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Optimal Monopoly Mechanisms with Demand Uncertainty

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Abstract. This paper analyzes a monopoly firm's profit-maximizing mechanism in the following context. There is a continuum of consumers with a unit demand for a good. The distribution of the consumers' valuations is given by one of two possible demand distributions/states. The consumers are uncertain about the demand state, and they update their beliefs after observing their own valuation for the good. The firm is uncertain about the demand state but infers it from the consumers' reported valuations. The firm's problem is to maximize profits by choosing an optimal mechanism among the class of anonymous, deterministic, direct revelation mechanisms that satisfy interim incentive compatibility and ex post individual rationality. We show that, under certain sufficient conditions, the firm's optimal mechanism is to set the monopoly price in each demand state. Under these conditions, Segal's optimal ex post mechanism is robust to relaxing ex post incentive compatibility to interim incentive compatibility.

Keywords: monopoly mechanism • correlated valuations • Bayesian incentive compatibility • ex post individual rationality

1. Introduction

Consider a monopoly firm that is trying to maximize profit in the presence of aggregate demand uncertainty. In each demand state, there is a distribution of consumer valuations, or a demand curve, in which each consumer is negligible relative to the market and desires at most one unit of the good. It is well known that, in the absence of demand uncertainty, there is no scope for price discrimination, and the monopolist's optimal mechanism is to charge the same optimal monopoly price to all consumers. Myerson [15] solves the optimal auction problem, and Bulow and Roberts [2] show that the monopoly problem is equivalent to the Myerson setting when consumer valuations are independent (in which case the demand curve would be known in a large economy). See also Harris and Raviv [8] and Riley and Zeckhauser [18].

On the other hand, when aggregate demand uncertainty is present and, therefore, consumer valuations are correlated, Crémer and McLean [4, 5] show that, under certain conditions, the monopolist can extract the entire consumer surplus using a Bayesian incentive compatible (BIC) and interim individually rational (IIR) mechanism. These mechanisms involve consumers participating in side bets with the firm, by which consumers must make/receive huge payments depending on the outcome of the bet. In particular, these payments can be far more than the consumers' valuation of the good being sold. However, in many monopoly situations, a consumer cannot be prevented from walking away from a deal when asked to pay more than the consumer's valuation of the good. Segal [19] accounts for this by requiring ex post individual rationality (EIR) along with ex post incentive compatibility. Segal [19] analyzes the model with a finite number of consumers and several possible distributions from which valuations are independently drawn. He shows that, when the number of buyers is large, the optimal mechanism converges to state-by-state monopoly pricing (SBSMP).

In the private values setting, Segal's [19] ex post incentive compatibility is equivalent to dominant strategy incentive compatibility (DIC). Imposing DIC rules out many forms of price discrimination. For example, given a profile of reported types, DIC requires any anonymous and deterministic mechanism to charge the same price to any consumer receiving the good. Therefore, to capture the situation in which consumers cannot be charged more than their valuation of the good but price discrimination is possible, we impose EIR as does Segal [19] but relax DIC to BIC.

We assume that there are two distributions, high and low, from which valuations are independently drawn. Appealing to the law of large numbers, these distributions also correspond to two possible realized demand curves. We first impose regularity conditions on the demand process. Under these assumptions and three additional conditions, we show that SBSMP is optimal among all anonymous, deterministic, EIR, and BIC mechanisms. One of the three additional conditions is similar to a "single crossing" condition for beliefs.

Peck and Rampal [17] provide a counterexample to SBSMP when this single crossing property does not hold. The other two conditions are concavity and a restriction over beliefs.

When the conditions of the SBSMP proposition are satisfied, it follows that Segal's characterization of SBSMP as the optimal ex post mechanism is robust to relaxing DIC to BIC. Specifically, we are referring to the case in which the number of consumers approaches infinity, there are two possible demand distributions, and we consider deterministic mechanisms.

To our knowledge, this is the first BIC–DIC equivalence result for environments with correlated types and EIR. Crémer and McLean [5] provide such a result under IIR and a spanning condition, which could require large payments from consumers (also see Kushnir [10]). Mookherjee and Reichelstein (1992), Manelli and Vincent [14], Gershkov et al. [7], and Kushnir and Liu [11] all consider BIC–DIC equivalence with independent types.

The literature considers settings in which the buyer's valuation depends on the "item type." The item type could refer to a characteristic of the good being sold, but in our context, it refers to the distribution of demand. Krähmer and Strausz [9] and Bergemann et al. [1] consider a sequential screening problem with one buyer, who first observes the distribution from which the buyer's valuation is drawn and later observes the buyer's valuation. When EIR is imposed, the seller does not engage in screening if a monotonicity condition is satisfied—instead setting a take-it-or-leave-it price. The no-screening result is similar to SBSMP in the sense that all consumers are offered the same deal. In the sequential screening literature, the seller does not observe the item type (i.e., the demand distribution); in the present paper, in contrast, the seller effectively observes the item type by aggregating the reports of the potential buyers.

In Daskalakis et al. [6], as in the present paper, the item type is observed by the seller and not by the buyer. They find that it is generally not optimal for the seller to reveal the item type to the buyer as would be the case for SBSMP. In Daskalakis et al. [6], a buyer of a given "bidder type" does not observe the buyer's valuation but rather a function specifying the buyer's valuation for each item type. Think of a bidder type as a type of firm bidding for ad space on a website, and think of the item type as the consumer who is visiting the website. The buyer knows the buyer's bidder type but not the item type. In the present paper, in contrast, the buyer knows the buyer's valuation, so the bidder type and the valuation are one and the same. It turns out that, when the buyer knows the buyer's valuation, this eliminates the benefit of hiding the item type from the buyer (under reasonable assumptions).

This paper is related to the literature on product bundling (see Manelli and Vincent [12, 13]) because consumption of the same good in different states of nature can be interpreted as different commodities. The counterexample to SBSMP in Peck and Rampal [17] is similar to an example in Carroll [3]. Daskalakis et al. [6] show that the optimal mechanism is equivalent to an optimal multi-item mechanism, with which hiding information about the item type can be interpreted as selling fractional bundles of those items. The fraction sold corresponds to the probability of the demand state. For this interpretation to be valid, it is assumed that the bidder type is uncorrelated with the item type. By contrast, in the present paper, the buyer and item types are correlated; in particular, the buyer observes the buyer's valuation for the realized item type, so buyers with different valuations update their beliefs differently and essentially expect different bundles.

In Section 2, the model is laid out, and some preliminary analysis is conducted. Section 3 contains the main result about SBSMP. Section 4 contains some concluding remarks. Proofs are contained in the appendix.

2. Model

A risk-neutral, profit-maximizing monopoly firm faces a continuum of consumers with a unit demand for a good. The firm has zero marginal cost of production. There are two demand states: low and high. Conditions 1–3 imply that the monopoly price in state 1 is strictly less than the monopoly price in state 2, so we refer to state 1 as the low state and state 2 as the high state.

The probability of the low state is α_1, and the probability of the high state is $\alpha_2 = 1 - \alpha_1$. For $i = 1, 2$, consumers' valuations in state i are distributed over $V = [\underline{v}, \overline{v}]$ according to the demand distribution $D_i(\cdot)$. In particular, $D_i(v)$ is the measure of consumers with valuation greater than v in state i. Think of the following process. First, nature selects the demand state according to the probabilities α_1 and α_2. Then, out of the measure of "potential" consumers, C, nature selects a consumer to be active in state i with probability $D_i(\underline{v})/C$. Finally, for the set of selected active consumers, nature independently selects valuations giving rise to the distribution, $D_i(v)$. See Peck [16] for more details and the derivation of (1) according to Bayes' rule. Consumers and the firm know the structure of demand but not the realization.

Because there is aggregate demand uncertainty, a consumer's valuation provides the consumer with significant information about the demand state. For a consumer whose valuation is v, the consumer's updated belief about the realized demand state is

$$\Pr(\text{Demand state is } i|\text{own valuation is } v) = \frac{\alpha_i\left(-D_i'(v)\right)}{\alpha_1\left(-D_1'(v)\right) + \alpha_2\left(-D_2'(v)\right)}. \tag{1}$$

In what follows, we assume that demand is twice continuously differentiable. The density of the downward-sloping demand distribution at valuation v is denoted as $(-D_i'(\cdot))$ in state $i = 1,2$. We assume that $(-D_i'(\cdot)) > 0$ holds at all $v \in V$ for $i = 1,2$.

We consider direct revelation mechanisms satisfying BIC and EIR. According to the revelation principle, consumers report truthfully without loss of generality. We appeal to the law of large numbers to conclude that the firm is able to infer the demand state perfectly from the profile of reported types. We restrict attention to deterministic mechanisms that specify, for each state, which valuation types consume and the amount paid by each type that consumes. Thus, we require anonymous mechanisms and rule out randomized mechanisms that specify a probability of consuming in state i. We are unable to solve the model without this restriction, but it may limit the firm's profit opportunities as shown by Peck and Rampal [17]. We also rule out introducing randomness indirectly by allowing consumption to depend on features of the profile of reports other than the inferred state.

The requirement that the payment scheme satisfy ex post individual rationality implies that the firm is not allowed to charge more than the reported valuation in any demand state and that, if a consumer is not given the good in some demand state, then the firm cannot elicit any positive payment from that consumer in that demand state. To summarize, the firm's problem is to maximize its expected revenue using an anonymous, deterministic, interim incentive compatible, and ex post individually rational mechanism, when facing a continuum of consumers who update about the demand state based on their private valuations. We state this problem formally in the next section.

2.1. The Monopoly Firm's Problem

Let $x_i(v)$ denote the probability with which the monopoly firm gives the good to valuation v in state i. As noted before, we restrict ourselves to the case in which the firm sells the good to v with probability one or zero. So $x_i(v) \in \{0,1\}$ for all v, and $i = 1,2$. Let $t_i(v)$ denote the payment required from v given that the demand state is i, conditional on v purchasing the good in state i. Thus, a mechanism offered by the monopoly firm is as follows:

$$\begin{aligned} &x_i(v) \in \{0,1\},\ \forall v \in V,\ i = 1,2, \\ &0 \le t_i(v) \le v,\ \forall v \in V,\ i = 1,2. \end{aligned} \tag{2}$$

For a given mechanism offered by the monopoly firm, let V_i denote the subset of valuations of V who consume only in state i. That is, for $i = 1,2$ and $j \neq i$, $v \in V_i$ if and only if $x_i(v) = 1$ and $x_j(v) = 0$ hold. So, by the EIR condition, for valuations in V_i, $t_i(v)$ can be positive but must be less than v, and $t_j(v)$ must be zero. Let V_{12} denote the subset of valuations of V that consume in both states, in which case $x_1(v) = 1$ and $x_2(v) = 1$ hold. So, by the EIR condition, for valuations in V_{12}, both $t_1(v)$ and $t_2(v)$ can be positive but must be less than v. Let $V_\varnothing$ denote the subset of valuations of V that do not consume the good in either state. That is, $V_\varnothing = V - [V_1 \cup V_2 \cup V_{12}]$.

We can state the simplified firm's problem as follows. The firm chooses the sets V_1, V_2, and V_{12} and the functions $t_i : V \to [0,\overline{v}]$ for $i = 1,2$ to solve

$$\max \int_{V_{12}} \left[t_1(v)\alpha_1\left(-D_1'(v)\right) + t_2(v)\alpha_2\left(-D_2'(v)\right)\right]dv + \int_{V_1} t_1(v)\alpha_1\left(-D_1'(v)\right)dv + \int_{V_2} t_2(v)\alpha_2\left(-D_2'(v)\right)dv. \tag{3}$$

Subject to (i) EIR (given by (2)) and (ii) BIC (as follows).

$$\begin{aligned} &(v - t_1(v))x_1(v)\alpha_1\left(-D_1'(v)\right) + (v - t_2(v))x_2(v)\alpha_2\left(-D_2'(v)\right) \\ &\quad \ge (v - t_1(\widehat{v}))x_1(\widehat{v})\alpha_1\left(-D_1'(v)\right) + (v - t_2(\widehat{v}))x_2(\widehat{v})\alpha_2\left(-D_2'(v)\right);\ \forall v,\ \widehat{v} \in V. \end{aligned} \tag{4}$$

The BIC condition is stated after canceling $[\alpha_1(-D_1'(v)) + \alpha_2(-D_2'(v))]$ from the denominator on both sides of the inequality.

