Figure 2.14 from page 63 of *Exploring the Heart of Matter*

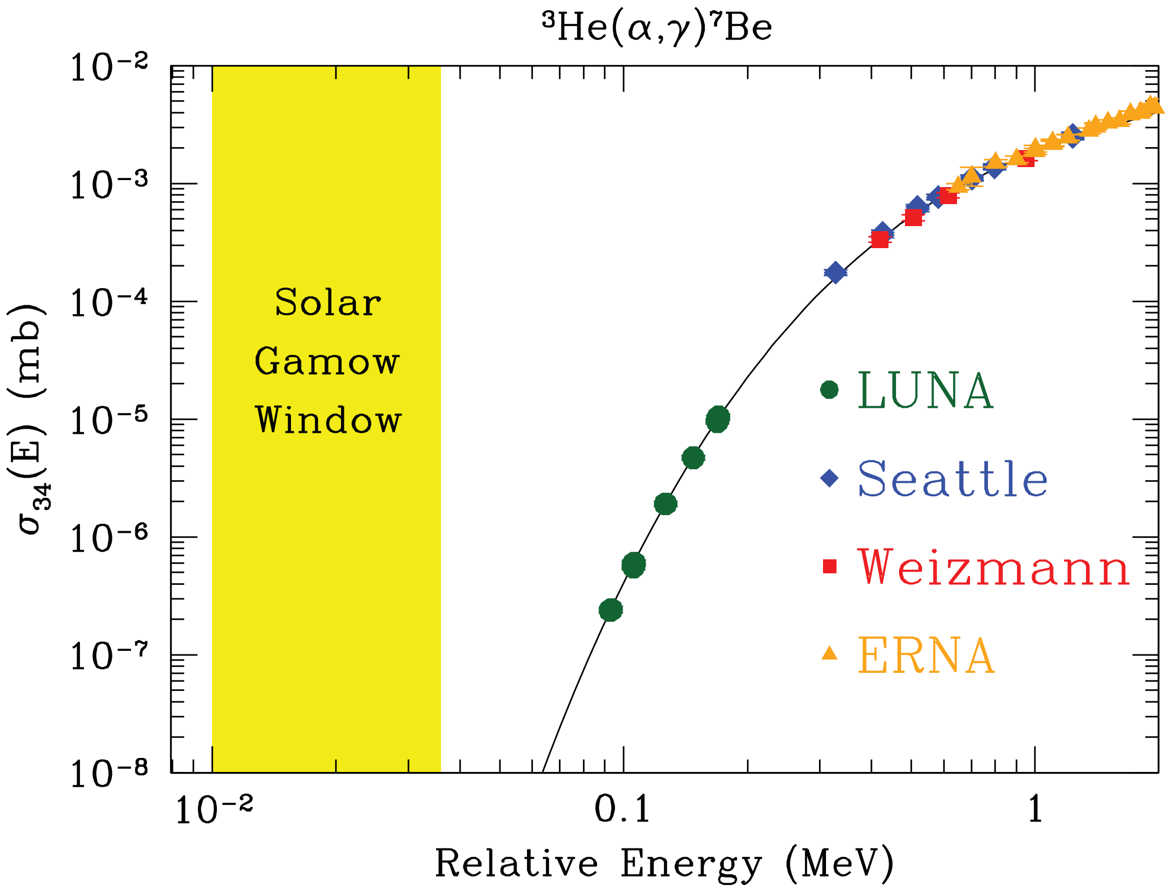
