A Theory of Fads, Fashion, Custom, and Cultural Exchange as Informational Cascades
by Bikhchandani, Hirshleifer, and Welch
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We often see convergence of behavior in large populations, but this convergence is often idiosyncratic (it depends on the population or culture in question) and error prone.

Sometimes, when behavior has converged but a little bit of new public information arrives suggesting a different action, the social equilibrium can radically shift (e.g., eat oat bran or drink red wine).

Although we see conformity of behavior, sometimes a fad can shift and we see conformity to a different behavior.

BHW attempt to explain these phenomena as information cascades-where it is optimal for an individual, observing the choices of all previous individuals, to ignore his own information.

## A Simple Model

There is an exogenous sequence of individuals who must decide whether to adopt or reject some behavior.

Individuals observe the history of choices of all previous individuals.

The cost of adopting is $C$, which for the simplest example is given by $C=\frac{1}{2}$.

The gain from adopting, $V$, is a random variable whose value is either 0 or 1 . Each state is equally likely.

Each individual $i$ receives a private signal, $X_{i} \in\{H, L\}$. Signals are conditionally independent across individuals (independent conditional on the state).

If $V=1$, we have

$$
\begin{aligned}
& X_{i}=H \quad \text { with probability } p_{i} \\
& X_{i}=L \quad \text { with probability } 1-p_{i}
\end{aligned}
$$

If $V=0$, we have

$$
\begin{aligned}
& X_{i}=H \quad \text { with probability } 1-p_{i} \\
& X_{i}=L \text { with probability } p_{i}
\end{aligned}
$$

For the simplest example, we assume $p_{i}=p$ for all $i$.

The expected gain to adopting is the posterior probability that $V=1$, denoted by $\gamma$,

$$
E(V \mid \text { history })=\gamma .
$$

## Equilibrium Behavior for the Simplest Example

Assume $p_{i}=p, \quad C=\frac{1}{2}$
Assume that an individual who is indifferent adopts or rejects with equal probability.
-Individual $i=1$ : choose $a$ if type $H$, choose $r$ if type $L$.
-Individual 2:
for history ( $a$ ) :
if type $H$, choose $a$,
if type $L$, choose $a$ and $r$ w.p. $\frac{1}{2}$.
for history $(r)$ :
if type $H$, choose $a$ and $r$ w.p. $\frac{1}{2}$,
if type $L$, choose $r$.
-Individual 3:

After the history $(a, a)$, it is profitable to adopt even for a type $L$. To see this,

$$
\begin{gathered}
\gamma=p r(V=1 \mid a, a, L)= \\
\frac{\frac{1}{2}(p)\left(p+(1-p) \frac{1}{2}\right)(1-p)}{\left[\frac{1}{2}(p)\left(p+(1-p) \frac{1}{2}\right)(1-p)\right]+\left[\frac{1}{2}(1-p)\left(1-p+\frac{p}{2}\right)(p)\right]} \\
=\frac{p+1}{3}
\end{gathered}
$$

which is more than one half. Thus, individual 3 always adopts. Nothing is learned from his behavior, so an Up cascade starts and all future individuals adopt.

After the history $(r, r)$, both types reject, so we start a Down cascade.

After the history $(a, r)$ [respectively, $(r, a)]$ the signals must have been $(H, L)$ [respectively, $(L, H)]$.

Therefore, using Bayes' rule we see that the posteriors are $\gamma=p$ if individual 3 is type $H$, and $\gamma=1-p$ if individual 3 is type $L$. We are in the same situation as individual 1. Adopt if type $H$ and reject if type $L$.

Whenever there is an imbalance of at least 2 between the number of individuals who adopt and the number of individuals who reject, there is a cascade in favor of the majority choice.

Just like "deuce" in tennis. A deuce game ends eventually, and here we reach a cascade eventually:

The ex ante probability of no cascade after two periods is the probability of different signals and individual 2 flipping a coin and deciding to choose the opposite action:

$$
2 p(1-p) \cdot \frac{1}{2}=p(1-p)
$$

Therefore, the ex ante probability of no cascade after $n$ periods ( $n$ even) is

$$
\begin{equation*}
[p(1-p)]^{n / 2} . \tag{1}
\end{equation*}
$$

Because of the symmetry of the problem, the probability of an Up cascade after $n$ periods is equal to the probability of a Down cascade, which is therefore

$$
\frac{1-[p(1-p)]^{n / 2}}{2} .
$$

As $n \rightarrow \infty$, the probability of no cascade approaches zero.