2.2. Conditions and Preliminary Results

In this section, we specify conditions on the demand process and establish preliminary results. We start with regularity conditions for the two demand states, that is, the "maintained assumptions" about demand. Then Fact 1 follows from BIC.

Condition 1 (Regularity).

i. *$D_1(v)$ and $D_2(v)$ are twice continuously differentiable.*

ii. *Demand is strictly downward sloping everywhere; that is, $D_i'(v) < 0$ holds for all $v \in V$ and $i \in \{1,2\}$.*

2.2.1. Fact 1. If the firm's mechanism satisfies BIC, then $t_i(v) = t_i(\widehat{v})$ must hold for all v, $\widehat{v}$ in V_i, where $i = 1, 2$.

2.2.2. Proof of Fact 1. For $i = 1,2$ and $j \neq i$, if $v, \widehat{v} \in V_i$, then $x_i(v) = x_i(\widehat{v}) = 1$ and $x_j(v) = x_j(\widehat{v}) = 0$ hold. Thus, the BIC condition (4) implies

$$(v - t_i(v))\alpha_i\big(-D_i'(v)\big) \geq (v - t_i(\widehat{v}))\alpha_i\big(-D_i'(v)\big),$$

which implies $t_i(v) \leq t_i(\widehat{v})$. Similarly, the BIC condition for $\widehat{v}$ with respect to v implies $t_i(\widehat{v}) \leq t_i(v)$. So Fact 1 holds. ■

Lemma 1 provides a first step toward characterizing the firm's optimal mechanism. Let v_1^*, v_2^*, and v_{12}^* denote the infimum valuations of the sets V_1, V_2, and V_{12}, respectively. The infima of these sets are well defined because they are bounded subsets of $\mathcal{R}$.

Lemma 1. *At the monopoly firm's optimal EIR and BIC mechanism, V_{12} is nonempty.*

The proof of Lemma 1 is given in the appendix.

Condition 2 (Information Effect). The ratio $Z(v) \equiv \frac{(-D_1'(v))}{(-D_2'(v))}$ *is strictly decreasing in v for all $v \in V$. That is, $Z'(v) < 0$ holds for all $v \in V$.*

Condition 2 specifies the information effect. Note that $\frac{\alpha_1}{\alpha_2}Z(v)$ is the ratio of the probability type v assigns to state 1 to the probability assigned to state 2. Thus, Condition 2 says that the greater the valuation of a consumer, the greater the probability the consumer assigns to the high demand state. The next step is to characterize the sets V_1, V_2, and V_{12}. In particular, the question is, given Conditions 1 and 2, whether the requirement that the firm's mechanism satisfy the BIC and EIR constraints implies that the firm's mechanism must order and structure the sets V_1, V_2, and V_{12} in a particular manner. Lemma 2 addresses this question.

Lemma 2. *Let v_i denote the arbitrary valuation of the set V_i for $i \in \{1,2,12\}$. Given Conditions 1 and 2, if the firm's mechanism satisfies the BIC and EIR constraints and the appropriate sets are nonempty, then we must have* (i) $v_1 < v_{12}$, *and* (ii) *at the firm's optimal mechanism, $v_2^* < v_{12}^*$ must hold.*

The proof of Lemma 2 is given in the appendix.

Given Lemma 1, it follows that the monopoly firm chooses a mechanism from among the following possible types of mechanisms: (1) with only V_{12} nonempty; (2) with V_1 and V_{12} nonempty and V_2 empty; (3) with V_2 and V_{12} nonempty and V_1 empty; (4) with V_1, V_2, and V_{12} all nonempty. In general, when the profit-maximizing monopoly price in the two demand states is different, it can be shown that the firm can improve upon a mechanism with just V_{12} nonempty (we show this in the proof of Proposition 1). Thus, the main question is going to be: which among (2)–(4) is optimal for the firm?

Note that, if the firm's mechanism gives the good to valuation v in state i or state j or both, then BIC implies that valuations greater than v are also given the good in some state because, otherwise, valuations greater than v can report their valuation as v, get the good in whichever state v gets the good, make a payment less than v (because, by EIR, the firm cannot charge more than the reported valuation to v), and earn a strictly positive surplus. Fact 1 implies $t_i(v_i) = t_i(v_i^*)$ for $i = 1,2$, and all $v_i \in V_i$. Lemma 3 further specifies the payment scheme.

Lemma 3. *If V_1 and V_2 are both nonempty and $v_i^* < v_j^*$ holds, or if only V_i and V_{12} are nonempty with $v_i^* < v_{12}^*$, then BIC, EIR, and Conditions 1 and 2 imply that $t_i(v_i^*) = v_i^* = t_i(v_i)$ holds for all $v_i \in V_i$.*

Proof of Lemma 3. First, note that, under the conditions of Lemma 3, valuations less than v_i^* are not given the good in either state; that is, valuations less than v_i^* belong to $V_\varnothing$. This holds by assumption if only V_i and V_{12} are nonempty with $v_i^* < v_{12}^*$ or if only V_1 and V_2 are nonempty with $v_i^* < v_j^*$. For the case in which V_1, V_2, and V_{12} are all nonempty and $v_i^* < v_j^*$ holds, this follows from Lemma 2(i). Next, recall that, by Fact 1, $t_i(v_i)$ must be constant for all $v_i \in V_i$. To see why $t_i(v_i^*) = v_i^*$ holds, note that, for any v such that $v < v_i^*$ holds, $t_i(v_i^*) \geq v$ must hold to satisfy BIC of v. So $t_i(v_i^*) \geq v_i^*$ must hold. The EIR constraint of v_i^* implies $t_i(v_i^*) \leq v_i^*$. Putting these two statements together, we have $t_i(v_i^*) = v_i^*$. ■

3. The Optimal Mechanism

Lemma 4 shows that, in the firm's optimal mechanism, it cannot be the case that V_1, V_2, and V_{12} are all nonempty. However, to prove Lemma 4, we require that demand be concave in both states.

Condition 3 (Concave Demand). *Demand is strictly concave; that is, $D_i''(v) < 0$ holds for all $v \in V$ and $i \in \{1,2\}$.*

Concavity is used in Lemma 4 to establish that the revenue function $pD_i(p)$ is concave for $i = 1, 2$, for which concavity is a sufficient condition but not necessary. For $i = 1, 2$, let p_i^m be the profit-maximizing monopoly price in demand state i. Concavity is also used as a sufficient condition (along with Conditions 1 and 2) to establish that p_1^m is lower than p_2^m (in Fact 2).

3.1. Fact 2

Conditions 1–3 imply

$$\frac{D_2(v^*)}{(-D_2'(v^*))} > \frac{D_1(v^*)}{(-D_1'(v^*))} \tag{5}$$

for all $v^* \in V$. And (5) implies p_1^m, which solves $p_1^m = -\frac{D_1(p_1^m)}{D_1'(p_1^m)}$, is strictly lower than p_2^m, which solves $p_2^m = -\frac{D_2(p_2^m)}{D_2'(p_2^m)}$.

The proof of Fact 2 is given in the appendix.

To show Lemma 4, we also require that, after observing their respective valuations, all types agree (as per their beliefs) about which state is more likely.

Condition 4 (Agreement over the More Likely State). *Either* (i) $\frac{\alpha_1}{\alpha_2} Z(v) < 1$ *holds for all* $v \in V$ *or* (ii) $\frac{\alpha_1}{\alpha_2} Z(v) > 1$ *holds for all* $v \in V$.

Lemma 4. *Suppose Conditions 1–4 hold; then, the optimal mechanism cannot have V_1, V_2, and V_{12} all nonempty.*

The proof of Lemma 4 is given in the appendix.

Condition 4 is needed in the proof of Lemma 4 to show that the profit from any mechanism with V_1, V_2, and V_{12} nonempty is bounded above by a mechanism that may not satisfy BIC but where V_1, V_2, and V_{12} are adjacent connected intervals; that is, the bounding mechanism has either $V_1 = [v_1^*, v_2^*)$, $V_2 = [v_2^*, v_{12}^*)$, and $V_{12} = [v_{12}^*, \overline{v}]$, or $V_2 = [v_2^*, v_1^*)$, $V_1 = [v_1^*, v_{12}^*)$, and $V_{12} = [v_{12}^*, \overline{v}]$. In particular, even with Conditions 1–4, there is no guarantee that all BIC and EIR mechanisms with V_1, V_2, and V_{12} nonempty have the interval property. However, the bounding mechanism satisfies the interval property, which allows us to express the upper bound profit in terms of the infima v_1^*, v_2^*, and v_{12}^*. We then show that the upper bound profit can be strictly increased by switching to a mechanism with only V_1 and V_{12} nonempty, which also satisfies BIC and EIR.

Lemma 4 yields that, under its conditions, in the optimal mechanism, either only V_{12} is nonempty or only V_1 and V_{12} are nonempty or only V_2 and V_{12} are nonempty. The SBSMP proposition establishes that, under Conditions 1–4, only V_1 and V_{12} are nonempty in the optimal mechanism. This, in turn, yields that the firm's optimal mechanism is to set the monopoly price in each demand state; that is, the firm's optimal mechanism is SBSMP.

Proposition 1 (SBSMP). *If Conditions 1–4 hold, then, within the class of deterministic BIC and EIR mechanisms, the monopoly firm's optimal mechanism is state-by-state monopoly pricing (SBSMP). The SBSMP mechanism is as follows:*

$$\begin{cases} t_1(v) = p_1^m, & \forall\, v \geq p_1^m \\ t_2(v) = p_2^m, & \forall\, v \geq p_2^m \\ x_1(v) = 1 \ \forall v \geq p_1^m, & x_1(v) = 0 \ \forall v < p_1^m, \\ x_2(v) = 1 \ \forall v \geq p_2^m, & x_2(v) = 0 \ \forall v < p_2^m. \end{cases} \tag{6}$$

Proof (Summary). By Lemma 4, mechanisms with V_1, V_2, and V_{12} all nonempty are suboptimal. The proof proceeds by ruling out the possibility that the optimal mechanism can be one with only V_{12} nonempty or one with only V_2 and V_{12} nonempty. This proves that the optimal mechanism must have only V_1 and V_{12} nonempty. Finally, we show that the optimal mechanism among mechanisms with only V_1 and V_{12} nonempty is SBSMP. The detailed proof is provided in the appendix.

4. Concluding Remarks

Under certain regularity conditions, we show that SBSMP is optimal among all anonymous, deterministic, EIR, and BIC mechanisms. The result is far from obvious as illustrated by the counterexample in Peck and Rampal [17], in which the regularity conditions are not satisfied. It would be nice to allow for randomized mechanisms, but much of the Myerson machinery is unavailable and very few results are available in the literature when types are correlated.

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Appendix. Proofs

A.1. Proof of Lemma 1.

We prove Lemma 1 by contradiction. Suppose, at the firm's optimal EIR and BIC mechanism, V_{12} is empty. We argue that each of the alternatives yields strictly lower profit than the profit from an EIR and BIC mechanism with V_{12} nonempty. The alternatives are as follows:

i. Only V_i nonempty for $i \in \{1, 2\}$. By Fact 1 and EIR, the firm's profit in this case is bounded above by $\alpha_i v_i^* D_i(v_i^*)$. If, instead, all $v_i \in V_i$ are given the good in both states (i.e., $V_{12} = V_i$) at price v_i^*, the profit would be $\alpha_i v_i^* D_i(v_i^*) + \alpha_j v_i^* D_j(v_i^*)$ for $i, j = 1, 2$ and $i \neq j$, which is strictly greater than $\alpha_i v_i^* D_i(v_i^*)$, and all EIR and BIC constraints would still be satisfied.

ii. Only V_i and V_j nonempty for $i, j = 1, 2$ and $i \neq j$. Without loss of generality, let $v_i^* \leq v_j^*$ hold. By BIC and EIR, using Fact 1, the firm's profit in this case is strictly lower than $\alpha_i v_i^* D_i(v_i^*) + \alpha_j v_j^* D_j(v_j^*)$ because V_i and V_j are disjoint sets by definition. However, $\alpha_i v_i^* D_i(v_i^*) + \alpha_j v_j^* D_j(v_j^*)$ is exactly the profit if, instead of only V_i and V_j nonempty, a different mechanism is used: one in which only V_i and V_{12} are nonempty, and (i) V_i equals $[v_i^*, v_j^*)$, which is empty if $v_j^* = v_i^*$ holds, and $t_i(v_i) = v_i^*$ holds for all $v_i \in V_i$; (ii) V_{12} equals $[v_{12}^*, \overline{v}]$ with $v_{12}^* = v_j^*$, and $t_i(v_{12}) = v_i^*$, $t_j(v_{12}) = v_{12}^*$ hold for all $v_{12} \in V_{12}$. Further, it is straightforward to verify that such a mechanism satisfies all EIR and BIC constraints. ■

A.2. Proof of Lemma 2.

