A Note on Competing Mechanisms and the Revelation Principle

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Abstract

When firms compete by choosing mechanisms, followed by consumers choosing firms, it is tempting to use the revelation principle to claim that we can restrict attention to mechanisms in which players report their initial types. This paper shows that this claim is false. In Example 1, consumers report their initial types only, and there is an equilibrium in which some player types lie, but there is no incentive-compatible direct T-revelation mechanism (where only initial types are reported) yielding the same equilibrium outcomes.

1. Introduction

The revelation principle states that, for any equilibrium of the Bayesian game defined by a particular mechanism, there is an incentive-compatible direct-revelation mechanism yielding the same equilibrium outcomes. Without loss of generality, we can ask agents to reveal their "type" and impose incentive-compatibility constraints. The revelation principle has been elaborated by Gibbard [1973], Myerson [1979, 1982], and others.[1] Of course, an agent's type can be quite complicated, as formalized by Mertens and Zamir [1985]. Besides describing intrinsic characteristics, the type must also describe an agent's information, including information about other agents' characteristics, information about other agents' information, etc.

When firms compete by choosing mechanisms, followed by consumers choosing firms, the type refers to the initial information about each others' characteristics, which we call the "initial type," and the new information consumers have about the mechanisms chosen by firms. It is tempting to claim that, without loss of generality, we can restrict attention to mechanisms in which players report their initial types. This paper shows that this claim is false. To correctly apply the revelation principle, a consumer must report the mechanisms chosen by other firms in addition to her initial information. In Example 1, consumers report their initial types only, and there is an equilibrium in which some player types lie, but there is no incentive-compatible direct T-revelation mechanism (where only initial types are reported) yielding the same equilibrium outcomes.

Unfortunately, if we are to recover the revelation principle by asking players to report their full types, some severe modelling problems arise. If the set of feasible mechanisms

[1] See Myerson [1989] for a survey on mechanism design.

includes all direct revelation mechanisms, then this set is not well defined unless the space of types is well defined. However, the space of types can only be defined if we know the set of feasible mechanisms, since these mechanisms are part of players' information.[2] For a practical example of the problem, each firm could try to offer a price slightly less than its rival's price. Although no attempt is made here to define a universal space of mechanisms, section 3 contains a discussion of how the model can be extended beyond the reporting of initial types to recover the revelation principle, while avoiding the infinite regress associated with fully reporting types.

2. The Model

The number of players or consumers is n. For $i = 1, \dots, n$, the private information of consumer i determines her *initial type*. Before mechanisms are announced, let T_i denote the set of possible initial types for consumer i, and let $T = T_1 \times \dots \times T_n$ denote the set of possible combinations of initial types. The initial type includes information about how much a consumer is willing to pay for a firm's product, how much other consumers know about one's own willingness to pay, and so on. Since firms will be announcing mechanisms and allowing consumers to choose which one to join, consumers will have additional information at the time they send messages to their firm.

The timing is as follows. First, consumers observe their own initial type, t_i. Second, each firm, $f = 1, \dots, F$, selects a mechanism (to be defined below). Each firm can randomize over which mechanism it selects. Third, consumers observe the mechanisms offered and commit to visit a single firm and abide by its mechanism. In choosing a firm and playing its mechanism, it

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[2] McAfee [1993] points out the infinite regress problem that arises when consumers are asked to report the mechanisms of other firms. See also Peters [1994].

is assumed that a consumer does not know who else has chosen the same firm. Let N_f denote the set of consumers joining firm f, let C denote the set of conceivable choices that can be made at a firm, and let $C_f(N_f) \subset C$ denote the set of all choices that can be made at firm f when the set of consumers is N_f. Consumer i's utility function is given by $u_i : C \times T \rightarrow \subset$. That is, consumer i's utility when joining firm f depends on the aggregate action taken at firm f and the initial types of all consumers, but not on the mechanisms of firms other than f.

Definition 2.1: A mechanism for firm f is a set of strategy spaces, $\{S_{i,f}\}|_{i=1}^{n}$ and a function, $\gamma_f : N \times S_{1,f} \times \ldots \times S_{n,f} \rightarrow C$, satisfying the feasibility condition $\gamma_f(N_f) \subset C_f(N_f)$ for all N_f.