What is the probability of a cascade being correct? (That is, an Up cascade when $V=1$ or a Down cascade when $V=0$.)

Assume w.l.o.g. that $V=1$.

$$
\begin{aligned}
& \operatorname{pr}(a, a \mid V=1)=p\left(p+\frac{1-p}{2}\right)=\frac{p(1+p)}{2} \\
& \operatorname{pr}(r, r \mid V=1)=(1-p)\left(1-p+\frac{p}{2}\right)=\frac{(2-p)(1-p)}{2} .
\end{aligned}
$$

The probability of the correct cascade is

$$
\begin{aligned}
& \frac{p r(a, a \mid V=1)}{p r(a, a \mid V=1)+p r(r, r \mid V=1)} \\
= & \frac{p(1+p)}{2\left(p^{2}-p+1\right)}
\end{aligned}
$$

and the probability of an incorrect cascade is

$$
\frac{(2-p)(1-p)}{2\left(p^{2}-p+1\right)}
$$

## General Model

Individuals $i=1,2, \ldots$ must sequentially choose from $\{a, r\}$. Each individual observes the history of all previous decisions.

Cost of adoption, $C$.

Gain from adoption, $V$. There are $S$ possible realizations of $V$ (states).
$V \in\left\{v_{1}, \ldots, v_{S}\right\}$, where we have $v_{1}<v_{2}<\ldots<v_{S}$. Also, nontrivial cases require $v_{1}<c<v_{S}$.

Priors are denoted as $\operatorname{pr}\left(V=v_{\ell}\right)=\mu_{\ell}$.

There are $R$ (conditionally independent) signals for each individual $i$. $X_{i} \in\left\{x_{1}, \ldots, x_{R}\right\}$, where $x_{1}<\ldots<x_{R}$.

The probability of signal $q$ given state $\ell$ is denoted as $p_{q \ell}$.

The solution concept is Perfect Bayesian Equilibrium. [Beliefs off the equilibrium path about $V$ ignore the action of an individual who should always have taken the opposite action. Note that off-equilibrium-path threats are irrelevant here, because there are no payoff externalities.]

Definition: An information cascade occurs if an individual's action does not depend on his private signal.

Because signals are conditionally independent, once a cascade starts, it will last forever.

Let $a_{i}$ denote individual $i$ 's action ( $a$ or $r$ ), and let $A_{i}$ denote the history, $\left(a_{1}, \ldots, a_{i}\right)$. Then $J_{i}\left(A_{i-1}, a_{i}\right)$ denotes the set of signals for which individual $i$ chooses action $a_{i}$.

Then the conditional expectation of $V$ for individual $n+$ 1 , denoted by $V_{n+1}\left(x_{q} ; A_{n}\right)$, is given by

$$
E\left[V \mid X_{n+1}=x_{q}, X_{i} \in J_{i}\left(A_{i-1}, a_{i}\right) \text { for all } i \leq n\right] .
$$

[Earlier individuals are assumed to have one of the signals consistent with their actions.]

Individual $n+1$ adopts if and only if $V_{n+1}\left(x_{q} ; A_{n}\right) \geq C$. This determines $J_{n+1}\left(A_{n}, a\right)$ and $J_{n+1}\left(A_{n}, r\right)$.

Note: Adopting when indifferent simplifies the analysis but does not change the basic results. Indifference does not occur generically, but our simple example with $C=\frac{1}{2}$ is a non-generic special case.

Results hinge on two regularity assumptions:

Assumption 1 (Monotone Likelihood Ratio Property): For all $\ell<S$, we have

$$
\frac{p_{q, \ell}}{p_{q+1, \ell}} \geq \frac{p_{q, \ell+1}}{p_{q+1, \ell+1}} \text { for all } q<R
$$

A higher signal means that the conditional distribution of states is higher, so the value of adoption is higher.