A.2.1. Proof of Part (i).

First, we show that $v_1 < v_{12}$ must hold. Suppose not; that is, let $v_1 > v_{12}$ hold (note that we cannot have v_1 equal to v_{12} because a valuation cannot belong to both V_1 and V_{12}). Consider the BIC constraints of v_{12} with respect to v_1 and of v_1 with respect to v_{12}:

$$(v_{12} - t_1(v_{12}))\alpha_1(-D_1'(v_{12})) + (v_{12} - t_2(v_{12}))\alpha_2(-D_2'(v_{12})) \geq (v_{12} - t_1(v_1))\alpha_1(-D_1'(v_{12}));$$
$$(v_1 - t_1(v_1))\alpha_1(-D_1'(v_1)) \geq (v_1 - t_1(v_{12}))\alpha_1(-D_1'(v_1)) + (v_1 - t_2(v_{12}))\alpha_2(-D_2'(v_1)) .$$

These can be rewritten as

$$(v_{12} - t_2(v_{12}))\frac{\alpha_2}{\alpha_1 Z(v_{12})} \geq t_1(v_{12}) - t_1(v_1), \tag{A.1}$$

and

$$t_1(v_{12}) - t_1(v_1) \geq (v_1 - t_2(v_{12}))\frac{\alpha_2}{\alpha_1 Z(v_1)}. \tag{A.2}$$

Together, (A.1) and (A.2) imply

$$(v_{12} - t_2(v_{12}))\frac{\alpha_2}{\alpha_1 Z(v_{12})} \geq (v_1 - t_2(v_{12}))\frac{\alpha_2}{\alpha_1 Z(v_1)}. \tag{A.3}$$

Next, note that $v_1 > t_2(v_{12})$ must hold. This is because, by EIR, $v_{12} \geq t_2(v_{12})$ must hold; further, $v_1 > v_{12}$ holds by assumption. Thus, $v_1 > v_{12} \geq t_2(v_{12})$ holds. Note that $v_1 > v_{12}$ implies $(v_1 - t_2(v_{12})) > (v_{12} - t_2(v_{12}))$. Further, by Condition 2, $1/Z(v)$ is increasing for all v. Thus, $\alpha_2/\alpha_1 Z(v_1) > \alpha_2/\alpha_1 Z(v_{12})$ holds, which implies

$$(v_{12} - t_2(v_{12}))\frac{\alpha_2}{\alpha_1 Z(v_{12})} < (v_1 - t_2(v_{12}))\frac{\alpha_2}{\alpha_1 Z(v_1)},$$

which is a contradiction of (A.3). Thus, it must be the case that $v_{12} > v_1$ holds.

A.2.2. Proof of Part (ii).

The aim is to show that $v_2^* < v_{12}^*$ must hold in the firm's optimal mechanism. The proof is by contradiction; that is, suppose $v_{12}^* \leq v_2^*$ holds. Given Lemmas 1 and 2(i), if $v_{12}^* \leq v_2^*$ holds, then either (a) $v_1^* < v_{12}^* \leq v_2^*$ holds and V_1, V_2, and V_{12} are all nonempty or (b) only V_2 and V_{12} are nonempty with $v_{12}^* \leq v_2^*$. We rule out both cases.

A.2.3. Ruling out (a).

Suppose mechanism A is an arbitrary mechanism with $v_1^* < v_{12}^* \leq v_2^*$ and V_1, V_2, and V_{12} all nonempty. In mechanism A, by the BIC of $v < v_1^*$ with respect to v_1^* (which implies $t_1(v_1^*) \geq v_1^*$) and by the EIR of v_1^*

(which implies $t_1(v_1^*) \le v_1^*$), we must have $t_1(v_1^*) = v_1^*$. Second, by Fact 1, all $v_1 \in V_1$ and $v_2 \in V_2$ must be charged v_1^* and $t_2(v_2^*)$, respectively. Third, by the BIC of $v \in V_{12}$ with respect to v_{12}^*, we have

$$(v - t_1(v))\alpha_1(-D_1'(v)) + (v - t_2(v))\alpha_2(-D_2'(v)) \ge (v - t_1(v_{12}^*))\alpha_1(-D_1'(v)) + (v - t_2(v_{12}^*))\alpha_2(-D_2'(v)),$$

which can be rearranged to

$$t_1(v)\alpha_1(-D_1'(v)) + t_2(v)\alpha_2(-D_2'(v)) \le t_1(v_{12}^*)\alpha_1(-D_1'(v)) + t_2(v_{12}^*)\alpha_2(-D_2'(v)). \tag{A.4}$$

The implication of (A.4) is that the profit from $v \in V_{12}$, given on the left side of (A.4) (see (3)), is bounded above by charging v the same payment scheme as offered to v_{12}^* (this follows from (A.4)). Fourth, by Lemma 2(i), $V_1 = [v_1^*, v_{12}^*)$ holds because no type with valuation above v_{12}^* can be in V_1. Given these four features of mechanism A, the profit from mechanism A is bounded above by

$$\begin{aligned}\pi_A = {} & \alpha_1[D_1(v_1^*) - D_1(v_{12}^*)]v_1^* + \alpha_1[D_1(v_{12}^*) - D_1(v_2^*)]t_1(v_{12}^*) + \alpha_2[D_2(v_{12}^*) - D_2(v_2^*)]t_2(v_{12}^*) + \alpha_1 t_1(v_{12}^*)\int_{[v_2^*,\bar{v}]-V_2}(-D_1'(v))dv \\ & + \alpha_2 t_2(v_{12}^*)\int_{[v_2^*,\bar{v}]-V_2}(-D_2'(v))dv + \alpha_2 t_2(v_2^*)\int_{V_2}(-D_2'(v))dv.\end{aligned} \tag{A.5}$$

In (A.5), we must have $t_2(v_2^*) \le t_2(v_{12}^*)$. To see this, note that the BIC of v_2^* with respect to v_{12}^* yields

$$(v_2^* - t_2(v_2^*))\alpha_2(-D_2'(v_2^*)) \ge (v_2^* - t_1(v_{12}^*))\alpha_1(-D_2'(v_2^*)) + (v_2^* - t_2(v_{12}^*))\alpha_2(-D_2'(v_2^*)), \tag{A.6}$$

and because $t_1(v_{12}^*) \le v_{12}^* \le v_2^*$ holds (first inequality holds by EIR and second holds by assumption of mechanism A), (A.6) implies $t_2(v_2^*) \le t_2(v_{12}^*)$.

Now, consider an alternative mechanism, labeled mechanism B, in which only the following changes are made to mechanism A: V_2 is set to be empty, all valuations in V_2 are allocated to V_{12}, and all valuations in V_{12} are charged $t_i(v_{12}^*)$ in state i for $i = 1, 2$ (the payment scheme offered to v_{12}^* in mechanism A). In mechanism B, the profit is

$$\pi_B = \alpha_1[D_1(v_1^*) - D_1(v_{12}^*)]v_1^* + \alpha_1 D_1(v_{12}^*)t_1(v_1^*) + \alpha_2 D_2(v_{12}^*)t_2(v_{12}^*), \tag{A.7}$$

which is clearly greater than π_A because valuations in V_2, which are in V_{12} for mechanism B, are charged a weakly greater amount in state 2, and the firm gets strictly positive revenue from them in state 1 as well. To complete the argument, note that, if BIC and EIR are satisfied in mechanism A, then they continue to hold in mechanism B. We first check that the BIC of all types in V_{12} with respect to types in V_1 is satisfied. Because the BIC of v_{12}^* with respect to v_1^* is satisfied in mechanism A, we have

$$\begin{gathered}(v_{12}^* - t_1(v_{12}^*))\alpha_1(-D_1'(v_{12}^*)) + (v_{12}^* - t_2(v_{12}^*))\alpha_2(-D_2'(v_{12}^*)) \ge (v_{12}^* - v_1^*)\alpha_1(-D_1'(v_{12}^*)), \text{ or} \\ v_{12}^* \ge (t_1(v_{12}^*) - v_1^*)\frac{\alpha_1}{\alpha_2}Z(v_{12}^*) + t_2(v_{12}^*)\end{gathered} \tag{A.8}$$

holds. Replacing v_{12}^* with any $v > v_{12}^*$ in (A.8) yields

$$v > (t_1(v) - v_1^*)\frac{\alpha_1}{\alpha_2}Z(v) + t_2(v), \tag{A.9}$$

because $t_1(v)$ and $t_2(v)$ are identical to $t_1(v_{12}^*)$ and $t_2(v_{12}^*)$ in mechanism B and because $Z(v) < Z(v_{12}^*)$ holds because of $v > v_{12}^*$ and Condition 2. Equation (A.9) can be rearranged to show that the BIC of v with respect to v_1^*, and thereby all types in V_1, is satisfied. The BIC of all $v_1 \in V_1$ with respect to V_{12} continues to hold in mechanism B because all valuations in V_{12} are offered the same payment scheme.

A.2.4. Ruling out (b). Suppose only V_2 and V_{12} are nonempty with $v_{12}^* \le v_2^*$. We first rule out $v_{12}^* = v_2^* = v^*$ being compatible with an optimal mechanism. By EIR, $t_2(v_2^*) \le v_2^*$ must hold, and by Fact 1, $t_2(v_2) = t_2(v_2^*)$ holds for all $v_2 \in V_2$. Further, valuations in V_2 pay zero in state 1. Following the arguments around (A.4), under BIC, the firm cannot extract more profit from V_{12} than by charging the same payment scheme to all $v_{12} \in V_{12}$. Further, by EIR, $t_i(v_{12}^*) \le v_{12}^*$ holds for $i = 1, 2$. Thus, profit from a mechanism with V_2 and V_{12} nonempty and $v_{12}^* = v_2^* = v^*$ is bounded above by

$$\int_{[v^*,\bar{v}]-V_2} v^*\{\alpha_1(-D_1'(v)) + \alpha_2(-D_2'(v))\}dv + \int_{V_2} v^*\alpha_2(-D_2'(v))dv, \tag{A.10}$$

where V_2 is a subset of $[v^*, \bar{v}]$. On the other hand, the BIC and EIR mechanism with only $V_{12} = [v_{12}^*, \bar{v}]$ nonempty, $t_i(v) = v_{12}^* = v^*$ for $i = 1, 2$ and all $v \in V_{12}$, yields profit equal to

$$\int_{[v^*,\bar{v}]} v^*\{\alpha_1(-D_1'(v)) + \alpha_2(-D_2'(v))\}dv,$$

which is clearly greater than (A.10). Thus, mechanisms with only V_2 and V_{12} nonempty with $v_{12}^* = v_2^*$ are ruled out.

Now, suppose only V_2 and V_{12} are nonempty with $v_{12}^* < v_2^*$. Given $t_2(v_{12}^*) \le v_{12}^*$ (by EIR), the BIC of v_2^* with respect to v_{12}^*, that is, (A.6), yields $t_2(v_2^*) < v_{12}^*$. This is because the first term on the right side of (A.6) is strictly positive because $(v_2^* - v_{12}^*) > 0$ holds by assumption, $\alpha_1(-D_1'(v_2^*)) > 0$ holds by Condition 1, and $t_1(v_{12}^*) \le v_{12}^*$ holds by EIR. But $t_2(v_2^*) < v_{12}^*$ contradicts the BIC of types with valuation slightly lower than v_{12}^* with respect to v_2^*. These types receive zero surplus because they don't consume in either state, but if they misreport their valuation as v_2^*, then they get the good in state 2 at a price lower than their valuation, which yields a positive surplus. ■

A.3. Proof of Fact 2.

