Tirole, Econom. Sept. 1982

Speculation

A speculator is someone willing to hold an asset with price risks.

Why? (not just different info.)
a) different beliefs
b) different risk aversion or initial position

A speculator is someone willing to pay more for an object than they would pay if they could not resell it.

— Capital Gains —

Main Result — With common priors and no insurance reasons for trading, there is no speculation under Rational Expectations.

Myopic REE traders base their decisions on the current price and the distribution of next period's price. No trader will expect gains from trade, but market value can be above "fundamental" value in infinite horizon models.

Fully Dynamic REE Myopic traders may all think they can realize their profits and leave the market, but that is inconsistent with a finite number of traders.

A sequence of self-fulfilling forecast functions for which each agent has a measurable stock holding strategy satisfying

1) $\forall t$, $\forall$ information holdings at time t, the strategy maximizes i's expected present discounted gain

(where the posterior is computed from common priors, information signal, and price information)

2) the market always clears in each period.

<u>Result</u> in a fully dynamic REE, price bubbles disappear.

OMIT

1) Price : p

2) Random return : $\tilde{p}$

3) X^i amount of transaction

4) $G^i = (\tilde{p} - p)\, X^i$ realized gain

5) utility $U^i(G^i)$

6) E : the set of potential realizations of $\tilde{p}$ (payoff relevant)

7) signal $s^i \in S^i$

8) $\Omega = E \times S$

9) common priors ν

$\nu^i(s^i \mid T)$ prob. of s^i, given $T \subset S$

Def a R.E.E. is a forecast function $\Phi : S \to P$

$\Phi(s) = p$ and a set of trades $X^i(p, S^i, S(p))$

relative to information s^i where $s \in S(p) \equiv \Phi^{-1}(p)$,

such that : 1) $X^i(p, S^i, S(p))$ maximizes expected utility conditional on s^i and $S(p)$, the information contained in prices.

2) market clearing $\sum_i X^i(p, S^i, S(p)) = 0$

budget constraint

~~note total gain must be zero by market clearing.~~

~~Assume no insurance motive to trade~~

$$\cancel{E\left(G^i \mid s^i, S(p)\right) \geqq 0 \qquad \forall s^i \text{ consistent with } S(p)}$$

~~Prop 1 In a REE, risk averse traders do not trade. Risk neutral traders do not expect any gain.~~

Dynamic Speculation

Can the price be above the market fundamental?

$t = 0, 1, 2, \ldots$

nonnegative dividend d_t is paid at the beginning of period t

d_t follows an exogenous stochastic process

p_t price of the stock in period t

finite set of traders

Preferences traders are risk neutral with discount factor γ. Complete capital markets. $E\left[\sum \gamma^t c_t\right]$

market clearing: $\sum_i x_t^i = \bar{x}$

Def

if no short sales: $x_t^i \geqq 0$

a trader is active if $x_t^i \neq x_{t-1}^i$ or

$$0 < x_t^i < \bar{x}$$

... traders are active

E set of potential stochastic processes for dividends.

S_t^i (private info. at time t) is an element of the partition F_t^i of the set $\mathcal{S}^i$. (includes history)

<u>assume</u> $F_t^i \subseteq F_{t+1}^i$ partition becomes finer

$\Omega = E \times \mathcal{S}$, with common priors

$\mathcal{S}_t \equiv \mathcal{S}_t(p_t)$ information contained in prices

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<u>Def</u> A <u>Myopic REE</u> is a sequence of self-fulfilling forecast functions $S_t \longrightarrow p_t = \Phi_t(S_t)$ s.t. there is a sequence of stock holdings $\{X_t^i(S_t^i, \mathcal{S}_t(p_t), p_t)\}$ satisfying:

$\mathcal{S}_t(p_t) = \Phi_t^{-1}(p_t)$

$s_t = (S_t^1 \ldots S_t^n)$

1) market clearing

no gains from ~~trade~~

2) $p_t = E\left[\gamma d_{t+1} + \gamma p_{t+1} \mid S_t^i, \mathcal{S}_t\right]$

maximization of expected short run gain
(slightly more complicated if short sales are disallowed)
(but Prop. 2 says that ② holds there as well.

p1171
No gains from trade

<u>Def</u> market fundamental

$$F(S_t^i, \mathcal{S}_t) \equiv E\left(\sum_{\tau=1}^{\infty} \gamma^\tau d_{t+\tau} \mid S_t^i, \mathcal{S}_t\right)$$

<u>bubble</u> $B(S_t^i, p_t) \equiv p_t - F(S_t^i, \mathcal{S}_t)$; speculation if the bubble is positive.

note — market fundamental can vary across traders.

Prop 3 p 1172 Finite horizon

proof by backward induction from the crash.

Prop 4 1173 Infinite Horizon

martingale $M_t = E_t(M_{t+1})$

$\boxed{\text{Example}}$ no uncertainty, $d_t = 1$ $\gamma = \frac{1}{2}$

∴ market fundamental $= 1$

REE: $P_t = (1 + P_{t+1})\frac{1}{2}$

$P_t = 1 + \alpha 2^t$

Initial Condition: $\alpha = 1$

$p_0 = 2$

$$\overbrace{2 - \tfrac{1}{2}(3)}^{\text{A's utility}} + \tfrac{1}{4}(5+1) - \tfrac{1}{8}(9) \cdots$$

$$\overbrace{-2 + \tfrac{1}{2}(3+1)}^{\text{B's utility}} - \tfrac{1}{4}(5) + \tfrac{1}{8}(9+1 \cdots$$

$A \xrightarrow{\text{sells}} B$ $t = 0$

$p_1 = 3$ $A \longleftarrow B$ 1

$p_2 = 5$ ⋮ 2

$p_3 = 9$ ⋮

⋆ Either trader does the best possible by leaving the market after selling, so it is reasonable to say

$G_A = 2$ $G_B = 0$

Payoffs are above the market fundamental!

p.d.v. of consumption $= 2 + 3/2 + 5/4 + 9/8 + \ldots = \infty$

No fully dynamic REE with bubbles
p. 1175

<u>Discussion</u>

① present discounted gains must be well defined, which is what rules out the bubbles.

② artifact of infinite lives (not just short period length)
bubbles if <u>real interest rate < real growth rate</u> -
<u>OG model</u> Sunspots literature (Money / Debt)

③ In more complicated models (endogenous d_t), defining an asset's "market fundamental" may be impossible.

④ <u>No arbitrage equation</u> means that there is no <u>incentive to aquire information.</u> No reason for equation to hold if traders are not price takers.
(Jackson & Peck trade due to speculative motives
endowment not ex ante P.O.)
No gains from trade is not general — depends on price taking