Department of Economics
The Ohio State University
Econ 8817 Game Theory

Fall 2014
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Homework #3–Due Thursday November 6

Directions: Answer all questions, and be neat. If you discuss the questions in study groups, list the members of your study group, and make sure that the writeup is your own work. In particular, do not look at the O-R solutions manual or any answers handed out in previous years.

1. O-R, exercise 217.3. You can formulate the game as defined in O-R, or as a carefully labelled game tree. Find all Nash equilibria, allowing for mixed strategies.

2. Player 1 is involved in an accident with player 2 with damages of \$12,000. Player 1 knows whether or not she is negligent. Player 2 does not know player 1's type, but instead assigns prior probability $\frac{1}{2}$ to each of the two types (negligent or not negligent). Player 1 must send player 2 a pre-trial take-it-or-leave-it offer, which must be either \$4000 or \$8000, after which player 2 decides either to accept or reject. If he accepts the offer, $m \in \{4000, 8000\}$, then player 1's payoff is $-m$ and player 2's payoff is m. If he rejects the offer, the case goes to trial, which player 1 wins if she is not negligent, and player 2 wins if player 1 is negligent. The loser pays the court costs of \$2000. Therefore, if the case goes to trial and player 1 is negligent, payoffs are $(-14000, 12000)$. If the case goes to trial and player 1 is not negligent, payoffs are $(0, -2000)$.

Formulate this as a signalling game and find all pure-strategy perfect Bayesian equilibria.

3. Consider the following cooperative game involving 5 political parties. The number of votes controlled by party i is denoted by x_i, where we have $x_1 = 45$, $x_2 = 45$, $x_3 = 3$, $x_4 = 3$, and $x_5 = 4$. Any coalition receiving a majority of the votes will be able to form a government and divide the surplus. Thus, the characteristic function is given by

$$\begin{aligned} v(S) &= 1 \quad \text{if} \quad \sum_{i \in S} x_i \geq 51 \\ &= 0 \quad \text{otherwise.} \end{aligned}$$

(a) Find the core of this game.

(b) Find the Shapley value of this game.

(c) What would be the Shapley value of the game in which the three smaller parties (3, 4, and 5) merged into a single party? If players receive their Shapley value allocation, it in their interest to merge?