To see why Conditions 1–3 imply (5), first rewrite (5) as

$$\frac{\int_{v^*}^{\bar{v}}(-D_2'(v))dv}{\int_{v^*}^{\bar{v}}(-D_1'(v))dv} > \frac{(-D_2'(v^*))}{(-D_1'(v^*))}. \tag{A.11}$$

Note that the left side of (A.11) is equal to

$$\frac{\int_{v^*}^{\bar{v}}\frac{(-D_1'(v))}{Z(v)}dv}{\int_{v^*}^{\bar{v}}(-D_1'(v))dv}.$$

$Z(v)$ and $(-D_1'(v))$ are nonnegative and strictly decreasing because of Conditions 1–3. Thus, it follows from Wang [20, lemma 2] (in Wang's notation, $x(\phi) = 1$, $y(\phi) = (-D_1'(v))$, and $z(\phi) = 1/Z(v)$) that we have

$$\frac{\int_{v^*}^{\bar{v}}\frac{(-D_1'(v))}{Z(v)}dv}{\int_{v^*}^{\bar{v}}(-D_1'(v))dv} > \frac{\int_{v^*}^{\bar{v}}\frac{1}{Z(v)}dv}{\int_{v^*}^{\bar{v}}dv}. \tag{A.12}$$

Because $Z(v)$ is strictly decreasing and we are considering $v \ge v^*$, it follows that the right side of (A.12) exceeds $1/Z(v^*)$. Therefore, we have

$$\frac{\int_{v^*}^{\bar{v}}\frac{(-D_1'(v))}{Z(v)}dv}{\int_{v^*}^{\bar{v}}(-D_1'(v))dv} > \frac{1}{Z(v^*)},$$

which implies (A.11) and its equivalent, (5). ■

A.4. Proof of Lemma 4: Ruling out V_1, V_2, and V_{12} All Nonempty

The proof of Lemma 4 relies on Lemmas A.1–A.4 detailed herein. First, we show Claim 1.

A.4.1. Claim 1. Suppose V_1 and V_2 are nonempty and BIC, EIR, and Conditions 1–3 hold. Then,

a. Condition 4(i), that is, $\frac{\alpha_1}{\alpha_2}Z(v) < 1$ for all v, implies $v_1^* < v_2^*$.
b. Condition 4(ii), that is, $\frac{\alpha_1}{\alpha_2}Z(v) > 1$ for all v, implies $v_2^* < v_1^*$.

A.4.2. Proof. To begin, we argue that $v_1^* = v_2^*$ is impossible under Condition 4. This is because, first, $v_1^* = v_2^* = v^*$, for some $v^* \in V$ implies (by Fact 1, EIR of v^*, and the BIC of valuations less than v^* with respect to v^*) that the price for consumption must be v^* for all types in V_1 and V_2, that is, $t_1(v_1) = t_2(v_2) = v^*$ holds for all $v_1 \in V_1$ and $v_2 \in V_2$, and second, Condition 4 implies that, given their updated beliefs, either all types consider state 2 more likely (Condition 4(i)) or all types consider state 1 more likely (Condition 4(ii)). Thus, under Condition 4(i), all types prefer reporting a valuation in V_2 (so V_1 has to be empty), and similarly under Condition 4(ii) V_2 must be empty.

Now, we consider strict contradictions of Claim 1a and b.

a. By contradiction, suppose $v_2^* < v_1^*$ and Condition 4(i) hold. By Lemmas 2(i) and 3, $t_2(v_2^*) = v_2^*$ holds. The BIC of v_1^* with respect to v_2^* yields

$$v_1^* - t_1(v_1^*) \ge (v_1^* - v_2^*)\frac{\alpha_2}{\alpha_1 Z(v_1^*)}, \text{ or}$$
$$t_1(v_1^*) \le v_1^* - (v_1^* - v_2^*)\frac{\alpha_2}{\alpha_1 Z(v_1^*)} < v_2^*, \tag{A.13}$$

where the last inequality holds because $\frac{\alpha_1}{\alpha_2}Z(v_1^*) < 1$ holds by Condition 4(i). But having $t_2(v_2^*) = v_2^*$ and $t_1(v_1^*) < v_2^*$ violates the BIC of v_2^* with respect to v_1^*.

b. By contradiction, suppose $v_1^* < v_2^*$ and Condition 4(ii) hold. By Lemmas 2(i) and 3, $t_1(v_1^*) = v_1^*$ holds. The BIC of v_2^* with respect to v_1^* yields

$$v_2^* - t_2(v_2^*) \ge (v_2^* - v_1^*)\frac{\alpha_1}{\alpha_2}Z(v_2^*), \text{ or}$$
$$t_2(v_2^*) \le v_2^* - (v_2^* - v_1^*)\frac{\alpha_1}{\alpha_2}Z(v_2^*) < v_1^*, \tag{A.14}$$

where the last inequality holds because $\frac{\alpha_1}{\alpha_2}Z(v_2^*) > 1$ holds by Condition 4(ii). But having $t_1(v_1^*) = v_1^*$ and $t_2(v_2^*) < v_1^*$ violates the BIC of v_1^* with respect to v_2^*. ■

The sketch of the proof of Lemma 4 is as follows. By Lemma 2 and Claim 1a (respectively, b), under Condition 4(i) (4(ii)), if V_1, V_2, and V_{12} are all nonempty in the optimal mechanism, then $v_1^* < v_2^* < v_{12}^*$ ($v_2^* < v_1^* < v_{12}^*$) holds. For any such choice of v_1^*, v_2^*, v_{12}^*, Lemma A.1 (Lemma A.3) specifies a mechanism that provides an upper bound for profits when $v_1^* < v_2^* < v_{12}^*$ ($v_2^* < v_1^* < v_{12}^*$) holds. Lemma A.2 (Lemma A.4) then demonstrates that this upper bound profit strictly increases as V_2 is "shrunk" until V_2 is empty, at which point the upper bound is also achievable using a mechanism that satisfies all BIC and EIR conditions. So it follows that the optimal mechanism cannot have V_1, V_2, and V_{12} all nonempty.

Lemma A.1. *Consider the class of BIC and EIR mechanisms in which V_1, V_2, and V_{12} are all nonempty, and v_1^*, v_2^*, and v_{12}^* are given. If Conditions 1–3 and 4(i) hold, then the profit under a mechanism in this class can be no greater than the profit that would result if all consumers report truthfully in the following mechanism:*

$$\begin{cases} V_1 = & [v_1^*, v_2^*) \\ V_2 = & [v_2^*, v_{12}^*) \\ V_{12} = & [v_{12}^*, \bar{v}] \\ t_1(v_1) = v_1^* & \forall v_1 \in V_1 \\ t_2(v_2) = v_2^* - (v_2^* - v_1^*)\frac{\alpha_1}{\alpha_2}Z(v_2^*) & \forall v_2 \in V_2 \\ t_1(v_{12}) = v_1^* & \forall v_{12} \in V_{12} \\ t_2(v_{12}) = t_2(v_2^*) + (v_{12}^* - v_1^*)\frac{\alpha_1}{\alpha_2}Z(v_{12}^*) & \forall v_{12} \in V_{12} \end{cases} \tag{A.15}$$

Proof of Lemma A.1. By Claim 1a, $v_1^* < v_2^*$ holds. Further, for the optimal mechanism with V_1, V_2, and V_{12} nonempty, by Lemma 2(ii), $v_2^* < v_{12}^*$ must also hold. So consider an arbitrary BIC and EIR mechanism with V_1, V_2, and V_{12} all nonempty and v_1^*, v_2^*, and v_{12}^* given such that $v_1^* < v_2^* < v_{12}^*$ holds. Label this mechanism as mechanism C. We argue that the profit from mechanism C is weakly lower than the profit from Mechanism (A.15) with v_1^*, v_2^*, and v_{12}^* the same as in mechanism C. Note that we do not require Mechanism (A.15) to satisfy BIC and EIR at this point in the argument.

Mechanism C must have the following features: By assumption $v_1^* < v_2^* < v_{12}^*$ holds. By Lemma 3, $t_1(v_1) = v_1^*$ must hold for all $v_1 \in V_1$. By Fact 1, $t_2(v_2) = t_2(v_2^*)$ holds for all $v_2 \in V_2$. By BIC, v_2^* must be indifferent with respect to reporting v_1^* because, if instead type v_2^* strictly prefers reporting v_2^* over reporting v_1^*, then, by continuity, for a valuation $v_1 \in V_1$ less than v_2^* but close enough to v_2^*, we have that v_1 also strictly prefers reporting v_2^* rather than v_1, which contradicts either the BIC of v_1 or the definition of v_2^* as the infimum of V_2. Thus, the following holds:

$$(v_2^* - t_2(v_2^*))\alpha_2(-D_2'(v_2^*)) = (v_2^* - v_1^*)\alpha_1(-D_1'(v_2^*)), \text{ or} \tag{A.16}$$

$$t_2(v_2^*) = v_2^* - (v_2^* - v_1^*)\frac{\alpha_1}{\alpha_2}Z(v_2^*). \tag{A.17}$$

Next, we argue that mechanism C must have $V_1 = [v_1^*, v_2^*)$; to show this, we need Claim 2.

A.4.3. Claim 2. For mechanism C, all types with valuation greater than v_2^* strictly prefer reporting v_2^* over reporting v_1^*.

A.4.4. Proof. By Lemma 3, $t_1(v_1) = v_1^*$ holds for all $v_1 \in V_1$. By Fact 1, $t_2(v_2) = t_2(v_2^*)$ holds for all $v_2 \in V_2$. By previous arguments, $t_2(v_2^*)$ is such that v_2^* is indifferent between reporting truthfully and reporting v_1^*. To prove Claim 2, we show that, for all types v such that $v > v_2^*$ holds, v strictly prefers reporting v_2^* over reporting v_1^*. Rewriting (A.16), the binding BIC of v_2^* with respect to v_1^*, yields

$$\frac{(v_2^* - t_2(v_2^*))}{(v_2^* - v_1^*)} = \frac{\alpha_1}{\alpha_2}Z(v_2^*). \tag{A.18}$$

Replacing v_2^* with v strictly greater than v_2^* in (A.18) yields

$$\frac{(v - t_2(v_2^*))}{(v - v_1^*)} \geq \frac{(v_2^* - t_2(v_2^*))}{(v_2^* - v_1^*)} = \frac{\alpha_1}{\alpha_2}Z(v_2^*) > \frac{\alpha_1}{\alpha_2}Z(v),$$

where the first inequality follows because $t_2(v_2^*) \geq v_1^*$ holds (by BIC), the last inequality follows because $Z(v)$ is strictly decreasing (by Condition 2), and $v > v_2^*$ holds by assumption. Thus,

$$\frac{(v - t_2(v_2^*))}{(v - v_1^*)} > \frac{\alpha_1}{\alpha_2}Z(v) \text{ holds.}$$

Cross-multiplying yields

$$(v - t_2(v_2^*))\alpha_2(-D_2'(v)) > (v - v_1^*)\alpha_1(-D_1'(v)).$$

Thus, for any type v such that $v > v_2^*$ holds, v strictly prefers reporting v_2^* over reporting v_1^*.

Given Claim 2, we have that mechanism C must have $V_1 = [v_1^*, v_2^*)$, $V_2 \subset [v_2^*, \overline{v}]$, $t_1(v_1) = v_1^*$ for all $v_1 \in V_1$, and $t_2(v_2) = t_2(v_2^*)$ for all $v_2 \in V_2$, where $t_2(v_2^*)$ is given by (A.17). Note that Mechanism (A.15) also has these same properties. In addition, Mechanism (A.15) specifies (a) that $V_2 = [v_2^*, v_{12}^*)$ and $V_{12} = [v_{12}^*, \overline{v}]$ hold and (b) the payment scheme over V_{12}. To finish the proof of Lemma A.1, we must argue that the features (a) and (b) of Mechanism (A.15) don't reduce its profit relative to the profit from mechanism C.

