omega' : q_1(\omega') > q_2(\omega')\}$. Suppose E is common knowledge at ω. Let M be the element of $\wp_1 \wedge \wp_2$ containing ω. Then $M = \bigcup_j P_1^j$, where we have the union of disjoint elements of $\wp_1$, and $M = \bigcup_j P_2^j$, where we have the union of disjoint elements of $\wp_2$.

Because E is common knowledge at ω, we must have $q_1(\omega') > q_2(\omega')$ for all $\omega' \in M$.

Therefore, for all $P_1^j \subseteq M$, and all $P_2^j \subseteq M$, we have

$$\frac{pr(A \cap P_1^j)}{pr(P_1^j)} > \frac{pr(A \cap P_2^j)}{pr(P_2^j)}$$

Cross multiplying, $pr(P_2^j)pr(A \cap P_1^j) > pr(P_1^j)pr(A \cap P_2^j)$.

Summing over (disjoint) $P_1^j \subseteq M$, we have $pr(P_2^j)pr(A \cap M) > pr(M)pr(A \cap P_2^j)$.

Summing over (disjoint) $P_2^j \subseteq M$, we have $pr(M)pr(A \cap M) > pr(M)pr(A \cap M)$, a contradiction.

3. O-R, exercise 146.1.

Answer: The minmax payoffs are given by $v_1 = v_2 = 1$, which implies that player 1 must receive a payoff of at least 1 in any subgame perfect equilibrium of the repeated game. Player 2's payoff exceeds player 1's payoff by at least 1 at any action profile of the stage game, so player 2 must receive a payoff of at least 2 in any subgame perfect equilibrium of the repeated game. Suppose $((A, A), (A, A), ...)$ is the outcome path of a subgame perfect equilibrium. Player 2's payoff is 3. By deviating to D in the first period, player 2 receives a payoff of 5 in period 1 and a continuation payoff of at least 2, because the contintuation strategies after the deviation must form a subgame perfect equilibrium. Therefore, the deviation yields a payoff of at least

$$(1-\delta)(5 + \sum_{t=1}^{\infty} \delta^t 2) = \frac{7}{2},$$

which is greater than 3.