Assumption 2 (No long-run Ties): $v_{\ell} \neq C$ for all $\ell$.

Proposition 1: If assumptions 1 and 2 hold, then as the number of individuals increases, the probability that a cascade eventually starts approaches one.

Intuition: If a cascade has not started before individual $n$ acts, then each previous choice reveals some information about his/her signal.

Applying the law of large numbers, the state $v_{\ell}$ can be learned with near certainty when $n$ is large, so $X_{n}$ has a negligible effect on that belief.

Since $v_{\ell} \neq C$, individual $n$ will choose the same action for all signal realizations, and a cascade starts.

## "Fashion Leaders"

Consider the binary signal model, where $p_{i}$ can depend on the individual, and assume $C=\frac{1}{2}$.

Result 1: (1) If $p_{1}>p_{n}$ for all $n>1$, then everyone copies individual 1 's action, starting a cascade.
(2) If $p_{n}=p$ is constant for all $n>1$, then all individuals $n>2$ are better off if $p_{1}=p-\varepsilon$ rather than $p_{1}=p+\varepsilon$ (for small $\varepsilon$ ).

Intuition for part (2) is that individual 2 will reveal her signal if it is more accurate than individual 1's signal.
-Result 1 indicates that a slight perturbation in information can have a big effect on social outcomes, and that payoffs can be non-monotonic in information quality.
-Less informed people will imitate those with a little better information.
-Sometimes it is more efficient to have the experts wait until a few others have moved.
-To bring about social change when the opinion or fashion leaders are the most informed, focus on influencing the leaders.

## Cascades are Fragile

## Public Release of Information

Result 2: The release of public information before the first individual's decision can make some individuals worse off, ex ante.

The first individual is always better off, but the second individual can be worse off. (Complicated example in appendix.)

Result 3: A small amount of public information can shatter a cascade in progress.

Consider the Simple Model with $p_{i}=p$ and $C=\frac{1}{2}$. A cascade forms after $\mid \#$ adopt $-\#$ reject $\mid=2$. Suppose an Up cascade starts when adoptions equal rejections after $n-1$ choices, followed by $a_{n}=a$ and $a_{n+1}=a$.

We infer that individual $n$ has signal $H$ and individual $n+1$ either has signal $H$ or signal $L$ with the right coin flip.

Now suppose that individual $n+100$ has signal $L$ and also that an independent public signal of $L$ is revealed. Then the probability of $V=1$ is strictly less than $\frac{1}{2}$, and the cascade is broken.

Even a slightly less accurate public signal of $L\left(p_{\text {public }}<\right.$ $p$ ) would break the cascade.

Proposition 3: If there is a positive probability, bounded above zero, of further public information release before each individual chooses (and the public information is conditionally independent, identically distributed, and satisfies assumptions 1 and 2), then individuals eventually settle on the correct cascade.

Intuition: The law of large numbers reveals $v_{\ell}$ based only on public disclosures.

Cascades are Fragile

Connection to Fads

An initial cascade can be based on relatively little information. If after a while, there is a small probability that $V$ changes, then the cascade can flip. One fad is replaced with another.

Cascade flips, not because people are convinced that $V$ has changed, but because they are slightly less sure about the state.

Example: $V$ stays constant for the first 100 periods. Then in period 101, we have a new draw w.p. $\frac{1}{10}$, and we keep $V$ w.p. $\frac{9}{10}$. Denoting the value in period 101 as $W$, we have

$$
\begin{aligned}
& W=V \quad \text { w.p. } 0.95 \\
& W=1-V \text { w.p. } 0.05
\end{aligned}
$$

Assume $C=\frac{1}{2}$ and $p=0.9$.

Clearly, under full information the probability of a switch in behavior is 0.05 . If previous signals are publicly observable, then the probability of a switch in behavior by, say, period 110 is roughly 0.05 .

When only actions are observable, the probability of a cascade reversal turns out to be greater than 0.0935, which is almost twice as likely as the probability of the state switch.

See also Moscarini, Ottaviani, and Smith (Economic Theory 1998) and Peck and Yang (IER forthcoming) for analysis of cycles bases on a constant probability of state switch.

