Bulow + Klemperer

"Willingness to pay" is not the same as a "demand curve" (derived from utility max s.t. budget constraint)

You can always decide to buy later. Makes willingness to pay very elastic.

Frenzy — a single purchase causes many others to come forward at the same price.

Crash — it becomes common knowledge that no further buyers will come forward unless the price drops significantly.

Model

K units for sale by a single seller.
K+L risk neutral buyers.

private values

valuations i.i.d. ~ $F(v)$ distribution
$F(v)$ strictly increasing and atomless on $[\underline{v},$

Seller starts at $p = \bar{v}$, lowering it continuously until all units are sold. That is,

① • when a purchase occurs, remaining buyers can change their mind and try to buy at the price.

- if demand does not exceed supply, these sales are made and back to 1
- if exactly all goods are sold, end.
- if no one is left who wants to buy, the price is continuously lowered.
- if more buyers simultaneously want to buy than the available supply, ($k+l$ buyers, k supply) then these $k+l$ play, with price starting at $\bar{V}$.

symmetric equilibria where no one bids more than his/her valuation.
[otherwise, everyone could bid $\bar{V}$, knowing that there will be excess demand and they don't have to buy.]

$\omega(v)$ expected price, conditional on receiving the object in a standard English auction.

$$\omega(v) = E(k+1^{st} \text{ valuation} \mid k+1^{st} \leq v)$$

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$$E\left(k+1^{st} \,\middle|\, k+1^{st} < V\right) =$$

$$\cancel{= \int \text{prob}\left(k+1^{st} < V \text{ and } k+1^{st} = x\right) \cdot x}$$

$$\int_{\underline{v}}^{\bar{v}} x \ \text{prob.}\left(k+1^{st} = x \,\middle|\, k+1^{st} < V\right) dx$$

$$\text{prob}(A|B) = \frac{\text{prob}(A \text{ and } B)}{\text{prob.}(B)}$$

$$\int_{\underline{v}}^{\bar{v}} \left[x \frac{f(x)\left[F(\bar{v}) - F(x)\right]^{k-1}\left[F(x) - F(\underline{v})\right]^{\ell-1}}{\int_{\underline{v}}^{\bar{v}} f(x')\left[F(\bar{v}) - F(x')\right]^{k-1}\left[F(x') - F(\underline{v})\right]^{\ell-1} dx'} \right] dx$$

- binomial terms (n choose k) cancel
- $\omega(V)$ really depends on $\underline{v}$ and $\bar{v}$, which depend on the play.

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facts:

1) optimal strategy is to offer to buy iff v is greater than some cutoff level.

2) information publicly revealed is that all valuations of remaining bidders lie between some $\underline{v}$ and $\bar{v}$

3) highest valuation bidders will receive the objects.

<u>Revenue Equivalence Thm</u> p7

Since it applies to all trading mechanisms satisfying (1) and (2), the ~~value~~ expected payment of continuing is $\omega(\hat{v})$

<u>Prop.</u> Offer to purchase iff $\omega(v) \geqq p$

<u>Proof</u>: suppose person with valuation $\tilde{v}$ is indifferent to bidding p. Either win now and pay p, or enter a new smaller lottery (auction) and get outbid. $\therefore$ expected payment/win is p.

by rev. eq.

$\therefore \ \omega(\tilde{v}) = p$

result follow from fact ①.

revenue equivalence

$\pi(v)$ prob. v receives a unit

$S(v)$ exp. surplus

$E(v)$ exp. payment, cond. on receiving a unit

$$S(v^a) = \pi(v^a)\left[v^a - E(v^a)\right] \geq \pi(v^b)\left[v^a - E(v^b)\right]$$

$$\pi(v^b)\left[v^a - E(v^b)\right] = S(v^b) + \pi(v^b)\left[v^a - v^b\right]$$

(take lim.. $v^a \to v^b$)

$\therefore$

$$S'(v) = \pi(v)$$

$$\therefore \quad S(v) = \int_{\underline{v}}^{v} \pi(x)\,dx + S(\underline{v})$$

