Dimensional Analysis Example

Here is a procedure for doing systematic dimensional analysis (on the left) with an example (on the right). In the example we are looking for the dependence on environmental variables of the speed of sound v in air (or any gas).

Procedure	Example
1. Determine the relevant quantities from *physics* considerations. This may mean identifying the equation(s) that determine how the system behaves (e.g., $F = ma$).	The wave speed v should depend on the ambient density ρ_0 (cf. mass of particle) and the ambient pressure p_0 (cf. restoring force on particle).
2. Determine the fundamental units ($[M]$, $[L]$, $[T]$) of each quantity. You can use an equation that contains the quantity (e.g., $F = ma$) if you know the units of everything else in the equation.	$v \sim [L][T]^{-1}$ $\rho_0 \sim \text{mass/volume} \sim [M][L]^{-3}$ $p_0 \sim \text{force/area} \sim ([M][L][T]^{-2})/[L]^2 \sim [M][L]^{-1}[T]^{-2}$
3. Postulate an equation relating the quantities, with unknown exponents (a, b, c, ...), which may be fractions.	$v = C\,(\rho_0)^a\,(p_0)^b$, with C a dimensionless constant.
4. Substitute units from 2 and combine exponents.	$[L][T]^{-1} \sim [M]^a[L]^{-3a}[M]^b[L]^{-b}[T]^{-2b}$ $\implies [L][T]^{-1} \sim [M]^{a+b}[L]^{-3a-b}[T]^{-2b}$
5. Equate exponents of $[M]$, $[L]$, $[T]$ on left and right sides of 4 and solve the resulting equations simultaneously.	$[M]$: $0 = a + b$ $[L]$: $1 = -3a - b$ $[T]$: $-1 = -2b$ $\implies b = 1/2 \quad \implies a = -1/2$ $\implies$ answer: $v = C\sqrt{p_0/\rho_0}$
6. *Always* check your results by plugging back into the equations.	$0 = -1/2 + 1/2$ ✓ $1 = -3 \times (-1/2) - 1/2 = 3/2 - 1/2$ ✓ $-1 = -2 \times 1/2$ ✓
If you have identified the most relevant quantities, the (undetermined!) dimensionless coefficient will typically be of order unity (e.g., between 1/3 and 3).	$C \approx 1.2$ for air!