Note that the BIC of v_i^* with respect to $v_{12} \in V_{12}$ implies $t_i(v_{12}) \geq t_i(v_i^*)$ for $i = 1, 2$ and all $v_{12} \in V_{12}$, which means assigning any type with valuation greater than v_{12}^* to V_2 instead of V_{12} only reduces profit. Thus, setting $V_1 = [v_1^*, v_2^*)$, $V_2 = [v_2^*, v_{12}^*)$, and $V_{12} = [v_{12}^*, \overline{v}]$ as in Mechanism (A.15) yields greater profit than from mechanism C if the payment scheme of Mechanism (A.15) also yields greater profit from V_{12} than the payment scheme of mechanism C.

So consider the payment scheme over the set $V_{12} = [v_{12}^*, \overline{v}]$. To finish the proof of Lemma A.1, Claim 3 demonstrates that the payment scheme in Mechanism (A.15) maximizes the firm's expected profit from V_{12} subject to a subset of BIC and EIR constraints. This means adding all the missing BIC and EIR constraints, as must be done for mechanism C, can only reduce profit from V_{12}.

A.4.5. Claim 3. Suppose Conditions 1–3 and 4(i) and $v_1^* < v_2^* < v_{12}^*$ hold with $V_{12} = [v_{12}^*, \overline{v}]$. Given $t_1(v_1^*) = v_1^*$ and $t_2(v_2^*)$ according to (A.17), the payment scheme

$$t_1(v_{12}) = v_1^* \quad \forall v_{12} \in V_{12},$$
$$t_2(v_{12}) = t_2(v_2^*) + (v_{12}^* - v_1^*)\frac{\alpha_1}{\alpha_2} Z(v_{12}^*) \quad \forall v_{12} \in V_{12},$$

maximizes profits from V_{12}, subject to (i) the BIC constraint of v_{12}^* with respect to v_2^*; (ii) the BIC constraint of types $v \in [V_{12} - \{v_{12}^*\}]$ with respect to v_{12}^*; (iii) the EIR constraint of v_{12}^*; (iv) the BIC constraint of v_1^* with respect to v_{12}^*, that is, $t_1(v_{12}^*) \geq v_1^*$; and (v) the BIC constraint of v_2^* with respect to v_{12}^*, that is, $t_2(v_{12}^*) \geq t_2(v_2^*)$.

A.4.6. Proof of Claim 3. The BIC of types $v \in [V_{12} - \{v_{12}^*\}]$ with respect to v_{12}^* can be rewritten as

$$t_1(v)\alpha_1(-D_1'(v)) + t_2(v)\alpha_2(-D_2'(v)) \leq t_1(v_{12}^*)\alpha_1(-D_1'(v)) + t_2(v_{12}^*)\alpha_2(-D_2'(v)). \tag{A.19}$$

The term on the left side of (A.19) is the contribution of v toward the firm's profit in (3). Thus, (A.19) shows that, for any given V_{12}, the firm cannot increase profits by charging different payment schemes to different types in V_{12}. Further, if the payment and good-allocation scheme is the same for all types within V_{12}, the BIC constraints of types $v \in [V_{12} - \{v_{12}^*\}]$ with respect to v_{12}^* are satisfied. So the maximization problem detailed in Claim 3 can be stated as follows:

$$\max_{t_1(v_{12}^*), t_2(v_{12}^*)} \quad t_1(v_{12}^*)\alpha_1 \int_{v_{12}^*}^{\overline{v}} (-D_1'(v))dv + t_2(v_{12}^*)\alpha_2 \int_{v_{12}^*}^{\overline{v}} (-D_2'(v))dv. \tag{A.20}$$

This is subject to the BIC constraint of v_{12}^* with respect to v_2^*:

$$(v_{12}^* - t_1(v_{12}^*))\alpha_1(-D_1'(v_{12}^*)) + (v_{12}^* - t_2(v_{12}^*))\alpha_2(-D_2'(v_{12}^*)) \geq (v_{12}^* - t_2(v_2^*))\alpha_2(-D_2'(v_{12}^*)). \tag{A.21}$$

The EIR constraints of v_{12}^* and the BIC constraints of v_1^* and v_2^* with respect to v_{12}^*

$$t_1(v_{12}^*) \leq v_{12}^*; \quad t_2(v_{12}^*) \leq v_{12}^*; \quad t_1(v_{12}^*) \geq v_1^*; \quad t_2(v_{12}^*) \geq t_2(v_2^*). \tag{A.22}$$

Rearranging (A.21) yields

$$t_1(v_{12}^*)\alpha_1(-D_1'(v_{12}^*)) + t_2(v_{12}^*)\alpha_2(-D_2'(v_{12}^*)) \leq v_{12}^*\alpha_1(-D_1'(v_{12}^*)) + t_2(v_2^*)\alpha_2(-D_2'(v_{12}^*)). \tag{A.23}$$

At the optimum, (A.23) binds. Further, by Fact 2, we have $\frac{D_2(v_{12}^*)}{(-D_2'(v_{12}^*))} > \frac{D_1(v_{12}^*)}{(-D_1'(v_{12}^*))}$, or

$$\frac{\int_{v_{12}^*}^{\overline{v}} (-D_2'(v))dv}{(-D_2'(v_{12}^*))} > \frac{\int_{v_{12}^*}^{\overline{v}} (-D_1'(v))dv}{(-D_1'(v_{12}^*))}. \tag{A.24}$$

Because of the linearity of the maximand (A.20) and the constraint (A.23) in $t_1(v_{12}^*)$ and $t_2(v_{12}^*)$, it follows from (A.24) that the solution is to set $t_1(v_{12}^*)$ as low as possible and $t_2(v_{12}^*)$ as high as possible subject to (A.23), $t_1(v_{12}^*) \geq v_1^*$ and $t_2(v_{12}^*) \leq v_{12}^*$. We claim that $t_1(v_{12}) = v_1^*$ and $t_2(v_{12}) = t_2(v_2^*) + (v_{12}^* - v_1^*)\alpha_1/\alpha_2 Z(v_{12}^*)$ for all $v_{12} \in V_{12}$ is optimal (where $t_2(v_{12})$ is derived using $t_1(v_{12}^*) = v_1^*$ in (A.23)).

To verify this claim, we show that setting $t_2(v_{12}^*) = t_2(v_2^*) + (v_{12}^* - v_1^*)\frac{\alpha_1}{\alpha_2} Z(v_{12}^*)$ satisfies $t_2(v_{12}^*) \leq v_{12}^*$. Using (A.17), we can express $(v_{12}^* - t_2(v_{12}^*))$ as

$$(v_{12}^* - v_2^*) + (v_2^* - v_1^*)\frac{\alpha_1}{\alpha_2} Z(v_2^*) - (v_{12}^* - v_1^*)\frac{\alpha_1}{\alpha_2} Z(v_{12}^*). \tag{A.25}$$

Evaluated at $v_{12}^* = v_2^*$, expression (A.25) is zero, so we are done with this claim if we show that the expression is nondecreasing in v_{12}^*. Differentiating with respect to v_{12}^* yields

$$1 - (v_{12}^* - v_1^*)\frac{\alpha_1}{\alpha_2}Z'(v_{12}^*) - \frac{\alpha_1}{\alpha_2}Z(v_{12}^*)\ . \tag{A.26}$$

Because $Z'(v_{12}^*) < 0$ and $(1 - \alpha_1/\alpha_2 Z(v_{12}^*)) \geq 0$ (Condition 4(i)) hold, the expression (A.26) is nondecreasing. This completes the proof of Claim 3 and Lemma A.1. ■

Lemma A.2. *If Conditions 1–3 and 4(i) hold, then at the firm's optimal mechanism within the class of BIC and EIR mechanisms, it cannot be the case that the sets V_1, V_2, and V_{12} are all nonempty.*

A.4.7. Proof of Lemma A.2. By Lemma 2 and Claim 1a, under Conditions 1–3 and 4(i), if the firm's optimal BIC and EIR mechanism has V_1, V_2, and V_{12} all nonempty, then $v_1^* < v_2^* < v_{12}^*$ must hold, which means (by Lemma A.1) that Mechanism (A.15) with the same v_1^*, v_2^*, v_{12}^* yields weakly greater profit than the optimal BIC and EIR mechanism under Conditions 1–3 and 4(i). However, Mechanism (A.15) may not satisfy BIC. To prove Lemma A.2, we consider the profit from an arbitrary mechanism (A.15) with $v_1^* < v_2^* < v_{12}^*$ and show that this profit strictly increases by appropriately reducing the gap between v_2^* and v_{12}^*, thereby making $V_2 = [v_2^*, v_{12}^*)$ smaller. Ultimately, when V_2 is empty and $V_1 = [v_1^*, v_{12}^*)$, $V_{12} = [v_{12}^*, \overline{v}]$, the resulting (A.15) mechanism also satisfies all BIC and EIR conditions. This rules out the possibility that, under Conditions 1–3 and 4(i), the firm's optimal BIC and EIR mechanism has V_1, V_2, and V_{12} all nonempty.

The profit from the mechanism given in (A.15) is

$$\pi(v_1^*, v_2^*, v_{12}^*) = \alpha_1 v_1^*[D_1(v_1^*) - D_1(v_2^*) + D_1(v_{12}^*)] + \alpha_2\left[(v_{12}^* - v_1^*)\frac{\alpha_1}{\alpha_2}Z(v_{12}^*)\right]D_2(v_{12}^*) + \alpha_2 D_2(v_2^*)\left(v_2^* - (v_2^* - v_1^*)\frac{\alpha_1}{\alpha_2}Z(v_2^*)\right).$$

It is convenient to write the last term in this expression, that is, $\alpha_2 D_2(v_2^*)(v_2^* - (v_2^* - v_1^*)\frac{\alpha_1}{\alpha_2}Z(v_2^*))$, as

$$\alpha_2 v_2^* D_2(v_2^*)\left(1 - \frac{\alpha_1}{\alpha_2}Z(v_2^*)\right) + \alpha_1 v_1^* Z(v_2^*)D_2(v_2^*).$$

Thus, we have

$$\pi(v_1^*, v_2^*, v_{12}^*) = \alpha_1 v_1^*[D_1(v_1^*) - D_1(v_2^*) + D_1(v_{12}^*)] + \alpha_2\left[(v_{12}^* - v_1^*)\frac{\alpha_1}{\alpha_2}Z(v_{12}^*)\right]D_2(v_{12}^*) + \alpha_2 v_2^* D_2(v_2^*)\left(1 - \frac{\alpha_1}{\alpha_2}Z(v_2^*)\right) + \alpha_1 v_1^* Z(v_2^*)D_2(v_2^*). \tag{A.27}$$

Taking the derivative of the profit expression in (A.27) with respect to v_2^* yields

$$\frac{\partial\pi}{\partial v_2^*} = -\alpha_1 v_1^* D_1'(v_2^*) + \alpha_2\frac{\partial(v_2^* D_2(v_2^*))}{\partial v_2^*}\left(1 - \frac{\alpha_1}{\alpha_2}Z(v_2^*)\right) - \alpha_1(v_2^* - v_1^*)D_2(v_2^*)Z'(v_2^*) + \alpha_1 v_1^* Z(v_2^*)D_2'(v_2^*).$$

Now, consider the case in which v_2^* is strictly less than the monopoly price in state 2 (henceforth p_2^m), that is, $v_2^* < p_2^m$ holds. By simplifying the derivative, for the case in which $v_2^* < p_2^m$ holds, we can see that

$$\frac{\partial\pi}{\partial v_2^*} = \alpha_2\frac{\partial(v_2^* D_2(v_2^*))}{\partial v_2^*}\left(1 - \frac{\alpha_1}{\alpha_2}Z(v_2^*)\right) - \alpha_1(v_2^* - v_1^*)D_2(v_2^*)Z'(v_2^*) > 0$$

holds. The last inequality follows because $\partial(v_2^* D_2(v_2^*))/\partial v_2^* > 0$ holds because of Condition 3 and because we are considering the case in which $v_2^* < p_2^m$ holds; further, $\alpha_1/\alpha_2 Z(v_2^*) \leq 1$ holds by Condition 4(i), $Z'(v_2^*) < 0$ holds by Condition 2, and $v_2^* > v_1^*$ holds by Claim 1a. The implication of $\partial\pi/\partial v_2^* > 0$ is that, whenever $v_1^* < v_2^* < v_{12}^*$ and $v_2^* < p_2^m$ hold, the firm can strictly increase profits from Mechanism (A.15) by increasing v_2^* toward v_{12}^* so that $V_2 = [v_2^*, v_{12}^*)$ shrinks.