$$S(\underline{v}) = 0$$

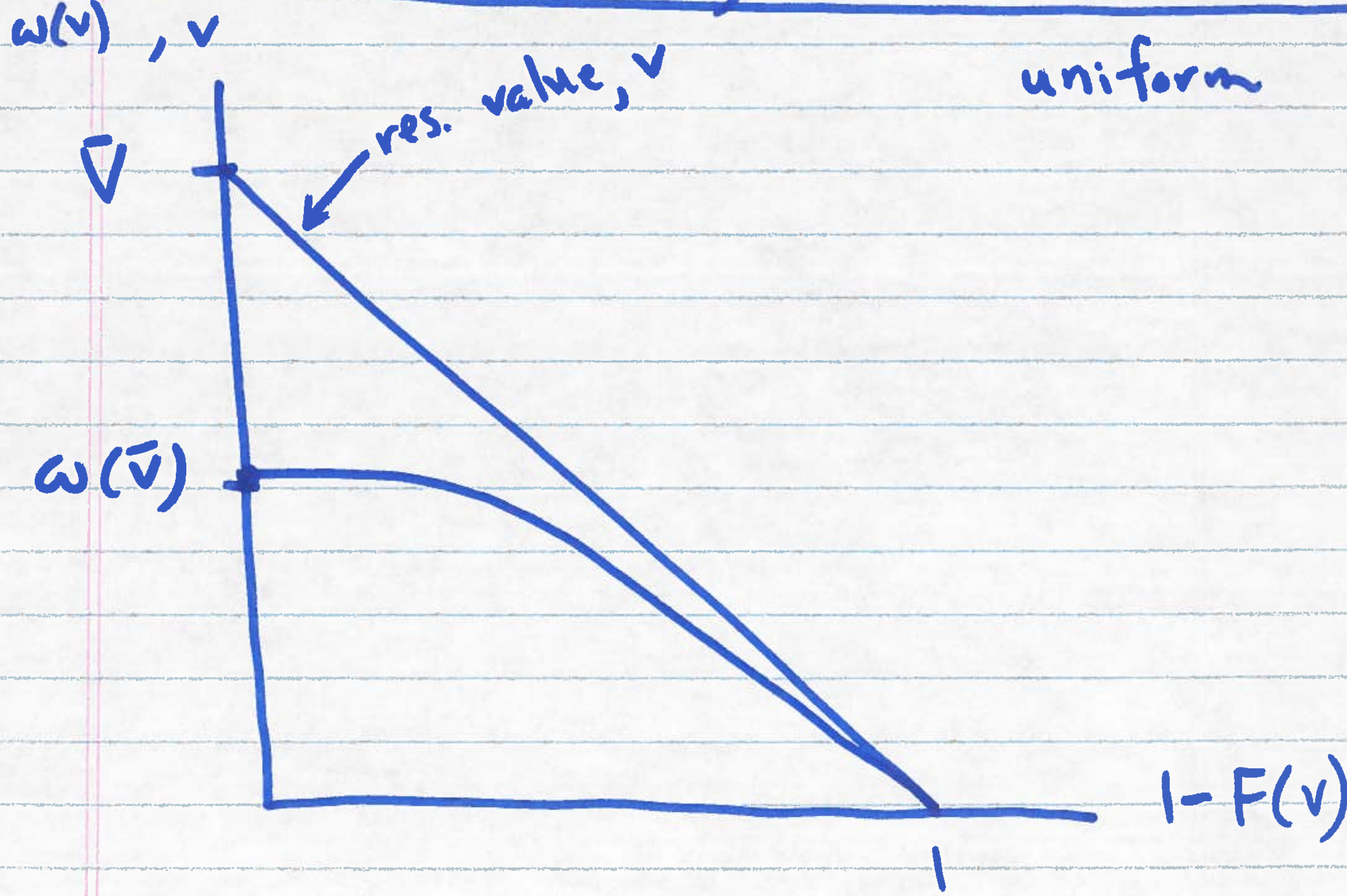
This only depends on the prob. that v's are one of the highest signals, not the mechanism

$$E(v) = v - \frac{S(v)}{\pi(v)}$$ does not depend on the mech.

Characterizing Price Paths

how does $\omega(v)$ respond to information?

initially, there is a mini crash until the price falls to $\omega(\bar{V})$ $\bar{v} = \bar{V}$.
As the price is lowered further with no sale, $\bar{V}$ is continuously lowered to $\omega^{-1}(P)$



The first sale goes to the highest valuation, V_1.
The price is $p = \omega(V_1)$.

$\bar{V}$ becomes V_1
# of objects becomes $k-1$.
" bidders " $k+l-1$

removing 1 object & 1 bidder increases $\omega(v)$.
$\therefore$ there is $\tilde{V} < V_1$ s.t. $\omega(\tilde{V}) = p$.

All bidders with $\tilde{V} < V < V_1$ participate in a frenzy. (say, j of them incl. first)

# of objects remaining becomes $k-j$
# of bidders becomes $k+l-j$
→ raises $\omega(v)$

$\bar{V}$ becomes $\tilde{V}$ → lowers $\omega(v)$

If j is low, 2nd effect dominates and we have a crash to $\omega(\tilde{V})$ or below

If j is large, 1st effect dominates and there might be a second round of frenzy.

