Grossman & Stiglitz

no arbitrage equations cannot always be satisfied, because no one will gather information.

R safe return

$u = \theta + \varepsilon$ risky return

θ is observable at cost c

informed demand depends on (θ, P)

uninformed " " " P — price

X supply of risky asset

λ fraction of agents that are informed

price function $P_\lambda(\theta, X)$

Rational Expectations uninformed & informed know the function $P_\lambda(\theta, X)$.

Uninformed do not observe X, or else they can invert P_λ to determine θ. NOISE

Equilibrium choice to become informed requires EU to be equated, which pins down λ.

Conjectures 1-6

Constant Absolute Risk Aversion

P price of risky asset

$$\max \quad V(W_{1i}) = -e^{-aW_{1i}} \qquad a>0$$

initial wealth $\quad W_{0i} \equiv \bar{M}_i + P\bar{X}_i$

trade: budget constraint is $\quad PX_i + M_i = W_{0i}$

Final wealth $\quad W_{1i} = RM_i + uX_i \quad$ is random.

Assume θ and ε have a multivariate normal distribution with

$$E(\varepsilon) = 0$$

$$E(\theta\varepsilon) = 0$$

$$Var(u|\theta) = Var(\varepsilon) \equiv \sigma_\varepsilon^2 > 0$$

$\therefore$ W_{1i} is normal, conditional on θ

(no limited liability!)

3/

Informed

$$E\left(V\left(W_1^*|\theta\right)\right) =$$

$$-\exp\left(-a\left\{E\left[W_1^*|\theta\right] - \frac{a}{2}\mathrm{Var}\left[W_1^*|\theta\right]\right\}\right)$$

$$RM + E(u^*|\theta)X$$

$$R(W_0 - XP) + E(u^*|\theta)X$$

$$RW_0 + X\left(E(u^*|\theta) - RP\right)$$

$$X^2\,\mathrm{Var}(u^*|\theta)$$

$$RW_0 + X_I(\theta - RP) \qquad X_I^2\sigma_\varepsilon^2 \qquad \text{eq. 7}$$

X_I informed demand

$\max_{X_I}$ (7) unconstrained yields

$$X_I(P,\theta) = \frac{\theta - RP}{a\sigma_\varepsilon^2}$$

Uninformed

$p^*(\theta, x)$

guess that u^* and p^* are jointly normally distributed.

$$E\left(V(W_1) \mid p^*\right) = -\exp\left[-a\left\{RW_0 + X_u\left(E[u^*/p^*] - RP\right) - \frac{a}{2} X_u^2 \, Var(u^*/p^*)\right\}\right]$$

demand $X_u(P; p^*)$

$$\frac{E\left[u^* \mid p^*(\theta, x) = P\right] - RP}{a \, Var\left[u^* \mid p^*(\theta, x) = P\right]}$$

Equilibrium

$$x = \lambda X_I\left(P_\lambda(\theta, x), \theta\right) + (1-\lambda) X_u\left(P_\lambda(\theta, x); P^*_\lambda\right)$$

$\forall x, \theta$

<u>define</u> $$W_\lambda(\theta, x) = \theta - \frac{a\sigma_\varepsilon^2}{\lambda}(x - Ex^*) \quad \lambda > 0$$

$$x \quad \lambda < 0$$

price is proportional to $(\theta + \text{noise})$

<u>Thm 1</u> p. 97

$$\mathrm{Var}\left[W_\lambda^* \mid \theta\right] = \frac{a^2 \sigma_\varepsilon^4}{\lambda^2} \mathrm{Var}(x^*)$$

<u>$a = 0$</u> risk neutrality — price has 0 variance, observe θ
$P_\lambda(\theta, x) = \alpha_1 + \alpha_2 \theta$

<u>$\sigma_\varepsilon^2 = 0$</u> informed observe u and return must equal safe return. Uninformed can observe θ and u.

<u>$\mathrm{Var}(x^*) = 0$</u> Uninformed can figure out θ from the price function, since they know x.

Information Market

Figure out utilities, and find λ such that traders are indifferent.

$$\rho_\theta^2 = \frac{1}{1+m}$$

(squared correlation coeff. between P_λ^* and θ^*) [informativeness of price system]

$$\frac{\mathrm{Var}(W_\lambda)}{\mathrm{Var}(\theta)} = m = \left(\frac{a\sigma_\varepsilon^2}{\lambda}\right)^2 \frac{\sigma_x^2}{\sigma_\theta^2}$$

$$n = \frac{\sigma_\theta^2}{\sigma_\varepsilon^2}$$

[quality of informed traders' info]

ρ_θ^2 squared correlation coeff. between P_λ^* and θ

$$\rho_\theta^2 = \frac{1}{1+n}$$

from Thm 2

$$(19b) \quad 1-\rho_\theta^2 = \frac{e^{2ac}-1}{n}$$

equil. informativeness of the price system

lower $1-\rho_\theta^2$, more informative prices.