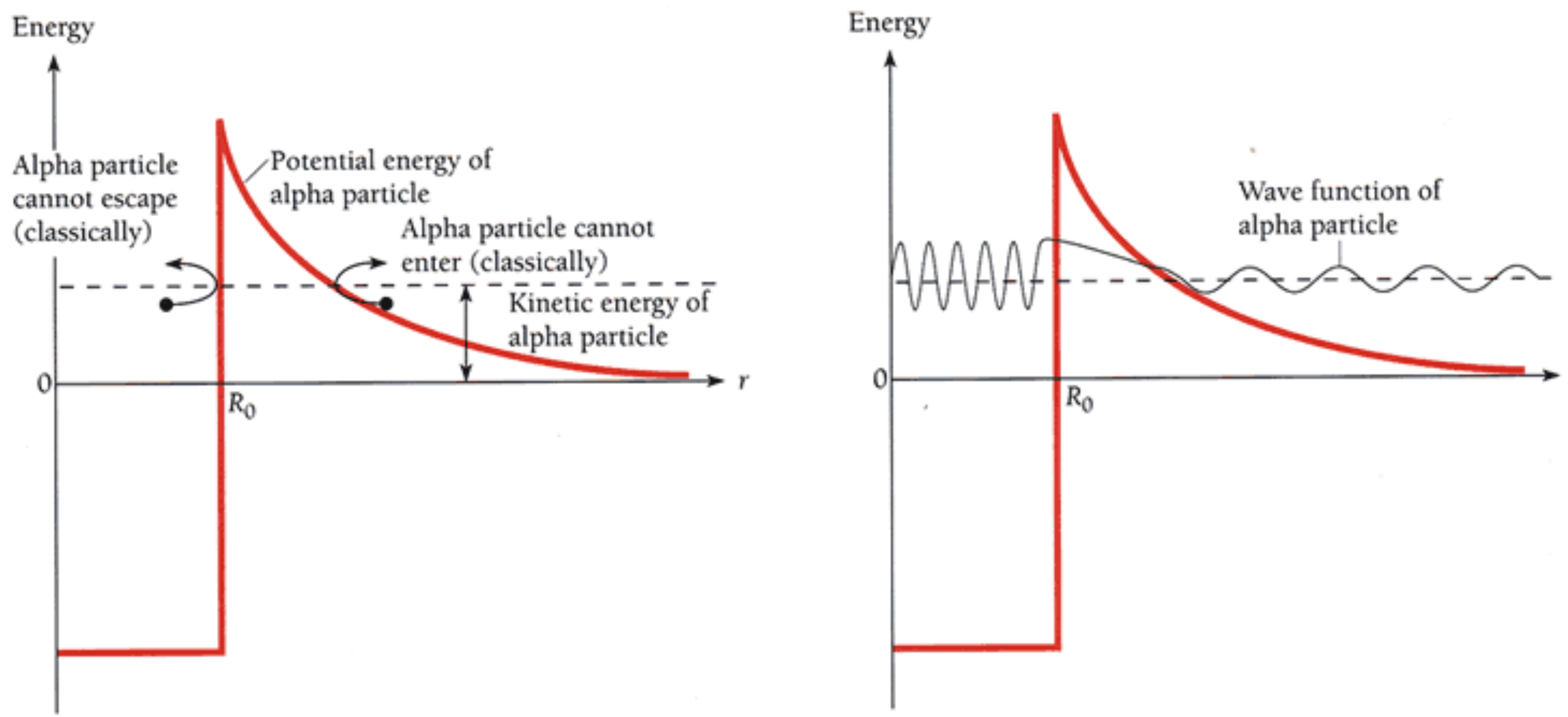
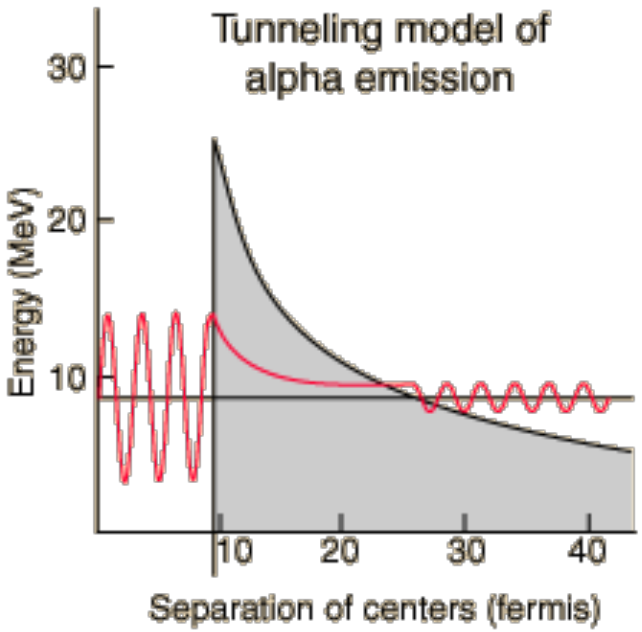


