The Ohio State University
Department of Economics

Econ 501.02
Winter 2009
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Midterm Examination Questions and Answers

Part I: Short Answer.

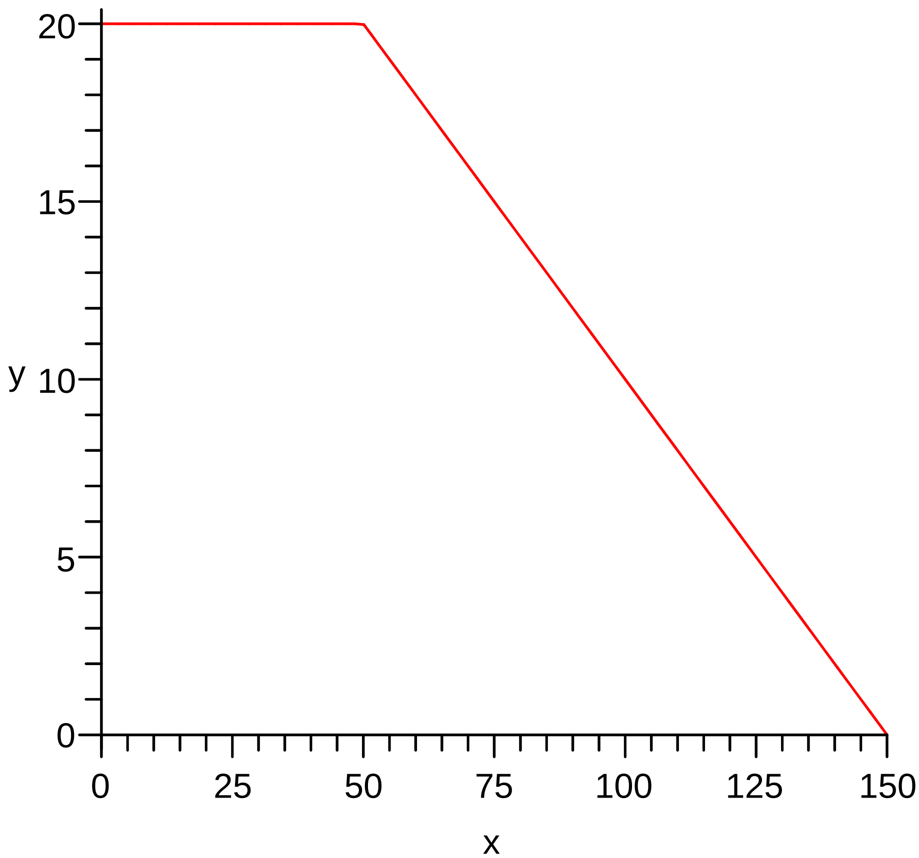
1. **(10 points)** Suppose a consumer derives utility from food (good x) and clothing (good y). The consumer has cash income, $M = 200$, and faces prices, $p_x = 2$ and $p_y = 10$. In addition to her cash income, the consumer receives from the government 100 in food stamps, which can be used to purchase food but not clothing. Using a carefully drawn and labeled graph, indicate the set of consumption bundles that the consumer can afford. (You need to put numbers on the axes so that I can see the amount of each good corresponding to any point in the graph.)

Answer:
The x intercept is 150, because both money and food stamps can be used to purchase food. The y intercept is 20, because only money can be used to purchase clothing However, the budget set's boundary is not a straight line. Starting at (0,20), the consumer can use food stamps to purchase food with no reduction in clothing consumption, so the budget frontier has a horizontal segment from (0,20) to (50,20). From (50,20), further increases in food consumption entail a reduction in clothing consumption, based on the price ratio 1/5, so the budget frontier has a segment with slope -1/5 from (50,20) to (150,0).

> $clothing := min\left(20, 30 - \frac{x}{5}\right);$

$$clothing := \min\left(20, 30 - \frac{1}{5}\,x\right) \tag{1}$$

> $plot(clothing, x = 0..150, y);$



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2. **(10 points)** True or False, and explain: Considering the "textbook" case in which a consumer's budget equation is $p_x x + p_y y = M$, then the income effect due to an increase in p_x will go in the opposite direction of the substitution effect if x is an inferior good.

Answer:
The statement is true. For all goods, the substitution effect of a price increase is to reduce the demand for good x, substituting away from the relatively more expensive good. Because purchasing power has gone down, the income effect is to increase the demand for good x, because demand for inferior goods increases as income falls.

3. **(10 points)** A developing country going through a food crisis is forced to ration its supplies of rice and water. Explain which of the following options will be better for its citizens. Briefly justify your answer.

option 1: Provide each adult citizen with 20 pounds of rice and 4 gallons of water, and prohibit trading of rations.

option 2: Provide each adult citizen with ration coupons which can be redeemed for 20 pounds of rice and 4 gallons of water, but allow people to trade their ration coupons.

Answer:
Option 2 is better, because it gives consumers an opportunity to find mutually beneficial trades and therefore makes everyone at least as well off as they would be in option 1, without the opportunity to trade. For example, some consumers might feel that 20 pounds of rice is more than they need, and trade some rice for more water, while other consumers have enough water and want more rice. If, by coincidence, all marginal rates of substitution were equal at (20,4), then the initial allocation would be Pareto optimal, but then the two options would be equivalent.