Next, consider the case in which $p_2^m \leq v_2^*$ holds. So, by definition of Mechanism (A.15), $p_2^m \leq v_2^* < v_{12}^*$ must hold. Consider the derivative of profit (A.27) with respect to v_{12}^*:

$$\begin{aligned}\frac{\partial\pi}{\partial v_{12}^*} &= \alpha_1 v_1^* D_1'(v_{12}^*) + \alpha_1\frac{\partial(v_{12}^* D_2(v_{12}^*))}{\partial v_{12}^*}Z(v_{12}^*) + \alpha_1(v_{12}^* - v_1^*)Z'(v_{12}^*)D_2(v_{12}^*) - \alpha_1 v_1^* Z(v_{12}^*)D_2'(v_{12}^*)\\ &= \alpha_1\frac{\partial(v_{12}^* D_2(v_{12}^*))}{\partial v_{12}^*}Z(v_{12}^*) + \alpha_1(v_{12}^* - v_1^*)Z'(v_{12}^*)D_2(v_{12}^*) < 0.\end{aligned}$$

The last inequality holds because $Z'(v_{12}^*) < 0$ and $\partial(v_{12}^* D_2(v_{12}^*))/\partial v_{12}^* < 0$ hold; the former holds by Condition 2, and the latter follows from concavity (Condition 3) and $v_{12}^* > p_2^m$. The implication of $\partial\pi/\partial v_{12}^* < 0$ is that, whenever $v_1^* < v_2^* < v_{12}^*$ and $v_2^* \geq p_2^m$ hold, the firm can strictly increase profits from Mechanism (A.15) by decreasing v_{12}^* toward v_2^* so that $V_2 = [v_2^*, v_{12}^*)$ becomes smaller.

To summarize, in either case, $v_2^* < p_m^2$ or $v_2^* \geq p_m^2$, the firm strictly increases profit from Mechanism (A.15) by either increasing v_2^* or decreasing v_{12}^*, thereby making the interval $V_2 = [v_2^*, v_{12}^*)$ smaller by reducing $(v_{12}^* - v_2^*)$. Therefore, for appropriately chosen values of v_1^* and v_{12}^*, Mechanism (A.15) with $(v_{12}^* - v_2^*) = 0$ and V_2 empty, that is, the mechanism given as follows

$$\begin{cases} V_1 = & [v_1^*, v_{12}^*) \\ V_{12} = & [v_{12}^*, \overline{v}] \\ V_2 = & \emptyset \\ t_1(v) = v_1^* & \forall v \geq v_1^* \\ t_2(v) = v_{12}^* & \forall v \geq v_{12}^* \end{cases} \tag{A.28}$$

yields higher profits than any mechanism (A.15) with $v_1^* < v_2^* < v_{12}^*$ and V_1, V_2, and V_{12} all nonempty.

To finish the proof, it is straightforward to verify that Mechanism (A.28) satisfies all BIC and EIR constraints. Therefore, under Conditions 1–3 and 4(i), it cannot be the case that, in the firm's optimal mechanism, V_1, V_2, and V_{12} are all nonempty. ■

Lemma A.3. *Consider the class of BIC and EIR mechanisms in which V_1, V_2, and V_{12} are all nonempty, and v_1^*, v_2^*, and v_{12}^* are given. If Conditions 1–3 and 4(ii) hold, then the profit of a mechanism in this class can be no greater than the profit that would result if all consumers report truthfully in the following mechanism:*

$$\begin{cases} V_2 = & [v_2^*, v_1^*) \\ V_1 = & [v_1^*, v_{12}^*) \\ V_{12} = & [v_{12}^*, \overline{v}] \\ t_2(v_2) = v_2^* & \forall v_2 \in V_2 \\ t_1(v_1) = v_1^* - (v_1^* - v_2^*)\frac{\alpha_2}{\alpha_1 Z(v_1^*)} & \forall v_1 \in V_1 \\ t_1(v_{12}) = v_1^* - (v_1^* - v_2^*)\frac{\alpha_2}{\alpha_1 Z(v_1^*)} & \forall v_{12} \in V_{12} \\ t_2(v_{12}) = v_{12}^* & \forall v_{12} \in V_{12}. \end{cases} \tag{A.29}$$

A.4.8. Proof of Lemma A.3. Suppose Conditions 1–3 and 4(ii) hold. Consider an arbitrary BIC and EIR mechanism with V_1, V_2, and V_{12} all nonempty and v_1^*, v_2^*, and v_{12}^* given; label this mechanism as mechanism D. We argue that the profit from mechanism D is weakly lower than the profit from the mechanism (A.29) in which v_1^*, v_2^*, and v_{12}^* are the same as in mechanism D. Note that mechanism D must satisfy BIC and EIR; however, Mechanism (A.29) is not required to satisfy either at this point in the argument.

Mechanism D must have the following features: By Claim 1b and Lemma 2(i) $v_2^* < v_1^* < v_{12}^*$ must hold. By Lemma 3, $t_2(v_2) = v_2^*$ must hold for all $v_2 \in V_2$. By Fact 1, $t_1(v_1) = t_1(v_1^*)$ must hold for all $v_1 \in V_1$. By BIC, v_1^* must be indifferent between reporting v_2^* and reporting v_1^* because, if instead type v_1^* strictly prefers reporting v_1^* over reporting v_2^*, then, by continuity, for a valuation v_2 less than v_1^* but close enough to v_1^*, we have that v_2 also strictly prefers reporting v_1^* rather than v_2^*, which contradicts either the BIC of v_2 or the definition of v_1^* as the infimum of V_1. Thus, we have

$$(v_1^* - t_1(v_1^*))\alpha_1(-D_1'(v_1^*)) = (v_1^* - v_2^*)\alpha_2(-D_2'(v_1^*)), \text{ or } t_1(v_1^*) = v_1^* - (v_1^* - v_2^*)\frac{\alpha_2}{\alpha_1 Z(v_1^*)}. \tag{A.30}$$

Therefore, mechanism D's payment scheme over V_1 and V_2 is determined by BIC. Note that Mechanism (A.29) has the same payment scheme over V_1 and V_2.

Now, we argue that setting V_2, V_1, and V_{12} as in Mechanism (A.29) indeed yields greater profit than from mechanism D. From (A.30), notice that $t_1(v_1^*) > v_2^*$ holds by Condition 4(ii). Because Condition 4(ii) and $t_1(v_1^*) > v_2^*$ hold, we have

$$\begin{aligned} &\frac{\alpha_1}{\alpha_2} Z(v) t_1(v_1^*) > v_2^* \ \ \forall v, \text{ or} \\ &\alpha_1 t_1(v_1^*)(-D_1'(v)) > \alpha_2 v_2^*(-D_2'(v)) \ \forall v. \end{aligned} \tag{A.31}$$

Inequality (A.31) implies that, for any type with valuation greater than v_1^*, the firm earns more profit from that type if it is in V_1 rather than in V_2. Further, by BIC (Lemma 2(i)), no valuation greater than v_{12}^* can be in V_1. Thus, the profit received from types below v_{12}^* is weakly higher in Mechanism (A.29) with $V_2 = [v_2^*, v_1^*)$, $V_1 = [v_1^*, v_{12}^*)$, and $V_{12} = [v_{12}^*, \overline{v}]$, than in mechanism D.

To show that Mechanism (A.29) yields higher overall profits than mechanism D, we must show that Mechanism (A.29) elicits weakly higher profit from types with valuation greater than v_{12}^*. To see this, first note that BIC implies $t_i(v_{12}) \geq t_i(v_i^*)$ for $i = 1, 2$ and all $v_{12} \in V_{12}$, which means assigning any type with valuation greater than v_{12}^* to V_2 rather than V_{12} only reduces profit. To complete the proof, Claim 4 demonstrates that the payment scheme of Mechanism (A.29) maximizes the firm's expected profit from V_{12} subject to a subset of BIC and EIR constraints. Thus, adding the missing BIC and EIR constraints, as must be done in mechanism D, can only reduce profit from V_{12} relative to Mechanism (A.29).

A.4.9. Claim 4. Suppose Conditions 1–3 and $v_2^* < v_1^* < v_{12}^*$ hold with $V_{12} = [v_{12}^*, \bar{v}]$. Given $t_2(v_2^*) = v_2^*$ and $t_1(v_1^*)$ according to (A.30), the payment scheme

$$t_1(v_{12}) = t_1(v_1^*) = v_1^* - (v_1^* - v_2^*)\frac{\alpha_2}{\alpha_1 Z(v_1^*)} \quad \forall v_{12} \in V_{12},$$
$$t_2(v_{12}) = v_{12}^* \quad \forall v_{12} \in V_{12},$$

maximizes profits from V_{12} subject to (i) the BIC constraint of v_{12}^* with respect to v_1^*; (ii) the BIC constraint of types $v \in [V_{12} - \{v_{12}^*\}]$ with respect to v_{12}^*; (iii) the EIR constraint of v_{12}^*; (iv) the BIC constraint of v_2^* with respect to v_{12}^*, that is, $t_2(v_{12}^*) \geq v_2^*$; and (v) the BIC constraint of v_1^* with respect to v_{12}^*, that is, $t_1(v_{12}^*) \geq t_1(v_1^*)$.

A.4.10. Proof of Claim 4. Repeating the arguments in Claim 3, (A.19) shows that, given V_1, V_2, and V_{12}, the firm cannot increase profits from V_{12} by charging different payment schemes to different types in V_{12}. Further, if the payment and good-allocation scheme is the same for all types within V_{12}, the BIC constraints of types $v \in [V_{12} - \{v_{12}^*\}]$ with respect to v_{12}^* are satisfied. So, the maximization problem detailed in Claim 4 can be stated as follows:

$$\max_{t_1(v_{12}^*), t_2(v_{12}^*)} \quad t_1(v_{12}^*)\alpha_1 \int_{v_{12}^*}^{\bar{v}} (-D_1'(v))dv + t_2(v_{12}^*)\alpha_2 \int_{v_{12}^*}^{\bar{v}} (-D_2'(v))dv. \tag{A.32}$$

This is subject to the BIC constraint of v_{12}^* with respect to v_1^*:

$$(v_{12}^* - t_1(v_{12}^*))\alpha_1(-D_1'(v_{12}^*)) + (v_{12}^* - t_2(v_{12}^*))\alpha_2(-D_2'(v_{12}^*)) \geq (v_{12}^* - t_1(v_1^*))\alpha_1(-D_1'(v_{12}^*)). \tag{A.33}$$

The EIR constraints of v_{12}^* and the BIC constraints of v_1^* and v_2^* with respect to v_{12}^* are

$$t_1(v_{12}^*) \leq v_{12}^*; \quad t_2(v_{12}^*) \leq v_{12}^*; \quad t_1(v_{12}^*) \geq t_1(v_1^*); \quad t_2(v_{12}^*) \geq v_2^*. \tag{A.34}$$

Rearranging (A.33) yields

$$t_1(v_{12}^*)\alpha_1(-D_1'(v_{12}^*)) + t_2(v_{12}^*)\alpha_2(-D_2'(v_{12}^*)) \leq t_1(v_1^*)\alpha_1(-D_1'(v_{12}^*)) + v_{12}^*\alpha_2(-D_2'(v_{12}^*)). \tag{A.35}$$

At the optimum, (A.35) binds. Fact 2 yields

$$\frac{\int_{v_{12}^*}^{\bar{v}} (-D_2'(v))dv}{(-D_2'(v_{12}^*))} > \frac{\int_{v_{12}^*}^{\bar{v}} (-D_1'(v))dv}{(-D_1'(v_{12}^*))}. \tag{A.36}$$

Because of the linearity of the maximand (A.32) and the constraint (A.35) in the choice variable $t_1(v_{12}^*)$ and $t_2(v_{12}^*)$, it follows from (A.36) that the solution is to set $t_1(v_{12}^*)$ as low as possible and $t_2(v_{12}^*)$ as high as possible subject to (A.35), $t_1(v_{12}^*) \geq t_1(v_1^*)$, and $t_2(v_{12}^*) \leq v_{12}^*$. Thus, $t_1(v_{12}) = t_1(v_1^*)$ and $t_2(v_{12}) = v_{12}^*$ for all $v_{12} \in V_{12}$ is optimal. It is straightforward to check that all constraints imposed in Claim 4 are satisfied.