FIGURE 2.14 The fusion probability of helium-3 (3He) and helium-4 (4He), an important reaction in stars affecting neutrino production in the sun, measured directly at various laboratories. The challenge is to measure the extremely small fusion rates at the low relative energies that the particles have inside stars. The reduced background in underground accelerator laboratories (LUNA data shown above in green) compared to aboveground laboratories (all other data) enables the measurement of fusion rates that are smaller by a factor of approximately 1,000. This reduces the error when extrapolating the fusion rate to the still lower stellar energies. SOURCE: Courtesy of Richard Cyburt, Michigan State University.


Energy
Alpha particle
cannot escape
(classically)
Potential energy of
alpha particle
Alpha particle cannot
enter (classically)
Kinetic energy of
alpha particle
0
R_0
r
Energy
Wave function of
alpha particle
0
R_0
r

$$P = \frac{|\chi_{III}|^2}{|\chi_I|^2} \propto \exp\left[-2\int_{r_1}^{r_2} k(r)\,dr\right] \qquad T \propto \frac{1}{P}$$



In the case of the Coulomb barrier, the above integral can be evaluated exactly.

$$\log T = a + \frac{b}{\sqrt{Q_\alpha}}$$

Geiger-Nuttall law of alpha decay 1911

For the Coulomb barrier above, derive the Geiger-Nuttal law. Assume that the energy of an alpha particle is $E=Q_\alpha$, and that the outer turning point is much greater than the potential radius.

Reaction Rate Definition

For a given relative velocity *v* with projectile number density n_p

$$\lambda = \sigma \cdot n_p \cdot v \left[s^{-1}\right]$$

reaction/target particle

energy/temperature dependent decay constant λ

$$R = \sigma \cdot n_p \cdot v \cdot n_T \cdot V \left[s^{-1}\right]$$

reaction rate in volume *V*

Reaction Rate in Stellar Environment

reaction rate per second and cm^3:

$$r = n_p \cdot n_T \cdot \sigma \cdot v$$

Reaction rate for particles with velocity distribution $\Phi(v)$

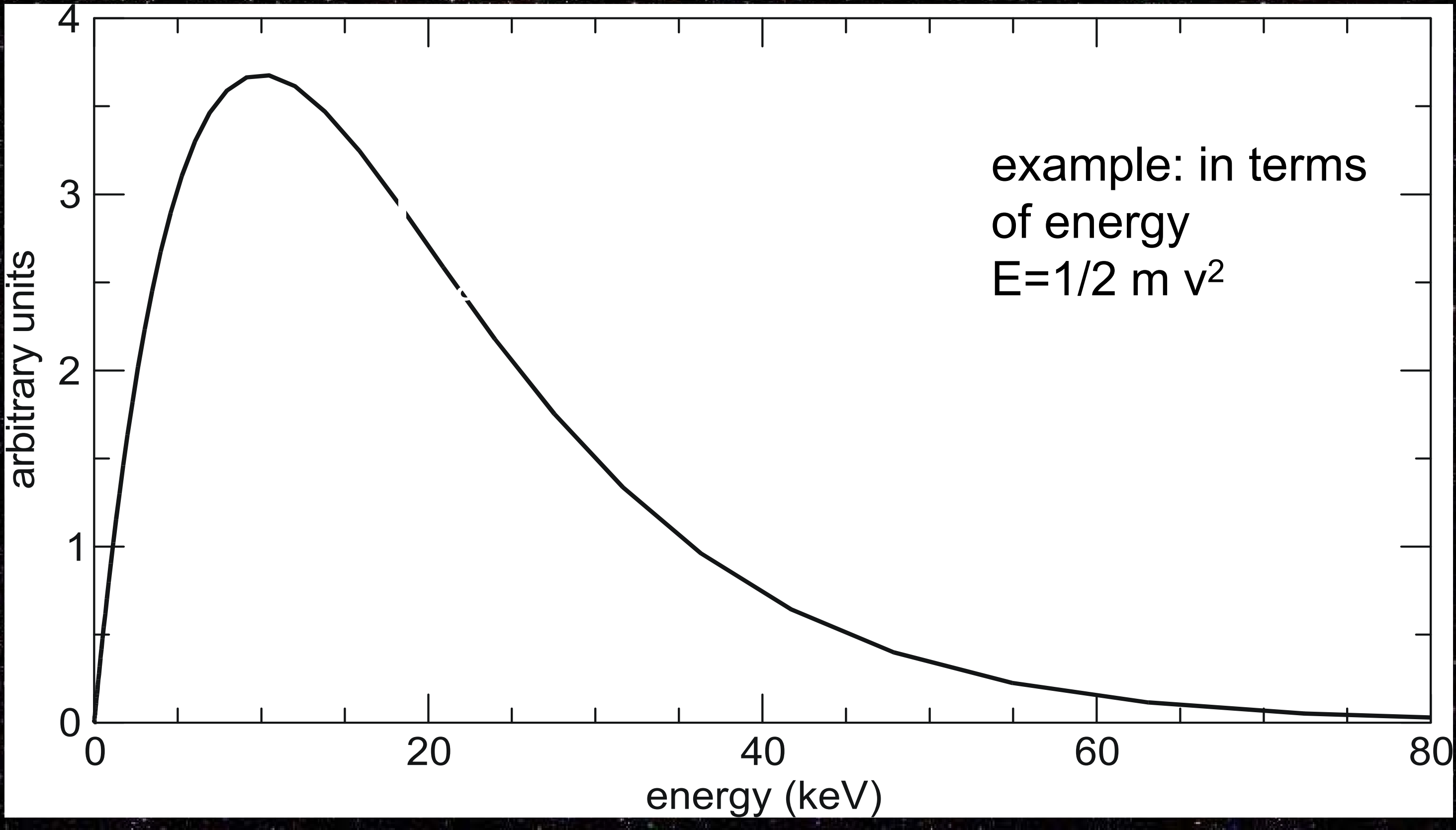
$$r = \frac{1}{1+\delta_{pT}} n_p \cdot n_T \cdot \int \sigma \cdot v \cdot \Phi(v) \cdot dv$$

