May the Strong Force be with you
The simplest contributions to the force between nucleons, as viewed from QCD. Here, the exchange of two colored gluons causes two quarks in each nucleon to change their colors (blue changes to green and vice versa in the case illustrated). This process produces a force without violating the overall color neutrality of the nucleons. The strength of the force depends on the separation of the different quark colors within each nucleon. On the other hand, low-energy nuclear physics measurements show clearly that the longest-range part of the force arises from the exchange of a single pi meson between two nucleons, as in In this low-energy view, the internal structure of each nucleon is generally attributed to three pseudo-quarks, which somehow combine the properties of the valence quarks, sea quarks, and gluons predicted by QCD.
The interaction between two nucleons is effected by the exchange of a particle. However, because the nucleon interactions appear to be short-ranged, the particle must have a finite mass. In fact, one can correlate the range and mass roughly by the quantum uncertainty principle, $r \sim \frac{1}{m}$, therefore, the mass of the quanta exchanged is about $\frac{1}{\text{fm}}$ which is about 200 MeV.
A realistic nuclear force force: schematic view

- Nucleon r.m.s. radius \( \sim 0.86 \text{ fm} \)
- Comparable with interaction range
- Half-density overlap at max. attraction
- \( V_{NN} \) not fundamental (more like intermolecular van der Waals interaction)
- Since nucleons are composite objects, three- and higher-body forces are expected.

\[
v_\pi(r) = \frac{f_{\pi NN}^2}{4\pi} \frac{m_\pi}{3} \tau_1 \cdot \tau_2 \left[ T_\pi(r) S_{12} + [Y_\pi(r) - \frac{4\pi}{m_\pi^3} \delta(r)] \sigma_1 \cdot \sigma_2 \right],
\]

where \( Y_\pi(r) \) and \( T_\pi(r) \) are dimensionless functions of \( m_\pi r \) defined as

\[
Y_\pi(r) = e^{-m_\pi r} \quad \text{Yukawa force}
\]
\[
T_\pi(r) = \left( 1 + \frac{3}{m_\pi r} + \frac{3}{m_\pi^2 r^2} \right) Y_\pi(r),
\]

and \( S_{12} \) is the tensor operator,

\[
S_{12} = 3 \sigma_1 \cdot \hat{r} \sigma_2 \cdot \hat{r} - \sigma_1 \cdot \sigma_2.
\]
There are infinitely many equivalent nuclear potentials!

\[ \hat{H}\Psi = E\Psi \]

\[ (\hat{U}\hat{H}\hat{U}^{-1})\hat{U}\Psi = E\hat{U}\Psi \]

Reid93 is from V.G.J. Stoks et al., PRC 49, 2950 (1994).

Three-body forces between protons and neutrons are analogous to tidal forces: the gravitational force on the Earth is not just the sum of Earth-Moon and Earth-Sun forces (if one employs point masses for Earth, Moon, Sun)
Resolution and Effective Field Theory

multipole expansion of electrostatic potential

\[ V(\vec{r}) = \frac{1}{4\pi \varepsilon_0} \sum_{n=0}^{\infty} \frac{1}{r^{n+1}} \int (r')^n P_n(\cos \theta') \rho(\vec{r}') d^3r' \]

\[ V(\vec{r}) = \frac{1}{4\pi \varepsilon_0} \left( \frac{Q}{r} + \frac{\vec{p} \cdot \vec{r}}{r^3} + \frac{1}{2} \sum_{i,j} Q_{ij} \frac{x_i x_j}{r^5} \cdots \right) \]

\[ \vec{p} = \int \vec{r}' \rho(\vec{r}') d^3r' = \sum_{k=1}^{N} q_k \vec{r}'_k \]

\[ Q_{ij} = \int (3x'_i x'_j - r'^2 \delta_{ij}) \rho(\vec{r}') d^3r' \]
Digital Resolution: Higher Resolution is Better (?)

- Computer screens, printers, digital cameras, TV’s …
- Higher resolution ➞ more pixels
- Pixel size ≪ characteristic scale ➞ greater detail
- Greater resolution ➞ more $$$
Consequences

- If system is probed at low energies, fine details not resolved
- Use low-energy variables for low-energy processes
- Short-distance structure can be replaced by something simpler without distorting low-energy observables
- Physics interpretation can change with resolution!

\[ \theta_{\text{min}} = \sin^{-1}(\lambda/a) \]

so if \( \lambda \geq a \), you don’t learn anything about the details of the slit

- \( \lambda \approx 10^{-10} \text{ m} \implies \) probe atoms
- \( \lambda \approx 10^{-14} \text{ m} \implies \) probe nucleus
- \( \lambda \approx 10^{-18} \text{ m} \implies \) probe quarks

\[ \text{de Broglie relation: } \lambda = \frac{h}{p} \]
The Hierarchy of Nuclear Forces

2N forces

Leading Order

\[ Q^0 \] (LO)

Next-to-Leading Order

\[ Q^2 \] (NLO)

Next-to-Next-to-Leading Order

\[ Q^3 \] (N^2LO)

Next-to-Next-to-Next-to-Leading Order

\[ Q^4 \] (N^3LO)

3N forces

4N forces

The Hierarchy of Nuclear Forces