Therefore, under Conditions 1–3 and 4(ii), mechanism (A.29) yields greater profit than any BIC and EIR mechanism with V_1, V_2, and V_{12} all nonempty (represented by the arbitrary mechanism D in this proof). ■

Lemma A.4. *If Conditions 1–3 and 4(ii) hold, then at the firm's optimal mechanism within the class of BIC and EIR mechanisms, it cannot be the case that the sets V_1, V_2, and V_{12} are all nonempty.*

A.4.11. Proof of Lemma A.4. Suppose Conditions 1–3 and 4(ii) hold and the firm's optimal BIC and EIR mechanism has V_1, V_2, and V_{12} all nonempty. By Lemma 2 and Claim 1b, BIC implies $v_2^* < v_1^* < v_{12}^*$. By Lemma A.3, the Mechanism (A.29) with the same v_2^*, v_1^*, v_{12}^* yields weakly greater profit than the optimal BIC and EIR mechanism with V_1, V_2, and V_{12} all nonempty. However, Mechanism (A.29) may not satisfy BIC. To prove Lemma A.4, we consider the profit from an arbitrary Mechanism (A.29) with $v_2^* < v_1^* < v_{12}^*$, and show that this profit strictly increases by appropriately reducing the gap between v_2^* and v_1^*, thereby making $V_2 = [v_2^*, v_1^*)$ smaller. Ultimately, when V_2 is empty and $V_1 = [v_1^*, v_{12}^*)$, $V_{12} = [v_{12}^*, \bar{v}]$, the resulting (A.29) mechanism also satisfies all BIC and EIR conditions. This rules out the possibility that, under Conditions 1–3 and 4(ii), the firm's optimal BIC and EIR mechanism has V_1, V_2, and V_{12} all nonempty.

The profit from Mechanism (A.29) is

$$\begin{aligned}\pi(v_2^*, v_1^*, v_{12}^*) &= \alpha_1 D_1(v_1^*)t_1(v_1^*) + \alpha_2[D_2(v_2^*) - D_2(v_1^*)]v_2^* + \alpha_2 D_2(v_{12}^*)v_{12}^* \\ &= \alpha_1 D_1(v_1^*)\left[v_1^* - (v_1^* - v_2^*)\frac{\alpha_2}{\alpha_1 Z(v_1^*)}\right] + \alpha_2[D_2(v_2^*) - D_2(v_1^*)]v_2^* + \alpha_2 D_2(v_{12}^*)v_{12}^*. \end{aligned}\tag{A.37}$$

Consider the derivative of the profit in (A.37) with respect to v_1^*. We have

$$\frac{\partial \pi(v_2^*, v_1^*, v_{12}^*)}{\partial v_1^*} = \alpha_1 D_1(v_1^*)\left[1 - \frac{\alpha_2}{\alpha_1}\left(\frac{Z(v_1^*) - (v_1^* - v_2^*)Z'(v_1^*)}{Z(v_1^*)^2}\right)\right] + \alpha_1 v_1^* D_1'(v_1^*) - \alpha_2 D_1'(v_1^*)\frac{(v_1^* - v_2^*)}{Z(v_1^*)} - \alpha_2 v_2^* D_2'(v_1^*). \tag{A.38}$$

Substituting $\frac{D_1'(v_1^*)}{Z(v_1^*)} = D_2'(v_1^*)$ into (A.38) and simplifying yields

$$\frac{\partial\pi(v_2^*, v_1^*, v_{12}^*)}{\partial v_1^*} = \alpha_1 D_1(v_1^*)\left[1 - \frac{\alpha_2}{\alpha_1}\left(\frac{Z(v_1^*) - (v_1^* - v_2^*)Z'(v_1^*)}{Z(v_1^*)^2}\right)\right] + \alpha_1 v_1^* D_1'(v_1^*) - \alpha_2 v_1^* D_2'(v_1^*).$$

Rearranging terms, we have

$$\frac{\partial\pi(v_2^*, v_1^*, v_{12}^*)}{\partial v_1^*} = \alpha_1[D_1(v_1^*) + D_1'(v_1^*)v_1^*] - \alpha_2\left[D_2'(v_1^*)v_1^* + \frac{D_1(v_1^*)}{Z(v_1^*)}\right] + \frac{\alpha_2 D_1(v_1^*)(v_1^* - v_2^*)Z'(v_1^*)}{Z(v_1^*)^2}. \tag{A.39}$$

Substituting $\frac{D_1'(v_1^*)}{Z(v_1^*)} = D_2'(v_1^*)$ into (A.39) and combining/rearranging terms, we have

$$\frac{\partial\pi(v_2^*, v_1^*, v_{12}^*)}{\partial v_1^*} = \alpha_1[D_1(v_1^*) + D_1'(v_1^*)v_1^*]\left[1 - \frac{\alpha_2}{\alpha_1 Z(v_1^*)}\right] + \frac{\alpha_2 D_1(v_1^*)(v_1^* - v_2^*)Z'(v_1^*)}{Z(v_1^*)^2}. \tag{A.40}$$

Because $v_1^* > v_2^*$ and $Z'(v_1^*) < 0$ hold, the last term in (A.40) is negative. By Condition 4(ii), $[1 - \alpha_2/\alpha_1 Z(v_1^*)] > 0$ holds. Thus, if $v_1^* \geq p_1^m$ holds, then the right side of (A.40) is negative, which implies that the firm can strictly increase profit from Mechanism (A.29) (given in (A.37)) by decreasing v_1^*, thereby reducing $(v_1^* - v_2^*)$ and shrinking V_2.

Next, suppose $v_1^* < p_1^m$ holds. Now, let us compute

$$\frac{\frac{\partial\pi(v_2^*, v_1^*, v_{12}^*)}{\partial v_2^*}}{\alpha_2} = \frac{D_1(v_1^*)}{Z(v_1^*)} - D_2(v_1^*) + D_2(v_2^*) + v_2^* D_2'(v_2^*). \tag{A.41}$$

When $\frac{\partial\pi(v_2^*, v_1^*, v_{12}^*)}{\partial v_2^*}$ is evaluated at $v_2^* = v_1^*$, the right side of (A.41) becomes

$$\frac{D_1(v_1^*)}{Z(v_1^*)} - D_2(v_1^*) + D_2(v_1^*) + v_1^* D_2'(v_1^*), \text{ or} \tag{A.42}$$

$$D_2'(v_1^*)\left[\frac{D_1(v_1^*)}{D_1'(v_1^*)} + v_1^*\right], \text{ or}$$

$$Z(v_1^*)[D_1(v_1^*) + D_1'(v_1^*)v_1^*]. \tag{A.43}$$

Note that (A.43) is strictly positive because $Z(v_1^*) > 0$ holds and because $[D_1(v_1^*) + D_1'(v_1^*)v_1^*]$, the marginal revenue in state 1, is strictly greater than zero because of Condition 3 and our supposition: $v_1^* < p_1^m$. From Condition 3, marginal revenue in state 2 is decreasing in v, so from (A.41), $\partial\pi(v_2^*, v_1^*, v_{12}^*)/\partial v_2^*$ is decreasing in v_2^*. Because we have shown that $\partial\pi(v_2^*, v_1^*, v_{12}^*)/\partial v_2^*$ is strictly positive when evaluated at $v_2^* = v_1^*$, it follows that $\partial\pi(v_2^*, v_1^*, v_{12}^*)/\partial v_2^*$ is strictly positive for all v_2^* strictly lower than v_1^*. In other words, when $v_1^* < p_1^m$ holds, the profit from Mechanism (A.29) (given in (A.37)) strictly increases as v_2^* is increased, and thereby $(v_1^* - v_2^*)$ is reduced and V_2 is shrunk.

To summarize, in either case, $v_1^* < p_m^1$ or $v_1^* \geq p_m^1$, the profit from Mechanism (A.29) can be strictly increased by either increasing v_2^* or decreasing v_1^*, thereby making the interval $V_2 = [v_2^*, v_1^*)$ smaller by reducing $(v_1^* - v_2^*)$. Therefore, for appropriately chosen values of v_1^* and v_{12}^*, the Mechanism (A.29) with $(v_{12}^* - v_2^*) = 0$ and V_2 empty, that is, the mechanism given as follows

$$\begin{cases} V_1 = & [v_1^*, v_{12}^*) \\ V_{12} = & [v_{12}^*, \bar{v}] \\ V_2 = & \emptyset \\ t_1(v) = v_1^* & \forall v \geq v_1^* \\ t_2(v) = v_{12}^* & \forall v \geq v_{12}^*, \end{cases} \tag{A.44}$$

yields higher profits than any Mechanism (A.29) with $v_2^* < v_1^* < v_{12}^*$ and V_1, V_2, and V_{12} all nonempty.

To finish the proof, it is straightforward to verify that Mechanism (A.44) satisfies all BIC and EIR constraints. Therefore, under Conditions 1–3 and 4(ii), it cannot be the case that, in the firm's optimal BIC and EIR mechanism, V_1, V_2, and V_{12} are all nonempty. ∎

Thus, Lemmas A.1–A.4 demonstrate that, given Conditions 1–4, BIC, and EIR, it is not optimal for the firm to choose a mechanism in which V_1, V_2, and V_{12} are all nonempty because even mechanisms that yield an upper bound over profit from BIC and EIR mechanisms with V_1, V_2, and V_{12} all nonempty can be improved upon by a mechanism in which V_2 is empty, and this mechanism also satisfies all BIC and EIR conditions. This concludes the proof of Lemma 4. ∎

A.5. Proof of the SBSMP Proposition

To prove the SBSMP proposition, we first show that the optimal mechanism cannot have only V_{12} nonempty or only V_2 and V_{12} nonempty. We finish by showing that the optimal mechanism with only V_1 and V_{12} nonempty is the SBSMP mechanism, which satisfies all BIC and EIR constraints.

A.5.1. Ruling out Mechanisms with Only V_{12} Nonempty. Suppose the firm chooses a mechanism such that only the set $V_{12} = [v^*_{12}, \bar{v}]$ is nonempty. The EIR constraint of v^*_{12} and the BIC constraint of valuations less than v^*_{12} imply $t_1(v^*_{12}) = t_2(v^*_{12}) = v^*_{12}$. From (3), it follows that the firm's profit from any valuation $v \in V_{12}$ is

$$\alpha_1 t_1(v)\big(-D_1'(v)\big) + \alpha_2 t_2(v)\big(-D_2'(v)\big).$$

But rearranging the BIC of v with respect to v^*_{12} yields

$$\alpha_1 t_1(v)\big(-D_1'(v)\big) + \alpha_2 t_2(v)\big(-D_2'(v)\big) \le \alpha_1 t_1\big(v^*_{12}\big)\big(-D_1'(v)\big) + \alpha_2 t_2\big(v^*_{12}\big)\big(-D_2'(v)\big).$$

So setting $t_1(v) = t_2(v) = v^*_{12}$ for all $v \in V_{12}$ achieves the maximum profit the firm can make from V_{12} under BIC and EIR. Thus, the profit for the case of only V_{12} nonempty is $\alpha_1 v^*_{12} D_1(v^*_{12}) + \alpha_2 v^*_{12} D_2(v^*_{12})$, which, for all $v^*_{12} \in V$, is strictly less than the profit from the SBSMP mechanism, $\alpha_1 p_1^m D_1(p_1^m) + \alpha_2 p_2^m D_2(p_2^m)$, because $p_1^m \neq p_2^m$ holds by Fact 2.