The function γ_f in Definition (2.1) specifies a game for each subset of N. The notation $\gamma_f(N_f)$ represents the outcome function of the game played by the player set N_f. The set of strategies available to player i at firm f, $S_{i,f}$, is independent of the remaining players at firm f. All consumers not joining firm f's mechanism, i N_f, are assumed to choose a "null action" that is not available to players joining firm f. Let Γ_f denote the set of all feasible mechanisms for firm f, which is assumed to be well-defined.

At the time players choose strategies in the mechanism, player i's type includes her initial type, t_i, as well as her knowledge of all the mechanisms chosen by all firms. Therefore, substantial modelling difficulties must be overcome in order to allow Γ_f to include all direct revelation mechanisms, in which players report their types and each firm f selects an outcome in $C_f(N_f)$. The difficulty is that player i's possible types include a specification of firm f's mechanism, which in turn cannot be defined without knowing player i's possible types. No attempt is made here to construct a space of "universal mechanisms" along the lines of Mertens and Zamir [1985], so Γ_f may embody constraints on feasibility beyond that in Definition (2.1). See Epstein and Peters [1996] for a major step in that direction.

Definition 2.2: A *direct T-revelation mechanism* for firm f is a mechanism in which $S_{i,f} = T_i$ for $i = 1, \ldots , n$.

We will assume that all direct T-revelation mechanisms satisfying the minimal feasibility condition in Definition (2.1) are included in Γ_f .

Let $p_i(t_{-i} \mid t_i)$ denote the probability that consumer i of initial type t_i attatches to other consumers' types being t_{-i} . These beliefs are assumed to be independent of the mechanisms chosen by firms, $\gamma = (\gamma_1 , \ldots , \gamma_F)$. Let $\eta_i(t_i , \gamma)$ denote the F-tuple of probabilities with which consumer i joins the various firms, as a function of her initial type and the mechanisms chosen by the firms. Then $\eta(t,\gamma)$, defined to be $(\eta_1(t_1,\gamma) , \ldots , \eta_n(t_n,\gamma))$, determines the probability of each N_f , given t and γ and given that consumer i joins firm f, denoted by $p_i(N_f \mid t,\gamma)$.[3]

Given γ, an equilibrium is a specification of the probability with which each consumer joins each firm and the strategy chosen there, as a function of the consumer's initial type, such that no consumer of any type has an incentive to switch firms or switch strategies. More formally, let $\sigma_{f,i}(t_i ,\gamma)$ be the strategy chosen by consumer i joining firm f, given t_i and γ. Let $\sigma_f(t,\gamma)$ denote the vector of strategy functions of all consumers at firm f given t and γ, and let $\sigma_{f,-i}(t_{-i} ,\gamma)$ denote the vector of strategy functions of all consumers other than i at firm f.

[3] Note that, while $p_i(t_{-i} \mid t_i)$ is a primitive, $p_i(N_f \mid t,\gamma)$ is not, since it depends on the mechanisms selected and on η.

Definition 2.3: Given γ, an *equilibrium, $e(\gamma)$,* is a specification of $\eta_i(t_i, \gamma)$ and $\sigma_{f,i}(t_i, \gamma)$ for each player i, firm f, and vector of feasible mechanisms γ, satisfying: for each player i and for each firm f chosen with positive probability, we have

$$\sum_{t_{-i}} p_i(t_{-i}|t_i) \sum_{N_f} p_i(N_f|t,\gamma)\ u_i(\gamma_f(N_f,\sigma_f(t,\gamma),t) \geq \sum_{t_{-i}} p_i(t_{-i}|t_i) \sum_{N_h} p_i(N_h|t,\gamma)\ u_i(\gamma_h(N_h,\sigma_{h,-i}(t_{-i},\gamma),s_i),t) \quad \text{for all } h=1,\dots,F \text{ and all } s_i \in S_{i,h}.$$

Definition 2.4: Given the mechanisms of other firms, γ_{-f}, and an equilibrium $e(\mu_f, \gamma_{-f})$, a direct T-revelation mechanism, μ_f, is *incentive compatible* if for all i joining firm f with positive probability, and for all types t_i and τ_i, we have

$$\sum_{t_{-i}} p_i(t_{-i}|t_i) \sum_{N_f} p_i(N_f|t,\gamma)\ u_i(\mu_f(N_f,t),t) \geq \sum_{t_{-i}} p_i(t_{-i}|t_i) \sum_{N_f} p_i(N_f|t,\gamma)\ u_i(\mu_f(N_f,t_{-i},\tau_i),t).$$

Let $z = (z_1, \dots, z_F)$ denote the "mixed choice" of mechanisms chosen by firms, where z_f is a distribution over Γ_f. Below, a realization of mechanisms, γ_{-f}, is assumed to be a realization occuring with positive probability under z_{-f}.