Accounting for reactions
Between identical particles

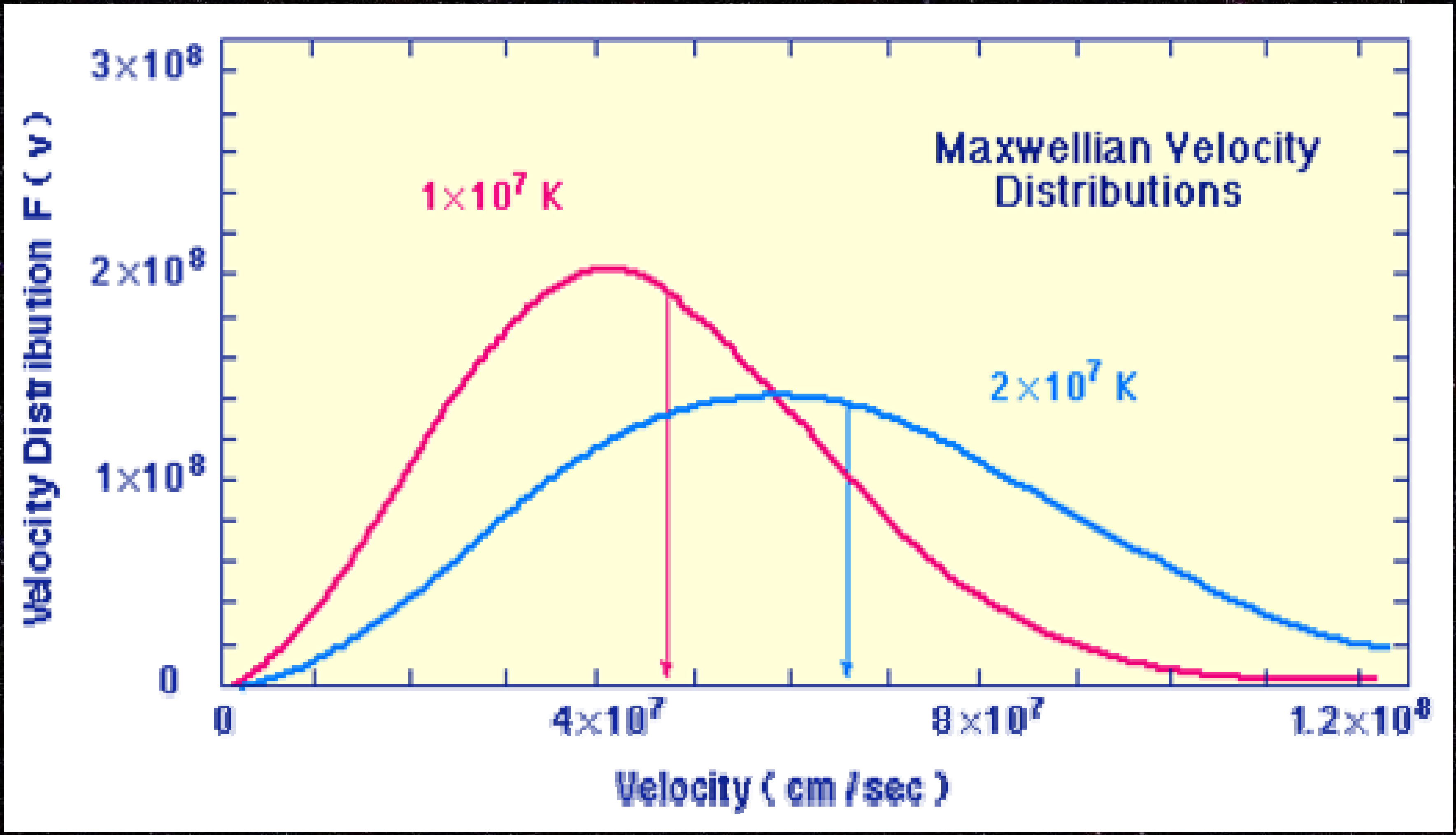
Maxwell Boltzmann Distribution

In stellar material of temperature T particles follow ideal gas law

$$\Phi(v) = 4\pi \left(\frac{m}{2\pi kT} \right)^{3/2} v^2 \, e^{-\frac{m v^2}{2kT}}$$ with $$\int \Phi(v) dv = 1$$



Temperature in Stars



Stellar reaction rates

$$r = \frac{1}{1+\delta_{pT}} Y_T Y_p \rho^2 \mathrm{N}_\mathrm{A}^2 < \sigma v >$$

reactions per s and cm^3

$$\lambda = \frac{1}{1+\delta_{pT}} Y_p \rho \mathrm{N}_\mathrm{A} < \sigma v >$$

reactions per s & Target nucleus

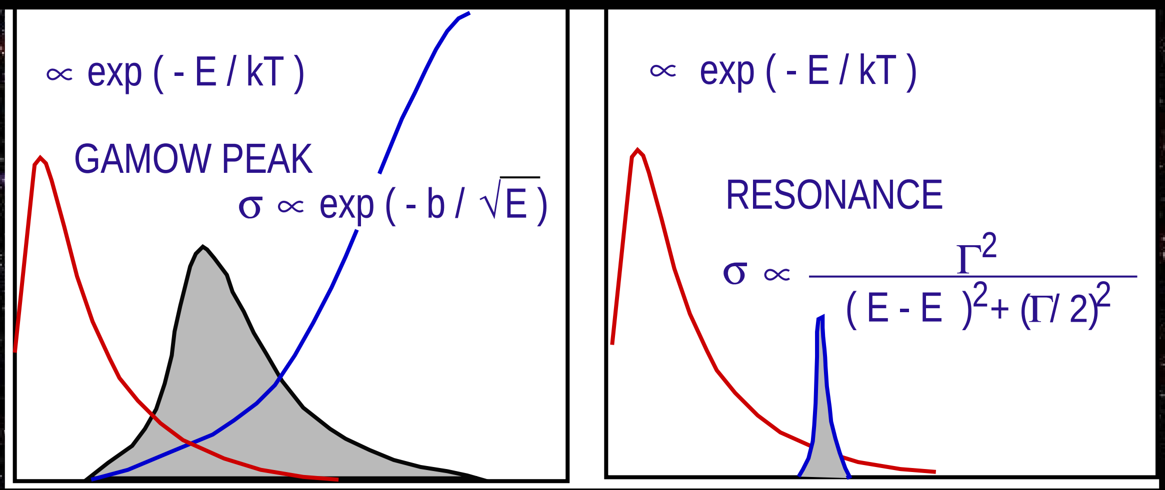
this is usually referred to as the **stellar reaction rate**

units of stellar reaction rate N_A<sv>: usually cm^3/s/mole

$$n_T = \rho \cdot N_A \cdot \frac{X_T}{A_T} = \rho \cdot N_A \cdot Y_T$$

X_T; mass fraction
Y_T: abundance

Gamow-Range & Reaction Rate



Nonresonant Reaction Contributions

$$N_A < \sigma v > \propto T^{-3/2} \int \sigma E \exp (- E / kT) \, dE$$

Resonant Reaction Rate

$$N_A < \sigma v > \propto T^{-3/2} \omega\gamma \exp (- E_R / kT)$$

σ: cross section

ωγ: res. strength

E_R: res. energy

The Gamow Range of Stellar Burning

The **Gamow window** or the range of relevant cross section for "non-resonant" processes is calculated:

