Dynamic Competition with Random Demand and Costless Search: A Theory of Price Posting

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Market Setting

–Sales Season

The good is offered for sale for a fixed length of time, after which is loses most of its consumption value.

–Aggregate Demand Uncertainty

The state of demand is unknown to consumers and firms.

–Intertemporal Substitution in Demand

Consumers optimize in the timing of their purchases.

–Learning about Demand

Consumers and firms observe the historical pattern of sales and make inferences about demand, based on which they adjust their purchase and pricing decisions over time.

Due to intertemporal substitution, some consumers will delay their purchases, which makes drawing inferences about demand more difficult.

–Production in Advance

Firms must produce in advance of the demand season.

–Low Search Costs

Consumers costlessly observe the prices on available output upon arriving at the market.

Applicability

–Think of Christmas season, Graduation season, or airline travel.

For the market for Chicago-NY flights on September 15:

(i) a ticket is worthless after September 15,

(ii) there is significant demand uncertainty,

(iii) consumers that first "arrive" on July 10 treat a ticket purchased on July 10 as a nearly perfect substitute for a ticket purchased on August 10, and try to optimize over the timing of their purchase,

(iv) consumers and firms learn about demand as seats are sold over the market period,

(v) capacity must be in place months in advance,

(vi) internet price searches and transactions are nearly costless.

–What price patterns do we see?

Gradual disappearance of low-priced tickets until airlines reevaluate demand conditions, followed by the reappearance of low-priced tickets.

Price patterns (as a function of the number of days until departure) on a particular flight are a highly non-monotonic stochastic process.

Prices are martingales up until about 2 weeks before departure.

Contributions of This Paper

–Modeling Aggregate Demand Uncertainty

Consumers know their own valuation and whether they are active. Hence, consumers possess private information.

Because there is aggregate demand uncertainty, information about demand is correlated across consumers.

Beliefs about the demand state differ across consumers and firms.

–Modeling Intertemporal Substitution by Consumers

Current literature on dynamic competition with demand uncertainty has no intertemporal substitution.

Durable goods literature has intertemporal substitution, but no aggregate uncertainty and does not look at perfect competition.

Two new forces determine which consumers purchase at the lowest available price and which consumers wait: option-value effect and information effect.

–Our equilibrium has consumers endogenously sorting themselves efficiently, with the higher valuation consumers purchasing and the lower valuation consumers waiting.

–The lowest available price at any point is a martingale, equaling the conditional expectation of the price that clears the aggregate market.

–Demand is revealed gradually over time. In the last period, all remaining output is priced at the market-clearing price.

–Thus, the economy gropes its way to the market-clearing price, without a Walrasian auctioneer.

Model

Continuum of potential firms, measure M.

Each firm can produce one unit of output at time $t = 0$, and has a production cost, $c > 0$.

Continuum of potential consumers, measure C.

Each active consumer has a valuation, v, and a time at which they first become active, t, where $\underline{v} \leq v \leq \bar{v}$.

There is perfect competition: $M > C$.

The market season consists of T periods.

For $t \in \{1, 2, ..., T-1\}$, demand activated in period t is denoted as $D_t(p, \alpha_t)$. This is the measure of newly active consumers with valuation $v \geq p$. $D_t(p, \alpha_t)$ is increasing in α_t.

The state of demand is $(\alpha_1, ..., \alpha_{T-1})$, distributed according to the continuous density function $f(\alpha_1, ..., \alpha_{T-1})$.

We assume that the random variables are affiliated: For all $t \neq t'$, we have

$$\frac{\partial^2 \ln f(\alpha_1, ..., \alpha_{T-1})}{\partial \alpha_t \partial \alpha_{t'}} > 0.$$

If a consumer of type (v, t) purchases at a price p in some period $t' \geq t$, she receives a net surplus of

$$v - p - (t' - t)\delta(v).$$

The delay cost satisfies

$$\delta(v) = 0 \quad \text{for} \quad v \leq \hat{v}$$

$$\delta(v) \quad \text{strictly increasing and differentiable for} \quad v > \hat{v}$$

Timing:

–At $t = 0$, firms decide whether to produce and nature chooses the set of active consumers (their valuations and when they become active).

–For $t = 1, ..., T$, firms that have unsold output post a price. Active consumers who chose not to purchase in previous periods are placed in a queue in random order. Then consumers newly active in period t arrive at the queue in random order.

–Consumers are sequentially released from the queue, observe the measure of sales so far in period t, a_t, and decide whether to purchase at the lowest available price, $p_t(a_t)$, or wait.

