The Extended Model: Robustness Results Online Appendix for "Bank Runs and the Optimality of Limited Banking"*

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In this appendix, we consider the extended model, which includes the feature that long-term investments made by banks might yield a different net return than outside investment. We introduce a parameter, ε, which would be positive if outside investment yields a higher return than bank investment, and negative if outside investment yields a lower return than bank investment. Bank investment held until period 2 yields the return, R, but outside investment held until period 2 now yields the return, $R+\varepsilon$.[1] A positive ε might be due to regulatory costs such as reserve requirements, or possibly because outside investment does not have to be monitored as closely as bank investment. A negative ε might arise if the bank is more efficient at monitoring its investments than individual consumers investing outside the bank. In Gertler and Kiyotaki (2015)[2], consumers could make loans directly, but they are less efficient in making and monitoring loans than the bank, so they deposit in the bank instead.

Our main result, Proposition 3, shows that the optimal banking system is partial, $d < 1$, and that it tolerates a positive probability of runs. The extended

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[1] For $\varepsilon \neq 0$, the equivalence result given in Proposition 1 no longer holds. Indeed, allocations yielding welfare higher than x^* (associated with $d = 1$) are feasible and incentive compatible if $\varepsilon > 0$ holds, since the patient benefit from the higher return on outside investment.

[2] Gertler, Mark and Nobuhiro Kiyotaki. 2015. "Banking, Liquidity, and Bank Runs in an Infinite Horizon Economy." *American Economic Review* 105(7): 2011-2043.

model demonstrates the robustness of Proposition 3 to small differences in the investment returns of the bank and outside investment. This result is not at all obvious when ε is negative, where bank investments yield a slightly higher return than outside investment.

Welfare in the extended model, conditional on a run not taking place, is denoted by $\widehat{W}(C,d,\varepsilon)$, given by

$$\begin{aligned}\widehat{W}(C,d,\varepsilon) &= \sum_{\alpha=0}^{N-1} f(\alpha)\left[\sum_{z=1}^{\alpha} u(x_1(z,d)) + (N-\alpha)u(x_2(\alpha,d,\varepsilon))\right] \\ &\quad + f(N)\left[\sum_{z=1}^{N} u(x_1(z,d))\right].\end{aligned}$$

The dependence on ε is due to the fact that $x_2(\alpha_1,d,\varepsilon) = (1-d)(R+\varepsilon)+c_2(\alpha_1,d)$ depends on ε. The incentive compatibility constraint is now given by

$$\sum_{\alpha=0}^{N-1} f_p(\alpha)\left[\frac{1}{1+\alpha}\sum_{z=1}^{\alpha+1} u((1-d)(R+\varepsilon-1)+x_1(z,d))\right] \leq \sum_{\alpha=0}^{N-1} f_p(\alpha)u(x_2(\alpha,d,\varepsilon)). \tag{0.1}$$

Welfare, conditional on a run taking place, is denoted by $W^R(C,d,\varepsilon)$, given by

$$W^R(C,d,\varepsilon) = \sum_{\alpha=0}^{N} f(\alpha)\left[\frac{\alpha}{N}\sum_{z=1}^{N} u(x_1(z,d)) + \frac{N-\alpha}{N}\sum_{z=1}^{N} u((1-d)(R+\varepsilon-1)+x_1(z,d))\right].$$

Overall welfare is given by $W(C,d,s,\varepsilon)$, where

$$\begin{aligned}W(C,d,s,\varepsilon) &= (1-s)\widehat{W}(C,d,\varepsilon) + sW^R(C,d,\varepsilon) && \text{if } (C,d,\varepsilon) \text{ allows a run equilibrium} \\ &= \widehat{W}(C,d,\varepsilon) && \text{if } (C,d,\varepsilon) \text{ does not allow a run equilibrium.}\end{aligned}$$

An s-optimal contract is a solution to the following problem

$$\begin{aligned}&\max_{C,d} W(C,d,s,\varepsilon) \\ &\text{subject to} \\ &(0.1),\ \text{resource constraints} \\ c_1(z,d) \geq\ &0.\end{aligned} \tag{0.2}$$

To avoid some technical issues surrounding a possible discontinuity in $W(C,d,s,\varepsilon)$ as we switch from contracts tolerating runs to contracts for which a run equilibrium does not exist, it will be useful to consider the following problem

$$\begin{aligned} & \max_{C,d}(1-s)\widehat{W}(C,d,\varepsilon)+sW^R(C,d,\varepsilon) \\ & \text{subject to} \qquad\qquad\qquad\qquad\qquad\qquad (0.3) \\ & (0.1), \text{ resource constraints} \\ c_1(z,d) \ \geq \ & 0. \end{aligned}$$

The following proposition shows that our main result is robust to the introduction of non-zero ε.

Proposition 4: *Suppose that any $(s,1)$-optimal contract (for the economy with $\varepsilon=0$) is such that (i) incentive compatibility is not binding and (ii) the post-deposit subgame has a run equilibrium. Then, for ε sufficiently small, when the deposit level is not constrained, the s-optimal contract entails less than full deposits, and a run occurs with positive probability on the equilibrium path.*

Proof. Consider a sequence of economies indexed by ν, such that $\varepsilon^\nu \to 0$ holds, and a sequence of solutions to (0.3). Denote the set of solutions to (0.3) by $C^*(s,\varepsilon^\nu)$. Assume without loss of generality that the sequence of solutions to (0.3) converges. That is, $(C^\nu,d^\nu)\in C^*(s,\varepsilon^\nu)$ converges to (C^0,d^0). The set of contracts solving the constraints in (0.3) is continuous and compact valued, and the objective is a continuous function. By the Theorem of the Maximum, $C^*(s,\varepsilon^\nu)$ is upper hemi-continuous in ε^ν. Therefore, (C^0,d^0) is a solution to (0.3) for the economy with $\varepsilon=0$. By Proposition 3, any s-optimal contract has a run equilibrium, so the solution to the modified problem (0.3) is a solution to the actual problem (0.2). That is, the optimal contract that tolerates runs yields higher welfare than the optimal contract that avoids runs. Therefore, we have $d^0<1$, and the contract (C^0,d^0) exhibits a run equilibrium. By continuity, it follows that, for ε^ν sufficiently small (positive or negative), we must have $d^\nu<1$, and the contract (C^ν,d^ν) exhibits a run equilibrium. ■