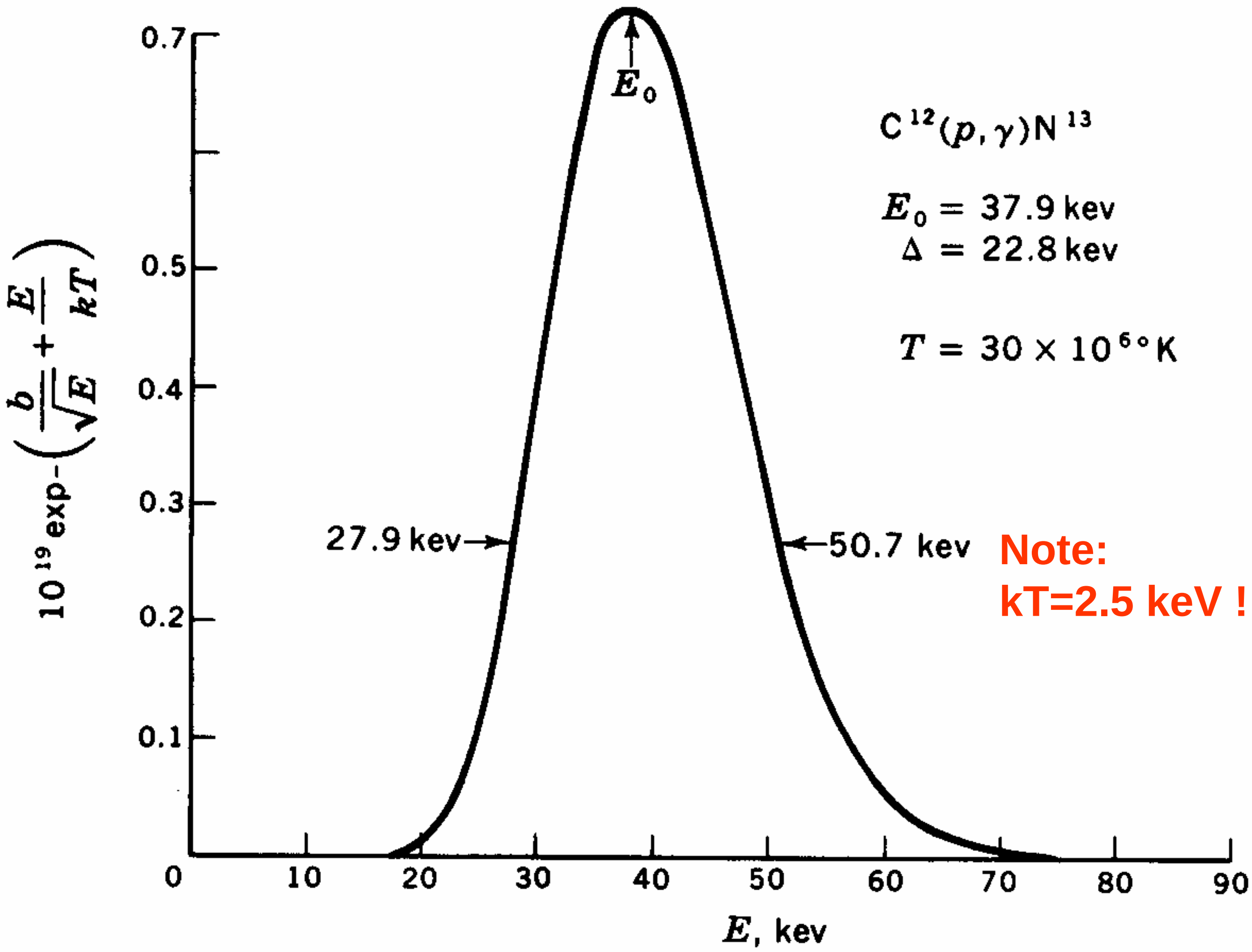
Check derivation in book

$$E_0 = \left(\frac{bkT}{2}\right)^{3/2} = 0.122 \cdot \left(Z_1^2 Z_2^2 A\right)^{1/3} T_9^{2/3} \text{ MeV}$$

$$\Delta E = \frac{4}{\sqrt{3}}\sqrt{E_0 kT} = 0.2368 \cdot \left(Z_1^2 Z_2^2 A\right)^{1/6} T_9^{5/6} \text{ MeV}$$