A sequential equilibrium will specify:

Firms: an aggregate production quantity q^*, and for each $t = 1, ..., T$, and each history of sales in prior periods $(a_1^r, ..., a_{t-1}^r)$ such that the market remains active in period t (i.e. such that $\sum_{\tau=1}^{t-1} a_\tau^r < q^*$), an aggregate quantity $a_t^{\max}(a_1^r, ..., a_{t-1}^r)$ offered for sale in period t, and a nondecreasing function $p_t(a_t; a_1^r, ..., a_{t-1}^r)$: $[0, a_t^{\max}] \to \mathbf{R}_+$ indicating the price of the a-th lowest priced unit offered for sale in period t.

Consumers: for each consumer type (v, t') with $t' \leq t$, and for each private history a_t^p, a function

$$\psi_t^{v,t'}(a_t; a_1^r, ..., a_{t-1}^r, a_t^p) : [0, a_t^{\max}] \to \{0, 1\}$$

indicating whether she will accept the price $p_t(a_t; a_1^r, ..., a_{t-1}^r)$ if when she arrives at the market a measure a_t of output has already been sold in period t.

Notation for the aggregate economy:

$$D(p, \alpha_1, ..., \alpha_{T-1}) = \sum_{t=1}^{T-1} D_t(p, \alpha_t),$$

Let $P(q, \alpha_1, ..., \alpha_{T-1})$ denote the inverse demand function associated with $D(p, \alpha_1, ..., \alpha_{T-1})$. Next, let q^e denote the efficient quantity, solving

$$E[P(q^e, \alpha_1, ..., \alpha_{T-1})] = c.$$

Finally, let $\bar{p}$ denote the highest possible market clearing price

$$\bar{p} = P(q^e, \bar{\alpha}_1, ..., \bar{\alpha}_{T-1}).$$

Assumption 4: $\hat{v} \geq \bar{p}$.

Assumption 5: *The logarithm of the hazard rate of any consumer with valuation* v *in period* t*, i.e.*

$$\ln\left(-\frac{\frac{\partial D_t}{\partial p}(v,\alpha_t)}{D_t(v,\alpha_t)}\right)$$

is (i) strictly decreasing in α_t*; and (ii) strictly supermodular in* (v,α_t)*.*

Theorem 1: There exists a revealing equilibrium, in which the output produced and its allocation across consumers is efficient. More precisely, $q^* = q^e$, and there exist unique functions $\bar{a}_t(\alpha_t; a_1^r, ..., a_{t-1}^r)$ and $v_t^*(a_t; a_1^r, ..., a_{t-1}^r)$ $\in (\widehat{v}, \bar{v})$, where $\bar{a}_t$ is strictly increasing in α_t, such that:

(i) Sales in period t, a_t^r, reveals α_t;

(ii) In state $(\alpha_1, ..., \alpha_{T-1})$ all period T output is offered at the price $P(q^e, \alpha_1, ..., \alpha_{T-1})$;

(iii) Prices are martingales, i.e.

$$p_t(a_t; a_1^r, ..., a_{t-1}^r) =$$

$$E[P(q^e, \alpha_1, ..., \alpha_{T-1})|hist, \alpha_t \geq \beta_t(a_t; a_1^r, ..., a_{t-1}^r)],$$

where $\beta_t(a_t; a_1^r, ..., a_{t-1}^r)$ is the lowest α_t consistent with the history;

(iv) A consumer of type (v, t) purchases in period t if and only if $v \geq v_t^*(a_t; a_1^r, ..., a_{t-1}^r)$;

(v) A consumer of type (v, t') with $t' < t$ purchases in period t if and only if $v \geq \bar{p}_t(a_1^r, ..., a_{t-1}^r)$, where $\bar{p}_t(a_1^r, ..., a_{t-1}^r) = P(q^e; \alpha_1^r, ..., \alpha_{t-1}^r, \bar{\alpha}_t, ..., \bar{\alpha}_{T-1})$;

(vi) For each t, the function $v_t^*(a_t; a_1^r, ..., a_{t-1}^r)$ is the solution in v to $\mathbf{\Delta}_t(v, a_t; a_1^r, ..., a_{t-1}^r) = 0$, where

$$\mathbf{\Delta}_t = \delta(v) - E[P(q^e, \alpha' s) | hist, \alpha_t \geq \beta_t(a_t; a_1^r, ..., a_{t-1}^r)] +$$

$$E[P(q^e, \alpha' s) | hist, \alpha_t \geq \beta_t(a_t; a_1^r, ..., a_{t-1}^r), \text{ type } (v, t)]$$

$$E[P(q^e, \alpha' s) | hist, \alpha_t \geq \beta_t(a_t; a_1^r, ..., a_{t-1}^r), \text{ type } (v,t)] =$$

$$\frac{\int P(q^e, \cdot) \left[\frac{-\frac{\partial}{\partial p} D_t(v, \alpha_t)}{D_t(v_t^*(a_t; a_1^r, ..., a_{t-1}^r), \alpha_t)} f(\cdot) \right] d\alpha_{T-1} ... d\alpha_t}{\int_{\beta_t(a_t; a_1^r, ..., a_{t-1}^r)}^{\bar{\alpha}_t} \int_{\underline{\alpha}_{t+1}}^{\bar{\alpha}_{t+1}} \cdots \int_{\underline{\alpha}_{T-1}}^{\bar{\alpha}_{T-1}} [\cdot] \, d\alpha_{T-1} ... d\alpha_t}$$