- increasing in quality of information, n
- decreasing in c
- decreasing in risk aversion (smaller positions)
- constant as we vary σ_x^2

thm 2

hard

$$1 = \frac{EV(W_I^\lambda)}{EV(W_u^\lambda)} = e^{ac}\sqrt{\frac{Var(u^*|\theta)}{Var(u^*|W_\lambda)}}$$

From 18

hard

$$1 = e^{ac}\sqrt{\frac{1+m}{1+m+nm}}$$

$$(1+m)e^{2ac} = 1+m+nm$$

$$e^{2ac}-1 = m\left[1+n-e^{2ac}\right]$$

19a $$m = \frac{e^{2ac}-1}{1+n-e^{2ac}}$$ or $$\boxed{1-\rho_\theta^2 = \frac{e^{2ac}-1}{n}}$$

19b

Thm 5 p104

No noise $\sigma_x^2 = 0$

all information is transmitted by the price. No one will become informed. If the cost is low enough, it pays to become informed when $\lambda = 0$. No equil.

When $W_\lambda = \theta$ $\lambda > 0$

easy to show $X_u(P_\lambda; P_\lambda^*) = X_I(P_\lambda; \theta)$

so market clearing $\Longrightarrow X_I(P_\lambda(\theta), \theta) = X$

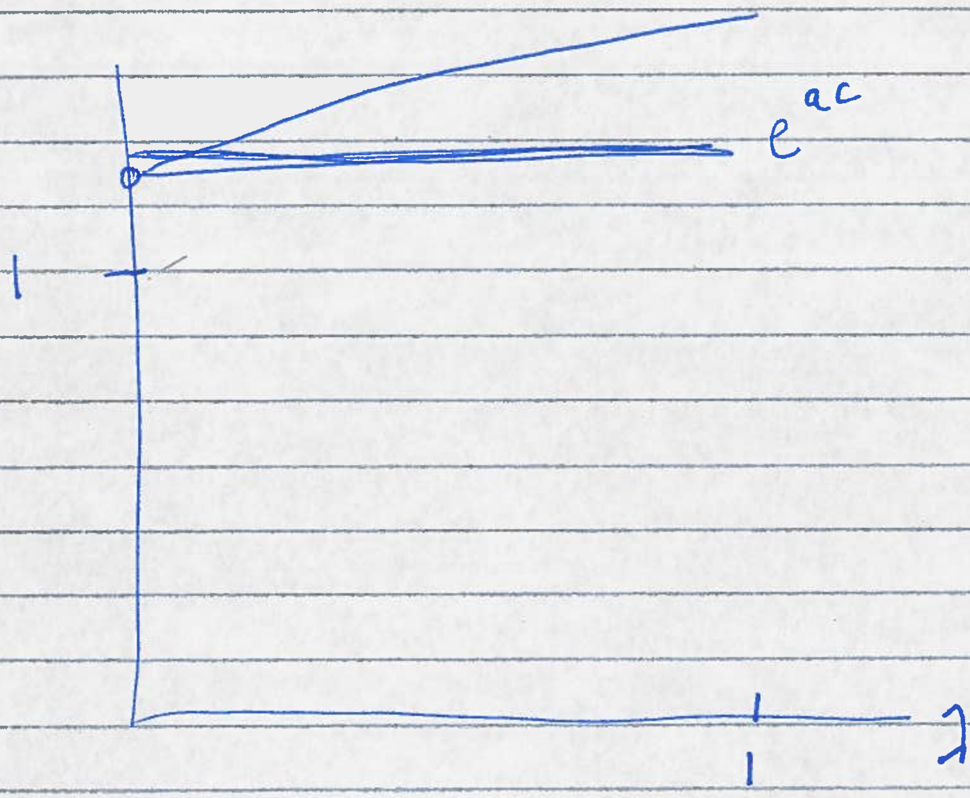
Informed demand is independent of θ, but

$$P_\lambda = \frac{\theta - x a \sigma_\varepsilon^2}{R} \quad \text{(from A.10)}$$

How does θ get into the price?

driven to imperfect comp.

Grossman–Stiglitz (no noise $\sigma_x^2 = 0$)



$$e^{ac}\sqrt{\frac{1+m}{1+m+nm}}$$

$\sigma_x^2 = 0$ implies

$m = 0$

so $\gamma(\lambda) = e^{ac} > 1$

for $\lambda > 0$

For $\lambda = 0$

$$\mathrm{Var}(u^*|w_\lambda) = \mathrm{Var}(u^*) = \sigma_\varepsilon^2 + \sigma_\theta^2$$

$$\therefore \frac{EV(W_I^\lambda)}{EV(W_u^\lambda)} = e^{ac}\sqrt{\frac{\sigma_\varepsilon^2}{\sigma_\varepsilon^2+\sigma_\theta^2}} = e^{ac}\sqrt{\frac{1}{1+n}}$$

If $e^{ac} < \sqrt{1+n}$, there is no equilibrium.

If $e^{ac} \geq \sqrt{1+n}$, there is an eq. where no one becomes informed