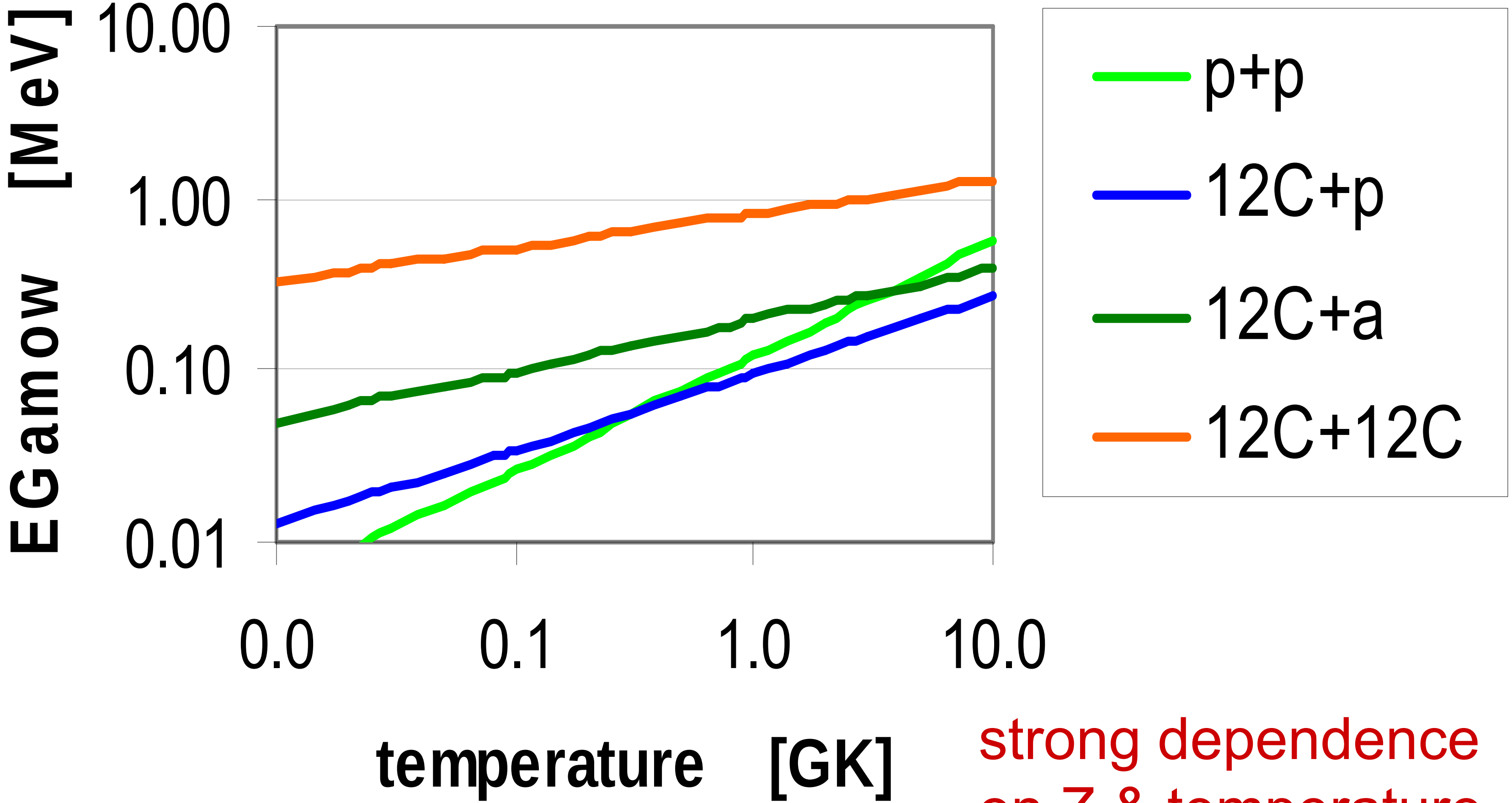
with A "reduced mass number" and T_9 the temperature in GK

The Gamow peak for $^{12}C(p,\gamma)^{13}N$



Note:
kT=2.5 keV !

Examples of Gamow window energies



strong dependence
on Z & temperature