Higher v are more likely to be active in higher α_t, correlated with higher demand in other periods. They expect higher future prices. Information effect gives incentive to buy now.

Cutoff type $v_t^*(a_t; a_1^r, ..., a_{t-1}^r)$ has the lowest expectation of future prices of those who purchase, negative information effect, but information effect balances delay cost.

Types lower than $v_t^*(a_t; a_1^r, ..., a_{t-1}^r)$ have the info effect working against them, have lower delay cost, and may have an option value of waiting.

Notice that given $(\alpha_1^r, ..., \alpha_{t-1}^r)$ the limiting probability of a consumer in the period t queue of new arrivals, taken at random, having a valuation between v and $v + \triangle v$ is proportional to $-\frac{\partial}{\partial p} D_t(v, \alpha_t)/D_t(\underline{v}, \alpha_t)$, and the probability of arriving in the queue when the transactions are between a_t and $a_t + \triangle a_t$ is inversely proportional to the rate at which transactions are occurring, $D_t(v_t^*(a_t; a_1^r, ..., a_{t-1}^r), \alpha_t)/D_t(\underline{v}, \alpha_t)$.

Under multiplicative uncertainty, $D_t(p, \alpha_t) = \alpha_t D_t(p)$, there is no information effect.

Proposition 5 gives closed-form solutions for the sequential equilibrium.

A consumer of type (v, τ) purchases in period $t \geq \tau$ if and only if $v \geq \bar{p}_t(a_1^r, ..., a_{t-1}^r)$. In particular, we have $v_t^*(a_t; a_1^r, ..., a_{t-1}^r) = \bar{p}_t(a_1^r, ..., a_{t-1}^r)$; all generation t customers with valuations above $\widehat{v}$ therefore purchase in period t, since $\widehat{v} \geq \bar{p} \geq \bar{p}_t(a_1^r, ..., a_{t-1}^r)$. The equilibrium is fully efficient, as the allocation is efficient and no consumer incurs a utility reduction from delaying purchase.

Example: Multiplicative Uncertainty and $T = 3$

Demand in period t (for $t = 1, 2$) is given by

$$D_t(p, \alpha_t) = \alpha_t(1 - p)$$

Therefore, the aggregate demand and inverse demand is given by

$$\begin{aligned} D(p, \alpha_1, \alpha_2) &= (\alpha_1 + \alpha_2)(1 - p) \\ P(q, \alpha_1, \alpha_2) &= 1 - \frac{q}{\alpha_1 + \alpha_2}. \end{aligned}$$

It follows that $\underline{v} = 0$ and $\overline{v} = 1$.

Let us set

$$\delta(v) = 0 \quad \text{for} \quad v < \frac{3}{4}$$
$$\delta(v) = \frac{1}{100}(v - \frac{3}{4}) \quad \text{for} \quad v \geq \frac{3}{4}.$$

It will be convenient to set the marginal production cost as follows

$$c = 6\ln(3) - 10\ln(2) + 1 \simeq 0.6602.$$

We assume that α_1 and α_2 are independent and identically distributed according to the uniform density on $[1, 2]$. We have

$$f(\alpha_1, \alpha_2) = 1 \ \text{ for all } (\alpha_1, \alpha_2) \in [1, 2] \times [1, 2].$$

Then the equilibrium quantity $q^* = q^e$ solves

$$\begin{aligned} & \int_{\alpha_1=1}^{2} \int_{\alpha_2=1}^{2} (1 - \frac{q}{\alpha_1 + \alpha_2}) d\alpha_2 d\alpha_1 \\ = \; & c = 6 \ln(3) - 10 \ln(2) + 1, \end{aligned}$$

yielding $q^* = q^e = 1$.