If $j > k$ (remaining) there is excess demand.
$\underline{V}$ becomes $\tilde{V}$
$\bar{V}$ becomes V_1
bidders becomes j

$\omega(v)$ shifts up, and the price might go up

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intuition for flat $\omega(v)$ p. 14

many in frenzy p. 16
crashes p. 16

Lower valuations could affect information flow and lead to higher revenues.

$V_1 \downarrow$ means a lower price, more people in the first frenzy, . . .

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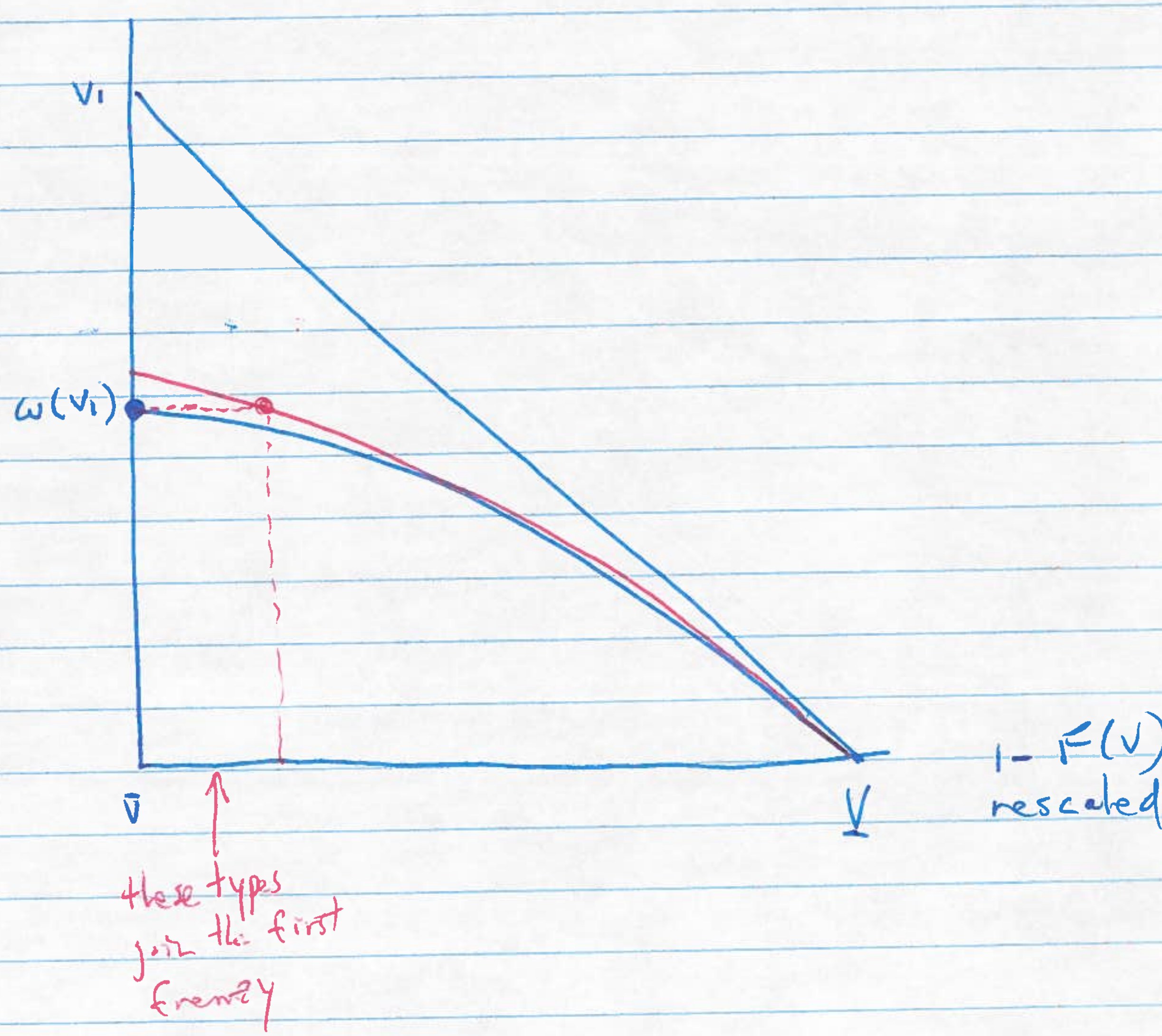
Common Values — decision to purchase raises your valuation, making frenzies more likely.