A.5.2. Ruling out Mechanisms with Only V_2 and V_{12} Nonempty.
By Lemma 2(ii), $v^*_2 < v^*_{12}$ must hold. By Lemma 3, $t_2(v_2) = v^*_2$ holds for all $v_2 \in V_2$. We first argue that mechanisms with only V_2 and V_{12} nonempty are suboptimal when Conditions 1–3 and 4(i) hold. So let Conditions 1–3 and 4(i) hold and consider the optimal BIC and EIR mechanism with only V_2 and V_{12} nonempty. Suppose the optimal v^*_2 and v^*_{12} in this mechanism are $v^*_2 = v^o_2$ and $v^*_{12} = v^o_{12}$. We first argue that the profit from such a mechanism has to be weakly lower than

$$\pi_u = \alpha_1 D_1\big(v^o_{12}\big)v^o_2 + \alpha_2 D_2\big(v^o_{12}\big)\left[v^o_2 + \big(v^o_{12} - v^o_2\big)\frac{\alpha_1}{\alpha_2} Z\big(v^o_{12}\big)\right] + \alpha_2\big[D_2\big(v^o_2\big) - D_2\big(v^o_{12}\big)\big]v^o_2.$$

To see why, note that (i) in π_u, as per Lemma 3, we have set $t_2(v_2) = t_2(v^o_2) = v^o_2$ for all $v_2 \in V_2$; (ii) by rearranging the BIC of valuations in V_{12} with respect to v^o_{12}, we obtain (A.19), from which it is clear that the firm cannot improve upon profits from V_{12} by charging different payment schemes to different valuations in V_{12}; (iii) by the arguments in the proof of Claim 3, because of Fact 2, the firm maximizes profits from $V_{12} = [v^o_{12}, \bar{v}]$ subject to a subset of BIC and EIR constraints by charging the highest price possible in state 2 and the lowest price possible in state 1; (iv) by EIR and BIC, $t_2(v^o_{12}) \le v^o_{12}$ and $t_1(v^o_{12}) \ge v^o_2$, respectively, must hold, and we have set $t_1(v^o_{12}) = v^o_2$, and consequently, the binding BIC of v^o_{12} with respect to v^o_2 yields

$$t_2\big(v^o_{12}\big) = v^o_2 + \big(v^o_{12} - v^o_2\big)\frac{\alpha_1}{\alpha_2} Z\big(v^o_{12}\big);$$

and (v) finally, we have set $V_{12} = [v^o_{12}, \bar{v}]$, in particular, we have not allowed any type with valuation greater than v^o_{12} to belong to V_2, which supports π_u being the upper bound because, by BIC, $t_1(v_{12}) \ge v^o_2$ and $t_2(v_{12}) \ge v^o_2$ hold for all v_{12} in V_{12}.

When Conditions 1–3 and 4(i) hold, the proof of Lemma A.2 shows that the profits from an arbitrary Mechanism (A.15) can be strictly increased by either increasing v^*_2 or decreasing v^*_{12}, thereby shrinking V_2 and making $(v^*_{12} - v^*_2)$ smaller. If we set $v^*_2 = v^o_2$, $v^*_{12} = v^o_{12}$, and $v^*_1 = v^o_2 - \epsilon$ in Mechanism (A.15), then for any $\delta > 0$, we can find $\epsilon > 0$ small enough such that the difference between π_u and the profit from the resulting Mechanism (A.15) is no more than δ. Notice that the gap between v^*_{12} and v^*_2 does not depend on δ.

The proof of Lemma A.2 shows that, relative to Mechanism (A.15), there is a profit advantage of shrinking the gap between v^*_{12} and v^*_2 to zero. For the $v^*_2 < p^m_2$ case, the profit advantage of increasing v^*_2 is

$$\frac{\partial \pi}{\partial v^*_2} = \alpha_2 \frac{\partial\big(v^*_2 D_2\big(v^*_2\big)\big)}{\partial v^*_2}\left(1 - \frac{\alpha_1}{\alpha_2} Z\big(v^*_2\big)\right) - \alpha_1\big(v^*_2 - v^*_1\big)D_2\big(v^*_2\big)Z'\big(v^*_2\big) > 0,$$

which is strictly positive even if $\epsilon = 0$. On the other hand, for the $v^*_2 \ge p^m_2$ case, the profit advantage of decreasing v^*_{12} is

$$\frac{\partial \pi}{\partial v^*_{12}} = \alpha_1 \frac{\partial\big(v^*_{12} D_2\big(v^*_{12}\big)\big)}{\partial v^*_{12}} Z\big(v^*_{12}\big) + \alpha_1\big(v^*_{12} - v^*_1\big)Z'\big(v^*_{12}\big)D_2\big(v^*_{12}\big) < 0.$$

That is, in both cases, the profit advantage is proportional to $v^*_{12} - v^*_2$, but this profit advantage is bounded above zero and does not depend on δ. Thus, the profit from Mechanism (A.29) with V_2 empty, $v^*_1 = v^o_2 - \epsilon$, and v^*_{12} appropriately chosen, is strictly greater than the profit π_u. Furthermore, it is straightforward to check that this mechanism satisfies all EIR and BIC conditions.

Next, Claim 5 deals with case in which Conditions 1–3 and 4(ii) hold.

A.5.3. Claim 5. Under Conditions 1–3 and 4(ii), the profit from the optimal BIC and EIR mechanism with only V_2 and V_{12} nonempty (and V_1 empty) is lower than the profit from the optimal BIC and EIR mechanism with only V_1 and V_{12} nonempty (and V_2 empty).

A.5.4. Proof of Claim 5. The mechanism with only V_1 and V_{12} nonempty that we consider is given by (A.44). It is straightforward to verify that Mechanism (A.44) satisfies all BIC and EIR conditions. To prove Claim 5, we show that, under Conditions 1–3 and 4(ii), for appropriately chosen v_1^* and v_{12}^*, Mechanism (A.44) also yields greater profit than the optimal BIC and EIR mechanism with only V_2 and V_{12} nonempty (and V_1 empty), which we denote by mechanism E. Mechanism E must have the following features. First, we must have $v_2^* < v_{12}^*$ (by Lemma 2(ii)) and $t_2(v_2) = v_2^*$ for all $v_2 \in V_2$ (by Lemma 3). Second, the profit from mechanism E is bounded above by the profit from an alternate mechanism, described as follows and denoted by mechanism F, that is only required to satisfy all the BIC and EIR constraints listed in Claim 3. Mechanism F is identical to mechanism E except (a) all types with valuation strictly greater than v_{12}^* that were in V_2 in mechanism E are instead allocated to V_{12}, and (b) these types are charged v_2^* in state 2 (as they were in mechanism E), and they are charged $t_1(v_{12}^*)$ in state 1. It is straightforward to verify that mechanism F satisfies all the constraints listed in Claim 3. Mechanism F has $V_{12} = [v_{12}^*, \overline{v}]$, and it yields greater profit than mechanism E because, if any type with valuation strictly greater than v_{12}^* belongs to V_2 in mechanism E, then such a type only yields v_2^* in state 2, and mechanism F assigns this type to V_{12}, elicits the same amount in state 2, and also elicits $t_1(v_{12}^*)$ from this type in state 1. Thus, the profit from mechanism E is less than the profit from mechanism F, given as follows

$$\begin{cases} V_2 = & [v_2^*, v_{12}^*) \\ V_{12} = & [v_{12}^*, \overline{v}] \\ V_1 = & \emptyset \\ t_2(v_2) = v_2^* & \forall v_2 \in V_2 \\ t_1(v_{12}), t_2(v_{12}) & \text{for } v_{12} \in V_{12}, \end{cases} \tag{A.45}$$

where v_2^*, v_{12}^* are identical to mechanism E, and the payment scheme over V_{12} is chosen as described earlier. To prove Claim 5, we show that, for appropriately chosen v_1^* and v_{12}^*, the mechanism given in (A.44) yields strictly greater profit than mechanism F, given in (A.45). The profit from mechanism F can be no more than

$$\pi_2 = \alpha_1 D_1(v_{12}^*)v_2^* + \alpha_2 D_2(v_{12}^*)v_{12}^* + \alpha_2[D_2(v_2^*) - D_2(v_{12}^*)]v_2^*. \tag{A.46}$$

To see why, note that (i) in π_2, by (A.45), $t_2(v_2) = v_2^*$ holds for all $v_2 \in V_2$; (ii) by the BIC of valuations in $V_{12} - \{v_{12}^*\}$ with respect to v_{12}^*, rearranged to (A.19), it is clear that the firm cannot improve upon profits from $V_{12} = [v_{12}^*, \overline{v}]$ by charging different payment schemes to different valuations in V_{12}; (iii) by the arguments in the proof of Claim 3, because of Fact 2, the firm maximizes profits from V_{12} subject to the constraints in Claim 3 (which are satisfied by mechanism F) by charging the highest price possible in state 2 and the lowest price possible in state 1; and (iv) finally, by the third and fifth constraints of Claim 3, $t_2(v_{12}^*) \le v_{12}^*$ (EIR of v_{12}^*) and $t_1(v_{12}^*) \ge v_2^*$ (BIC of v_2^* with respect to v_{12}^*), respectively, must hold, and we have set $t_2(v_{12}^*) = v_{12}^*$ and $t_1(v_{12}^*) = v_2^*$ to calculate π_2.

Now consider the mechanism (A.44) in which (a) we set v_1^* equal to v_2^* from mechanism F and (b) we set the value of v_{12}^* in Mechanism (A.44) equal to the value of v_{12}^* in mechanism F. The profit from Mechanism (A.44) with these values of v_1^* and v_{12}^* is given by

$$\pi_1 = \alpha_1 D_1(v_{12}^*)v_2^* + \alpha_2 D_2(v_{12}^*)v_{12}^* + \alpha_1[D_1(v_2^*) - D_1(v_{12}^*)]v_2^*. \tag{A.47}$$

Note that π_1 is strictly greater than π_2 because we have

$$\alpha_1[D_1(v_2^*) - D_1(v_{12}^*)] > \alpha_2[D_2(v_2^*) - D_2(v_{12}^*)], \text{ or}$$
$$\int_{v_2^*}^{v_{12}^*} \alpha_1(-D_1'(v))dv > \int_{v_2^*}^{v_{12}^*} \alpha_2(-D_2'(v))dv,$$

because, by Condition 4(ii), $\alpha_1(-D_1'(v)) > \alpha_2(-D_2'(v))$ or $\frac{\alpha_1}{\alpha_2}Z(v) > 1$ holds for all v. ■

A.5.5. Mechanism with Only V_1 and V_{12} Nonempty. Consider the firm's optimal mechanism in which only V_1 and V_{12} are nonempty. By Lemma 2, it follows that $v_1^* < v_{12}^*$, and $V_1 = [v_1^*, v_{12}^*)$, $V_{12} = [v_{12}^*, \overline{v}]$ hold. By Lemma 3, $t_1(v_1^*) = v_1^*$ must hold. The question is what payment scheme should be charged from V_{12}. This is answered in Claim P.

A.5.6. Claim P. Suppose only V_1 and V_{12} are nonempty with $V_1 = [v_1^*, v_{12}^*)$ and $V_{12} = [v_{12}^*, \overline{v}]$. Given Conditions 1–3 and $t_1(v_1^*) = v_1^*$, the payment scheme

$$t_1(v_{12}) = v_1^* \quad \forall v_{12} \in V_{12},$$
$$t_2(v_{12}) = v_{12}^* \quad \forall v_{12} \in V_{12},$$

maximizes profits from V_{12}, subject to all EIR and BIC constraints.

A.5.7. Proof of Claim P. Note that $t_1(v_1) = v_1^*$ for all $v_1 \in V_1$ is implied by Lemma 3. To see why the payment scheme in Claim P maximizes profits from V_{12}, note that Claim 4 also solves the same maximization problem except with a different value for $t_1(v_1^*)$ and with an additional constraint, the BIC of v_2^* with respect to v_{12}^*, which doesn't bind there. Thus, it suffices to replace $t_1(v_1^*) = v_1^*$ in Claim 4 and to verify that the mechanism in Claim P satisfies all BIC and EIR constraints, which is straightforward. ■

Claim P implies that the optimal profit in the case of only V_1 and V_{12} nonempty is

$$\pi = \alpha_1 D_1(v_1^*)v_1^* + \alpha_2 D_2(v_{12}^*)v_{12}^*,$$

where v_1^* and v_{12}^* should be chosen to maximize π. By Conditions 1 and 3, the first-order conditions with respect to v_1^* and v_{12}^* yield the unique profit-maximizing values: $v_1^* = p_1^m$ and $v_{12}^* = p_2^m$. Note that this mechanism satisfies all BIC and EIR constraints and yields the maximized profit equal to

$$\pi^* = \alpha_1 D_1(p_1^m)p_1^m + \alpha_2 D_2(p_2^m)p_2^m.$$

This completes the proof of the SBSMP proposition. ■

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