Since $\overline{\alpha}_t = 2$, we have

$$\overline{p} = 1 - \frac{1}{2+2} = \frac{3}{4}.$$

In period 1, all consumers with $v \geq \overline{p}$ purchase, so total purchases in period 1 will be $\alpha_1(1-\overline{p})$. This allows us to infer the minimum possible period-1 demand state, as a function of period-1 transactions a_1, as follows:

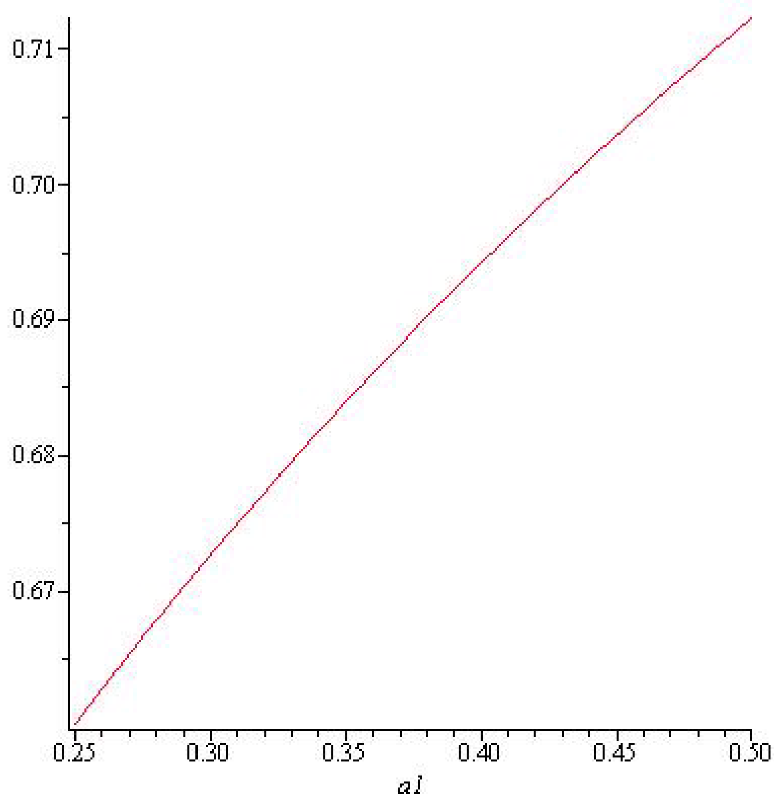
$$\begin{aligned} \beta_1(a_1) &= 1 \quad \text{for} \quad a_1 \leq \underline{\alpha}_1(1-\overline{p}) = \frac{1}{4} \\ \beta_1(a_1) &= 4a_1 \quad \text{for} \quad \frac{1}{4} < a_1 \leq \overline{\alpha}_1(1-\overline{p}) = \frac{1}{2}. \end{aligned}$$

Prices in period 1 are given by

$$\begin{aligned} p_1(a_1) &= c = 6\ln(3) - 10\ln(2) + 1 \quad \text{for} \quad a_1 \leq \frac{1}{4} \\ p_1(a_1) &= \frac{\int_{4a_1}^{2}\int_{1}^{2}(1-\frac{1}{\alpha_1+\alpha_2})d\alpha_2 d\alpha_1}{\int_{4a_1}^{2}\int_{1}^{2} d\alpha_2 d\alpha_1} \quad \text{for} \quad \frac{1}{4} < a_1 \leq \frac{1}{2}. \end{aligned}$$

This yields a closed-form expression (too long to write here).

Here is a plot of $p_1(a_1)$:



Based on Proposition 5, we can compute a closed-form expression for prices in period 2,

$$p_2(a_2; a_1^r) = 1 + \frac{\ln(a_2 + a_1^r)}{2 - a_2(4a_1^r + 2) + 2a_1^r(1 - 2a_1^r)}.$$

Moving on to period 3, all transactions occur at the market clearing price for the realized demand state,

$$P(1, \alpha_1^r, \alpha_2^r) = 1 - \frac{1}{\alpha_1^r + \alpha_2^r}. \qquad (1)$$

Conclusions

Under our regularity conditions, consumers endogenously ration themselves efficiently, and the price in the last period is the market-clearing price for the hypothetical economy where everyone shows up at once.

Demand is gradually learned over time, in a recurring pattern of increasing transactions prices within a period, reflecting increased optimism by firms regarding the demand state, followed by a markdown at the start of the next period, reflecting the drying up of sales.

Non-monotonic prices and the martingale feature is a close fit to airline ticket prices.

Under multiplicative uncertainty, examples are easy to compute: the cutoff valuation is the highest possible market-clearing price given the history,

$P(q^e; \alpha_1^r, ..., \alpha_{t-1}^r, \bar{\alpha}_t, ..., \bar{\alpha}_{T-1}).$

Why not just have an auction? Delay costs. Maybe some consumers do not treat sellers' products as perfect substitutes.

What happens if delay costs approach zero? Fewer and fewer consumers purchasing during the period they first become active. Inferring the state becomes less plausible.

What if firms have heterogeneous costs? Not much difference.

What happens without our regularity conditions or with new demand in period T?