4. (**10 points**) True or False, and explain: If the average product of labor is higher for Firm 1 than for Firm 2, then it must follow that Firm 1 is more productive than Firm 2, in the sense that for a given amount of inputs Firm 1 can produce more output.

Answer:
The statement is false. First of all, even if Firm 1 and Firm 2 are using the same bundle of inputs, it does not follow that Firm 1 is necessarily more productive at all other input bundles. Secondly, the two firms might be using different input bundles, which by itself would explain the difference in average products. For example, if Firm 1 has a lot of capital for each worker and Firm 2 has very little capital for each worker, then Firm 1 would likely have a higher average product of labor, even if Firm 2 would produce more output if the inputs were equalized.

Part II: Longer Answer.

5. **(30 points)** A consumer has the utility function over goods x and y,

$$u(x, y) = x^{1/2} + y^{1/2}.$$

Let the price of good x be given by p_x, let the price of good y be given by p_y, and let income be given by M.

(a) Derive the consumer's generalized demand function for good y.
(b) Is good y normal or inferior?
(c) If we have $p_x = 2$, $p_y = 1$, and $M = 12$, compute the utility maximizing consumption bundle of goods x and y.

Answer:
(a) Set up the Lagrangean,

$$L = \sqrt{x} + \sqrt{y} + \lambda[M - p_x x - p_y y].$$

Differentiating with respect to x, y, and λ, we have

$$\begin{aligned} \frac{1}{2}x^{-1/2} - \lambda p_x &= 0 & (1) \\ \frac{1}{2}y^{-1/2} - \lambda p_y &= 0 & (2) \\ M - p_x x - p_y y &= 0. & (3) \end{aligned}$$

From (1) and (2), we have

$$\lambda = \frac{\frac{1}{2}x^{-1/2}}{p_x} = \frac{\frac{1}{2}y^{-1/2}}{p_y}$$

After simplifying, squaring both sides, and solving for x in terms of y, we have

$$x = \frac{(p_y)^2 y}{(p_x)^2}. \tag{4}$$

Plugging (4) into the budget constraint, (3), we have

$$M - p_x \frac{(p_y)^2 y}{(p_x)^2} - p_y y = 0$$

Simplifying, putting the y terms on the right side, and factoring out y, we have

$$M = y[p_y + \frac{(p_y)^2}{(p_x)}]. \tag{5}$$

Solving (5) for x, we have the generalized demand function

$$y = \frac{M}{[p_y + \frac{(p_y)^2}{(p_x)}]} = \frac{Mp_x}{p_y(p_x + p_y)} \tag{6}$$

(b) From (6), we can see that the demand for y is an increasing function of M, and therefore a normal good. You can verify this by showing that the derivative with respect to M is

$$\frac{\partial y}{\partial M} = \frac{p_x}{p_y(p_x + p_y)}$$

which is positive.

(c) This problem can be solved by setting up the Lagrangean expression to solve the utility maximization problem, but since we already did this in part (a), it is easier to substitute $p_x = 2$, $p_y = 1$, and $M = 12$ into (6), yielding $y = 8$. From the budget equation, (3), it follows that $x = 2$.

6. **(30 points)** Buckeye Gear is a local seller of officially licenced OSU sweater vests. They have determined that the market is composed of two segments, students and nonstudents, and that the demand curve for each segment is as follows:

$$\begin{aligned} \text{students} \quad &: \quad x_S = 200 - 4p \\ \text{non-students} \quad &: \quad x_{NS} = 340 - 5p, \end{aligned}$$

where p is the price of a sweater vest in dollars.

(a) If $p = 35$, what is the quantity demanded by students and the quantity demanded by non-students?

(b) What is the market demand function (total demand as a function of p)?

(c) If $p = 35$, what is the price elasticity of demand (for the entire market)?

(d) What price should be charged if the goal is to maximize total revenue for the entire market?

Answer:

(a) Substituting $p = 35$ into the students' demand function, we have $x_S = 60$, and substituting $p = 35$ into the non-students' demand function, we have $x_{NS} = 165$.

(b) Students' demand will be zero if $p > 50$, and non-students' demand will be zero if $p > 68$, so the market demand function is given by

$$\begin{aligned} x_{Market} &= 540 - 9p \qquad \text{if } p < 50 \\ &= 340 - 5p \qquad \text{if } 50 \leq p < 68 \\ &= 0 \qquad \qquad \text{if } 68 \leq p. \end{aligned}$$

(c) The elasticity formula is

$$\varepsilon^d = \frac{dx}{dp_x}\left(\frac{p_x}{x}\right).$$

From the market demand equation, this formula becomes

$$-9(\frac{35}{540-9(35)}),$$

which equals -1.4.

(d) The easiest way to find the revenue maximizing price is to find the price at which the price elasticity of demand is -1. The other way is to derive the total revenue function and maximize it. To find the total revenue as a function of x, we first solve the demand function for inverse demand:

$$\begin{aligned} x &= 540 - 9p \\ 9p &= 540 - x \\ p &= \frac{540-x}{9}. \end{aligned}$$

Therefore, the total revenue function is

$$TR(x) = \frac{(540-x)x}{9}$$

Setting the derivative equal to zero, we have:

$$\begin{aligned} \frac{540-2x}{9} &= 0 \\ x &= 270. \end{aligned}$$

Substituting $x = 270$ into the inverse demand function, we have $p = 30$.