Definition 2.5: Given $\gamma_f \in \Gamma_f$ and z_{-f}, the *revelation principle is satisfied relative to T for firm f* if, for each selection of equilibrium for each γ_{-f}, $e(\gamma_f, \gamma_{-f})$, there is an incentive compatible, direct T-revelation mechanism μ_f and selection of equilibrium for each γ_{-f}, $e(\mu_f, \gamma_{-f})$, such that (i) consumers continue to choose firms with the same probabilities, $\eta_i(t_i, \gamma) = \eta_i(t_i, \gamma_{-f}, \mu_f)$ for all γ_{-f} and (ii) the outcome remains the same for all players of all types (including players joining other firms) for all γ_{-f}.

The placement of the qualifier “for all γ_{-f}” in Definition (2.5) is important. Since firm f does not know the other firms’ mechanisms, a single μ_f should be used in place of γ_f. We now present the main theorem, which shows by example that the revelation principle may fail if consumers can only report their initial types. Example 1 shows that there may be an equilibrium where consumers sometimes lie to firm 1, but there is no incentive compatible direct T-revelation mechanism yielding the same equilibrium outcomes.

Theorem: *The revelation principle is not always satisfied relative to T. In particular, it is possible to have a mechanism in which some player sometimes lies about her type, but where there is no equivalent mechanism in which she always reports truthfully.*

Proof: Consider Example 1 below, of a direct T-revelation mechanism in which some consumers sometimes lie, but where there is no equivalent direct T-revelation mechanism which is incentive compatible. ■

Example 1: There are three consumers, n = 3, and for each consumer there are three possible initial types, $t_i \in \{a,b,c\}$. There are two firms, each with one unit of output. Feasible allocations at firm 1 involve at most one consumer receiving the output and paying a price of either \$1 or \$2. Feasible allocations at firm 2 involve painting the output the favorite color of either type a, b, c, or no one, and then giving the output to at most one consumer at a price of zero. Type a values firm 1's output at \$0, type b values firm 1's output at \$1.50, and type c values firm 1's output at \$x, where $5/2 < x < 6$. All types value firm 2's output at \$5 if it is painted their favorite color, and \$0 otherwise.

The distribution of types, (t_1, t_2, t_3), is (a,b,c) with probability 1/6, (a,c,b) with probability 1/6, (b,a,c) with probability 1/6, (b,c,a) with probability 1/6, (c,a,b) with probability 1/6, and (c,b,a) with probability 1/6. In other words, there is exactly one consumer of each type, and the identities of the types are random.

Firm 1's mechanism is as follows. Consumers report their type, and the outcome is the following function of reports:

Report (permutation of:)		Outcome
(a,0,0) or (b,0,0) or (c,0,0)	$\# N_1 = 1$	single consumer purchases at price of \$2
(a,b,0)	$\# N_1 = 2$	consumer reporting (b) purchases at price \$1
(a,c,0) or (b,c,0)	$\# N_1 = 2$	consumer reporting (c) purchases at price \$2
(a,a,0) or (b,b,0) or (c,c,0)	$\# N_1 = 2$	no one purchases
(a,b,c)	$\# N_1 = 3$	consumer reporting (c) purchases at price \$2
otherwise	$\# N_1 = 3$	no one purchases

Table 1: Firm 1's mechanism

Firm 2 employs a mixed strategy over which mechanism to select. With probability 1/6, firm 2 chooses mechanism A: the output is painted the favorite color of type a and given to someone announcing type a. If no one announces type a, then no one receives the output, and if several people announce type a, then the selection is made randomly. With probability 1/6, firm 2 chooses mechanism B, identical to mechanism A except that "b" replaces "a". With probability 1/3, firm 2 chooses mechanism C, identical to mechanism A except that "c" replaces "a". With probability 1/3, firm 2 chooses mechanism D, in which the output is destroyed.