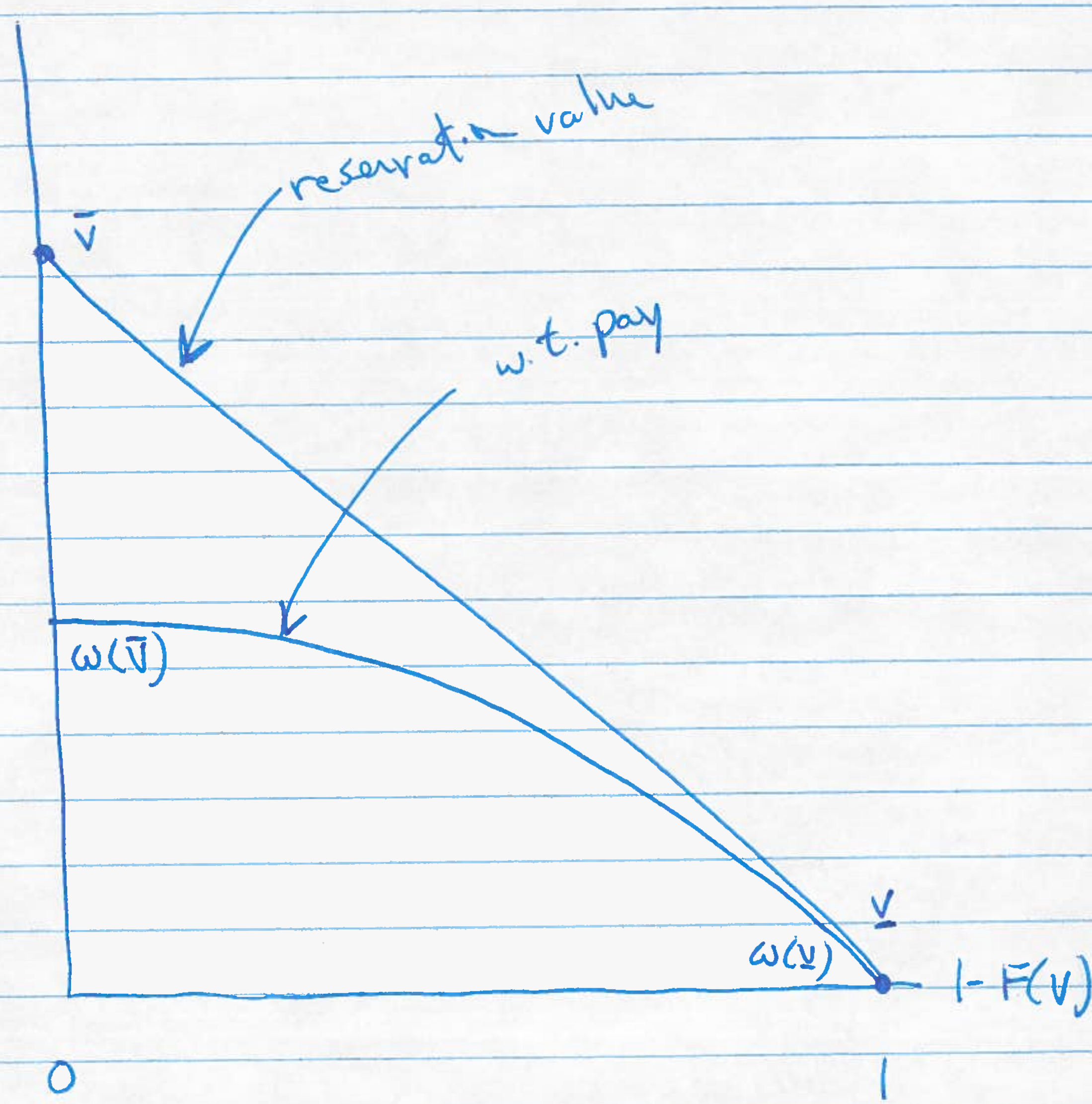
"Herd behavior" relies on common values. This work does not.

Assumptions

1) 1 seller
2) only buyers have asymmetric info
* 3) Once a sale is made, others can simultaneously say yes or no at that price.

As $p \downarrow$, below $\omega(\bar{V})$, $\bar{v}$ gradually falls and $\omega(v)$ gradually shifts down. First sale occurs when $p = \omega(v_1; [\underline{V}, v_1])$




reservation value
w.t. pay
$\bar{v}$
$\omega(\bar{v})$
$\underline{v}$
$\omega(\underline{v})$
$1-F(v)$
0
1