1) Two spin-1/2 particles are governed by \( H = \frac{J^2}{2I} + \omega_0 J_z \), where \( J = J_1 + J_2 \).
At time \( t = 0 \) the total angular momentum is measured and found to be 0.
(a) At time \( t \), \( J_{1z} \) is measured. What are the possible results and what is the probability of each? (b) At time \( 2t \) the total angular momentum is measured, \( J^2 \). What are the possible results (follow all possible branches from the measurement at time \( t \)) and what is the probability for each result?

2) \( H_0 \) is the Coulomb Hamiltonian for hydrogen. \( H' = -eE_0 z \). Compute the first-order energy shift for the \( n = 1 \) state and for a complete set of \( n = 2 \) states.

3) \( H_0 \) is the 2-dimensional well, with \( 0 < x < a \) and \( 0 < y < a \). \( H' = \alpha \delta(x - a/2) \delta(y - a/2) \). (a) Compute \( E^{(1)}_{1mn} \). (b) Demonstrate whether \( E^{(2)} \) is finite or not.

4) A particle with spin 1 (\( s_1 = 1 \)) interacts with a particle with spin 1/2 (\( s_2 = 1/2 \)). The Hamiltonian is:
\[
H = \lambda \mathbf{S}_1 \cdot \mathbf{S}_2
\]
(a) Give a complete set of eigenstates (show all quantum numbers and say what each is in words; e.g., \( s \) is the total spin) and the energy eigenvalue for each state.
(b) At time \( t = 0 \) the z-component of the first particle’s spin is measured to be 0 and the z-component of the second particle’s spin is measured to be 1/2. At a later time \( t_1 \), the z-component of the first particle’s spin is measured again. What are the possible results and their respective probabilities?

6) A particle is confined to an infinite one-dimensional square well so that the wave function is non-zero only for \( 0 < x < a \). A perturbing potential, \( V(x) = -\alpha \delta(x - a/2) \), is added. (a) Compute the first-order shift in all energies. (b) Compute the second-order shift in the ground state energy, leaving your answer as a sum but clearly stating or showing which terms are zero (if any). Is this second-order shift finite or infinite?

7) Two non-identical spin-1/2 particles (\( s_1 = 1/2, s_2 = 1/2 \)) are fixed in space, so that we only need their spin state. To leading order:
\[ H_0 = \omega S_1 \cdot S_2, \]

where \( \omega > 0 \). (a) What are the eigenstates and eigenvalues? (b) A perturbing potential, \( V = \lambda S_{1z} \), is added. Note that only the first particle is involved in \( V \). Compute the first-order perturbative shifts of the energies for each eigenstate. (c) Compute the second-order shift in the energy of the ground state.

8) Four electrons are confined to a sphere of radius \( a \). In addition to the confining interaction the interaction \( \delta H = b \mathbf{L} \cdot \mathbf{S} \) is added. (a) If \( b > 0 \) what is the ground state? Give the shell configuration and a complete set of allowed quantum numbers of your choice to define the ground state. (b) Same as (a) for \( b < 0 \).

9) Two spin-1/2 particles are governed by \( H = \frac{J^2}{2I} + \omega_0 J_z \), where \( J = J_1 + J_2 \). At time \( t = 0 \) the total angular momentum is measured and found to be 0. (a) At time \( t \), \( J_{1z} \) is measured. What are the possible results and what is the probability of each? (b) At time \( 2t \) the total angular momentum is measured, \( J^2 \). What are the possible results (follow all possible branches from the measurement at time \( t \)) and what is the probability for each result?

10) Use degenerate first-order perturbation theory with \( H_0 \) being the Coulomb Hamiltonian for hydrogen. The perturbation is \( H' = -eE_0 z \). Compute the first-order energy shifts for the \( n = 1 \) state and for a complete set of \( n = 2 \) states. There are five results.

11) A spin-1/2 particle has \( H_0 = \omega_0 S_z, H' = \omega_1 S_x \). Compute the first order shifts in the energies of the spin-up and of the spin-down states using first-order perturbation theory. Compute the exact eigenvalues and compare with your perturbative result.

12) [extra credit] \( H_0 \) is the 2-dimensional well, with \( 0 < x < a \) and \( 0 < y < a \). \( H' = \alpha \delta(x - a/2) \delta(y - a/2) \). (a) Compute \( E_{mn}^{(1)} \). (b) Demonstrate whether \( E^{(2)} \) is finite or not.