It is easy to check that the following constitutes an equilibrium:

- When firm 2 selects mechanism A, the type a consumer chooses firm 2 and truthfully announces her type. Consumers of types b and c choose firm 1 and truthfully announce their types. The type c consumer purchases the output at a price of \$2.
- When firm 2 selects mechanism B, the type b consumer chooses firm 2 and truthfully announces her type. The type a consumer visits firm 1 and truthfully announces her type. The type c consumer announces type b (a lie) in order to pay \$1 for the output.
- When firm 2 selects mechanism C, the type c consumer chooses firm 2 and truthfully announces her type. Consumers of types a and b choose firm 1 and truthfully announce their types. The type b consumer purchases the output at a price of \$1.
- When firm 2 selects mechanism D, consumers of types a and b choose firm 1 with probability ½ and firm 2 with probability ½ . The type c consumer chooses firm 1 and all consumers truthfully report their types. The type c consumer purchases the output for \$2. Since $x > 5/2$, type c prefers to be truthful.

Now suppose that there is an equivalent direct T-revelation mechanism for firm 1, μ_1 , that is incentive compatible. Without loss of generality, suppose that consumer $i = 3$ is the type c consumer. Consider the situation when $N_f = \{1,3\}$ and consumer 1 is of type a. When firm 2 selects mechanism D, consumer 3 must purchase the output for \$2. However, when firm 2 selects mechanism B, consumer 3 must purchase the output for \$1. Assuming consumers 1 and 3 always tell the truth, it is impossible for firm 1 to distinguish between firm 2 selecting mechanism D and mechanism B, so the outcome under μ_1 must be the same in both cases, a contradiction.

Remark 2.6: Example 1 clearly relies on firm 2 randomizing over which mechanism it selects. Indeed, if all firms are deterministically selecting mechanisms, then for any given equilibrium there is a direct revelation mechanism, one for each firm, which yields an equilibrium in which agents' arrival probabilities are unaffected, outcomes are unaffected, and all mechanisms are incentive compatible. Consumers do not receive any new information from the announcement of mechanisms, and the standard revelation principle argument applies. However, notice that it may be impossible for one firm to unilaterally switch to an equivalent direct revelation mechanism. The problem is that, if firm 1 switches to a direct revelation mechanism, outcomes are affected if firm 2's mechanism asks whether or not firm 1 is using a direct revelation mechanism.[4]

Remark 2.7: The example illustrates how players can provide information to firms by their mere presence or absence, in addition to any messages they may send. Here, if the type a player (who is indifferent) can be induced to visit firm 1 when firm 2's mechanism is B and visit firm 2 when firm 2's mechanism is C, then firm 1 could extract the maximum possible surplus while only asking players their valuation, by charging a price of $1 only when one player arrives.

3. Discussion

How, then, can the message space be extended beyond the reporting of initial types to recover the revelation principle while avoiding the infinite regress that may be associated with the full reporting of types? One approach is simply to have *all firms* ask their players to report

[4] I am grateful to Lars Stole for helping me clarify this point. Martimort and Stole [1997] reach similar conclusions with respect to the revelation principle in the context of common agency games.

the strategy they would have chosen in the original mechanism, γ_f. For example, instead of having an auction, ask the players to report (their initial types and) the bids they would have made had the mechanism been an auction. This may seem like a meaningless distinction, but notice that the distinction could matter if some firms ask players to report details of other firms' mechanisms. For example, suppose that firm 2's mechanism gives one outcome if players report that firm 1 has players bid, and firm 2's mechanism gives another outcome if players report that firm 1 has players report the bids they would have made. Then firm 1's movement from the auction γ_f to the revelation mechanism μ_f affects the outcome at firm 2, thereby indirectly affecting its own outcome if players change their choices of which firm to visit. It is clear, though, that if all firms ask players to report the strategy they would have played in the original mechanism, and if firms then choose the corresponding outcome that would have obtained, there is no incentive for any player to switch firms or to falsely report the strategy they would have chosen.

Instead of reporting the strategies consumers would have played in γ_f, it is interesting to consider the alternative of reporting the probabilities of each N_f, $p_i(N_f \mid t, \gamma_{-f}, \mu_f)$. Rather than reporting a strategy corresponding to a specific game, consumers would report probability distributions. While such a specification allows us to prove the revelation principle when γ_f has a unique equilibrium, an overall equilibrium may involve seemingly irrelevant details about other firms' mechanisms to select among multiple equilibria to γ_f.[5] Messages may then be insufficient to allow firm f to condition the outcome on the selected equilibrium of γ_f.

[5] This is reminiscent of the sunspots literature. See Shell [1987] for references.

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